# ELEC ENG 3BB3:

# Cellular Bioelectricity

## Notes for Lecture 5

Thursday, January 16, 2014

## *Parallel conductance model:*

- The two major results developed thus far are the *Nernst-Planck equation* and the *Nernst potential*.
- The latter utilizes the former to derive an expression for the equilibrium potential of each ion species.
- We would like to apply the Nernst-Planck equation to ion flow through a channel when it is *not* at the Nernst equilibrium. However, the exact concentration and electric potential gradients along the length of a channel are typically not known.

## *Parallel conductance model (cont.):*

- Simplifying assumptions can be made about the concentration and electric potential gradients, but unfortunately the resulting mathematical expressions do *not* accurately describe the behaviour of many ion channels.
- Consequently, a phenomenological description of current flow in ionic channels is typically used. This **parallel conductance model** does incorporate three earlier results:
  1. the *capacitance* of the plasma membrane,
  2. the *conductive* nature of ion flow, and
  3. the *equilibrium potential* for each ion.

# *Parallel conductance model (cont.):*

Assuming independent conductance channels for $K^+$, $Na^+$ and $Cl^-$, the electric circuit for a *membrane patch* is:

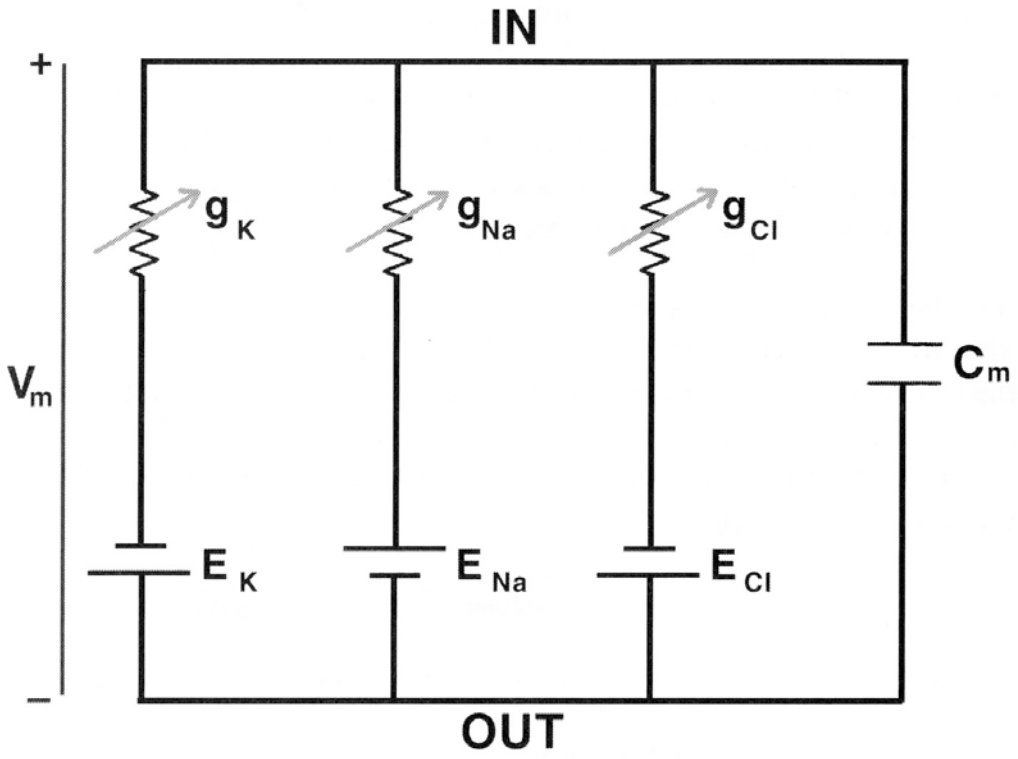


**Figure 3.3.** The Parallel-Conductance Model of an Excitable Membrane (IN = intracellular, OUT = extracellular). Independent conductance channels are present for $K^+$, $Na^+$, and $Cl^-$. Transmembrane potential $V_m$ is positive when the inside has higher potential than the outside. The battery polarity is chosen to show that usually the Nernst potentials of $E_K$ and $E_{Cl}$ are negative (inside more negative than outside) and that of $E_{Na}$ is positive (inside more positive than outside).

## *Ionic currents:*

The current for the $p^{th}$ ion is assumed to be proportional to how far the membrane potential $V_m$ deviates from the equilibrium potential $E_p$, with the constant of proportionality $g_p$ corresponding to the instantaneous conductance of the channel.

For the three ionic channels shown in Fig. 3.3, we have:

$$I_{\mathrm{K}} = g_{\mathrm{K}} (V_m - E_{\mathrm{K}}) \tag{3.26}$$

$$I_{\mathrm{Na}} = g_{\mathrm{Na}} (V_m - E_{\mathrm{Na}}) \tag{3.27}$$

$$I_{\mathrm{Cl}} = g_{\mathrm{Cl}} (V_m - E_{\mathrm{Cl}}) \tag{3.28}$$

*Capacitive current:*

The capacitive current is:

$$I_C = C\frac{\mathrm{d}V_m}{\mathrm{d}t}, \qquad (3.29)$$

where C is the capacitance for the patch of membrane.

Importantly, at rest (i.e., at steady state), $I_C = 0$ because $dV_m/dt = 0$.

## *Resting* $V_m$ *at steady-state:*

The total transmembrane current is:

$$I_m = I_C + I_{\mathsf{K}} + I_{\mathsf{Na}} + I_{\mathsf{Cl}}.$$

Assuming that no current is being injected into the intra- or extra-cellular space, the total transmembrane current must be zero, such that at steady state:

$$I_m = 0 = 0 + I_{\mathsf{K}} + I_{\mathsf{Na}} + I_{\mathsf{Cl}}$$

$$\Rightarrow g_{\mathsf{K}}\left(V_m - E_{\mathsf{K}}\right) + g_{\mathsf{Na}}\left(V_m - E_{\mathsf{Na}}\right) + g_{\mathsf{Cl}}\left(V_m - E_{\mathsf{Cl}}\right) = 0. \qquad (3.30)$$

*Resting* $V_m$ *at steady-state (cont.):*

Solving for $V_m$ to obtain the *resting transmembrane potential* $V_{rest}$ gives:

$$V_{\text{rest}} = \frac{g_{\text{K}} E_{\text{K}} + g_{\text{Na}} E_{\text{Na}} + g_{\text{Cl}} E_{\text{Cl}}}{g_{\text{K}} + g_{\text{Na}} + g_{\text{Cl}}}. \quad (3.31)$$

That is, the resting membrane potential is the weighted sum of the equilibrium potentials, where the weightings depend on the resting values of the ionic conductances.

*Example resting* $V_m$*:*

Assuming the following equilibrium potentials and resting ionic conductances for the squid axon:

$$
\begin{aligned}
E_{\mathrm{K}} &= -74.7\ \mathrm{mV}, & g_{\mathrm{K}} &= 0.367\ \mathrm{mS/cm^2}, \\
E_{\mathrm{Na}} &= \ \ 54.2\ \mathrm{mV}, & g_{\mathrm{Na}} &= 0.010\ \mathrm{mS/cm^2}, \\
E_{\mathrm{Cl}} &= -65.8\ \mathrm{mV}, & g_{\mathrm{Cl}} &= 0.582\ \mathrm{mS/cm^2},
\end{aligned}
$$

from Eqn. (3.31) we find that the resting membrane potential is $V_m = ¡ 68.0$ mV.

## *Contributions from chloride:*

Although the resting membrane potential in the previous example is quite close to the chloride equilibrium potential, it is primarily potassium that determines the resting potential (which would be ¼ ¡ 71.0 mV if the chloride channel were ignored).

This is because the intracellular chloride concentration is so small that it undergoes large percentage changes with just small amounts of chloride influx or efflux.

Consequently, the chloride equilibrium potential tends to track the resting potential determined by potassium.

# *Contributions from chloride (cont.):*

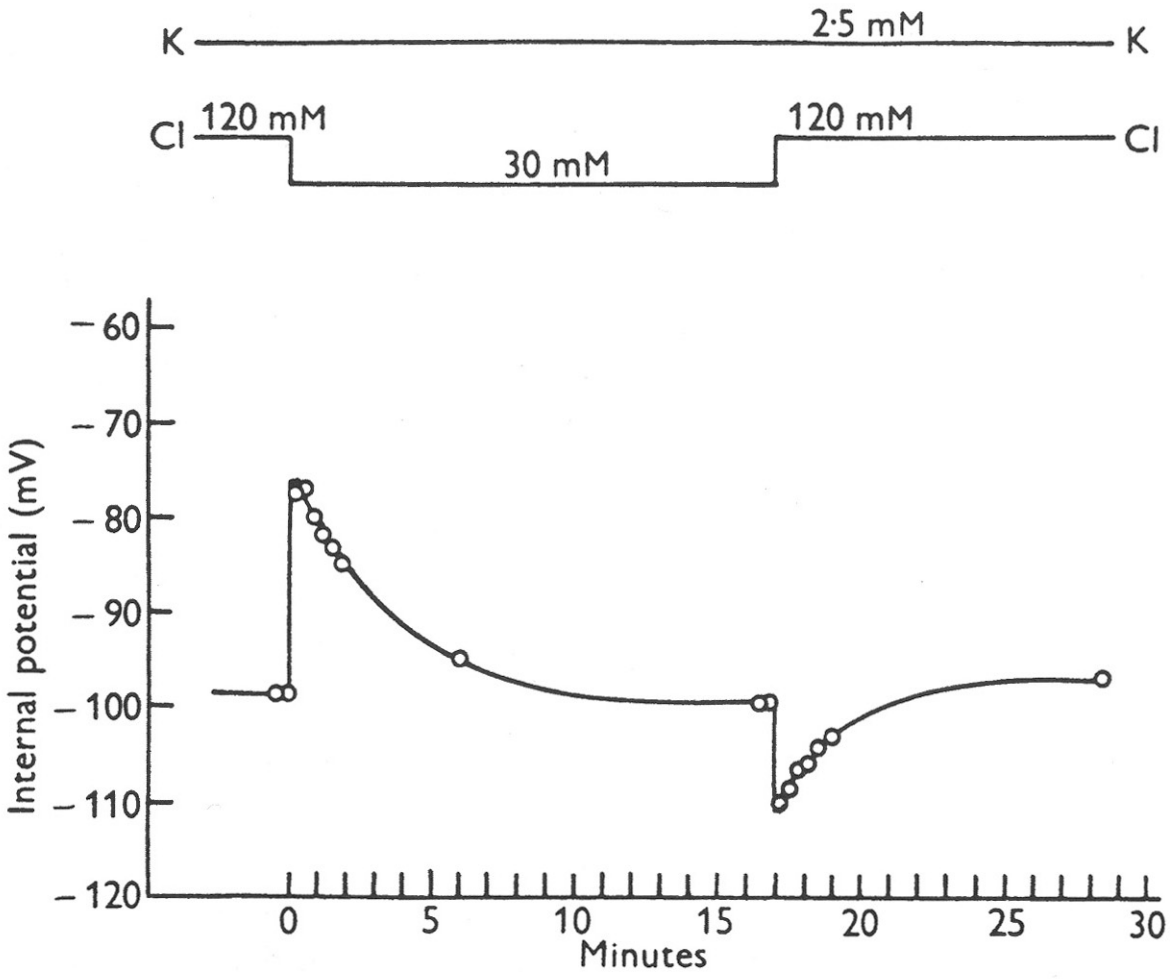


*Figure 3.4.* The effect of a sudden reduction in the external chloride concentration on the membrane potential of an isolated frog muscle fiber [Reprinted with permission from A. L. Hodgkin and P. Horowicz, The influence of potassium and chloride ions on the membrane potential of single muscle fibers, *J. Physiol.* **148**:127–160 (1959).]

*Membrane conductance/resistance at rest:*

If the membrane potential is at rest, then the total resting membrane conductance G (or its reciprocal, the total resting membrane resistance R) can be determined from the resting values of the ionic conductances according to:

$$G = \frac{1}{R} = g_{\mathrm{K}} + g_{\mathrm{Na}} + g_{\mathrm{Cl}}.$$

# *Equivalent circuit near rest:*

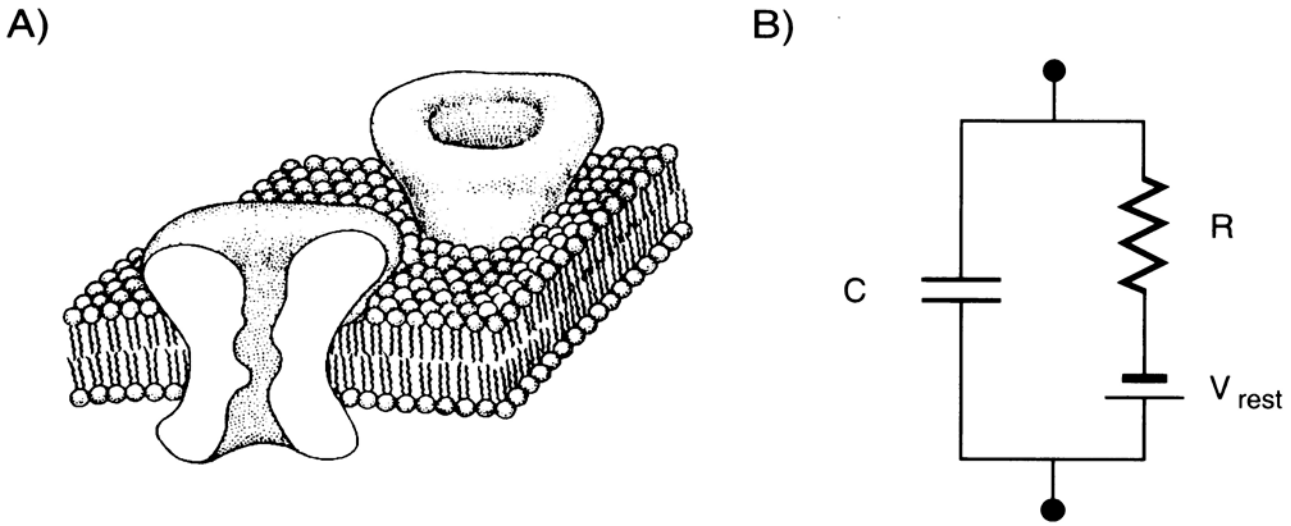


**Fig. 1.1 NATURE OF THE PASSIVE NEURONAL MEMBRANE** **(A)** Schematic representation of a small patch of membrane of the types enclosing all cells. The 30–50 Å thin bilayer of lipids isolates the extracellular side from the intracellular one. From an electrical point of view, the resultant separation of charge across the membrane acts akin to a capacitance. Proteins inserted into the membrane, here ionic channels, provide a conduit through the membrane. Reprinted by permission from Hille (1992). **(B)** Associated lumped electrical circuit for this patch, consisting of a capacitance and a resistance in series with a battery. The resistance mimics the behavior of voltage-independent ionic channels inserted throughout the membrane and the battery accounts for the cell's resting potential $V_{rest}$.

(from Koch)