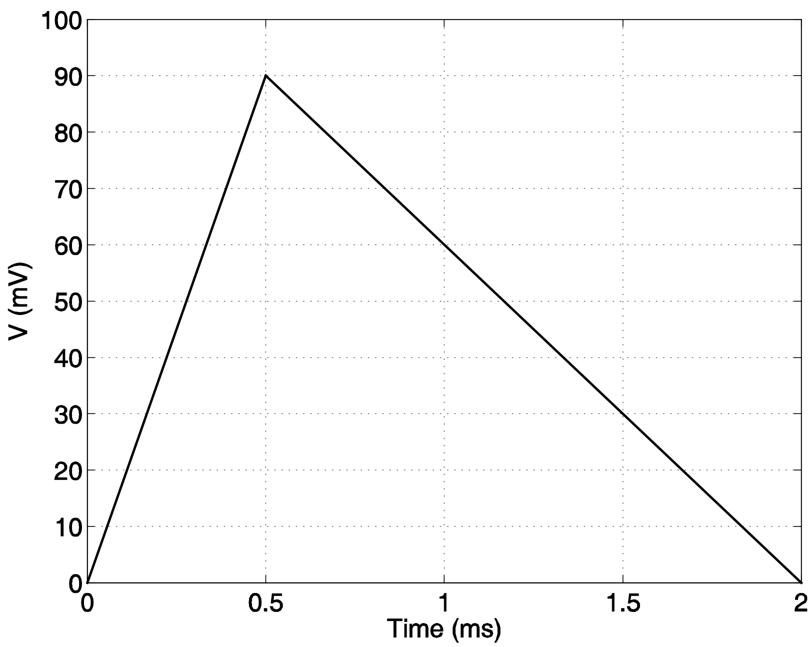
ELEC ENG 3BB3 – Cellular Bioelectricity (2014)

Tutorial 4

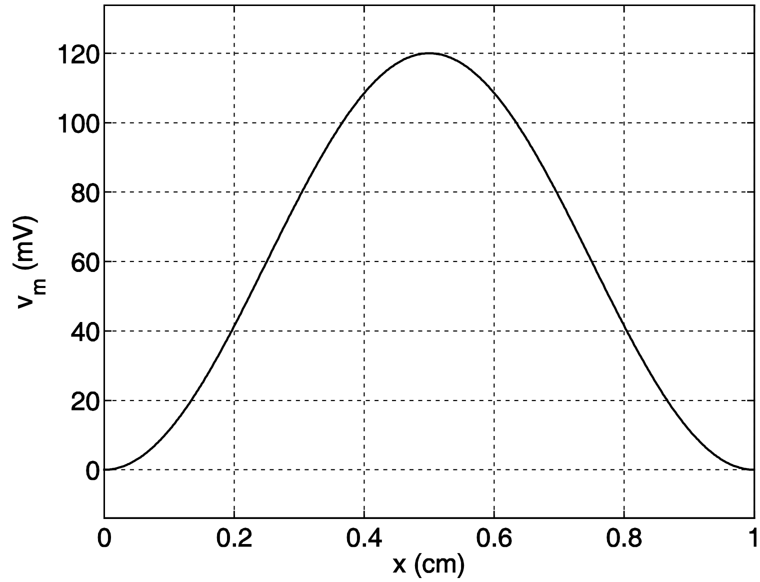
1. Consider a voltage-clamp experiment performed on the potassium channels found in frog nodes of Ranvier, which can be described by the *Frankenhaeuser–Huxley* potassium channel model—Eqns. (12.27)–(12.32) of Plonsey & Barr (3^{rd} Edition). However, instead of the experiment being conducted at the standard temperature of $T = 295.18^\circ\text{K}$, it is done at $T = 300^\circ\text{K}$. Assume that the kinetics of this channel have a temperature-scaling coefficient of $Q_{10} = 2.7$.

   If the membrane is held at a holding potential $V_h = -70\,\text{mV}$ for time $t < 0$ and is stepped to a clamp potential of $V_c = -40\,\text{mV}$ at time $t = 0$, derive an expression for the potassium permeability $P_\text{K}(t)$ for all times $t \geq 0$.

2. Consider an action potential with a waveform that can be approximated by a triangle, as given in the figure below. Assume that the *tail* of the action potential spatial waveform is at position $x = 0$, that no currents are being injected into the intra- or extra-cellular space from external sources, and the intra- and extra-cellular resistances per unit length are $r_i = 1.25\ \text{M}\Omega/\text{cm}$ and $r_e = 12.5\ \text{k}\Omega/\text{cm}$, respectively. If this action potential is propagating (without dissipation) along an unmyelinated axon in the $+x$ direction with velocity 6 m/s, calculate the resulting local circuit currents, i.e., the transmembrane current per unit length i_m and the axial intra- and extra-cellular currents I_i and I_e, respectively, *as a function of position* x.



3. Consider an action potential with the *spatial* waveform illustrated in the figure below.



The relative transmembrane potential at time $t = 0$ can be described by:

$$v_m(x) = \begin{cases} 0, & \text{for } x < 0 \ \text{cm}, \\ 120\sin^2\left(\dfrac{2\pi}{d}x\right), & \text{for } 0 \le x \le 1 \ \text{cm}, \\ 0, & \text{for } x > 1 \ \text{cm}, \end{cases}$$

where x is in units of cm, v_m is in units of mV, and $d = 2$ cm.

Assume that no currents are being injected into the intra- or extra-cellular space from external sources and the intra- and extra-cellular resistances per unit length are $r_i = 1\ \text{M}\Omega/\text{cm}$ and $r_e = 10\ \text{k}\Omega/\text{cm}$, respectively.

a. Calculate the local circuit currents, i.e., the transmembrane current per unit length i_m and the axial intra- and extra-cellular currents I_i and I_e, respectively, *as a function of position* x.

b. If the action potential is propagating stably with a velocity of $-12\ \text{m}\cdot\text{s}^{-1}$, find an expression for the *temporal* action potential waveform $v_m(t)$ at the position $x = 0.2$ cm, and sketch this waveform.