EE 791 EEG-6
Measures of Dynamic Properties (cont'd)

Coherence and Phase Synchronization

Spatial analysis of EEG means the analysis of a joint observation of time series {$V_{mk}(t)$ consisting of k =1,K observations in m = 1, M data channels. Coherence between pairs of electrodes provides an entry point to examine the spatial properties of the stochastic source activity. Coherence is defined as *a linear correlation coefficient* that primarily estimates the amount of phase synchronization between any two data channels. Figure 9-9 shows 3 different examples of phase synchrony or otherwise between two sine waves with the same frequency. In (a) the sine waves have the same amplitude, phase and phase difference for each trial (a deterministic signal, coherence 1); (b) signals have variable phase between trials but always a 45 degree phase difference (coherent stochastic process, coherence 1); (c) variable phase and variable phase difference (incoherent stochastic process).

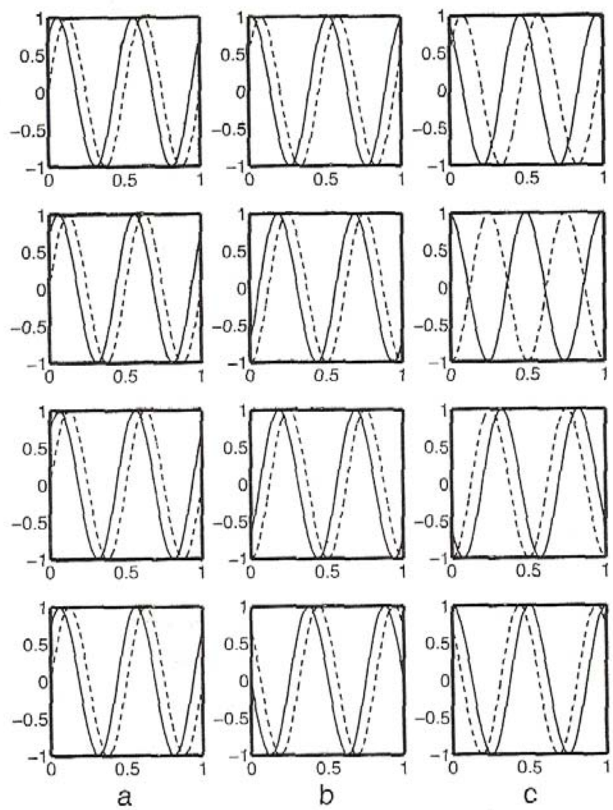

Figure 9-9 Examples of two simulated sine waves with different kinds of phase relationships, shown for four epochs. (a) The two sine waves have the same phase on every trial. (b) The two sine waves have random phase across the trials, but the phase difference is fixed at 45°. (c) The two sine waves have random phase and random phase difference across the trials.

All figures are from Nunez and Srinivasan, 2006, "Electric Fields of the Brain", 2nd Edition, Oxford University Press

Coherence is a measure very similar to squared cross correlation coefficient which describes the amount of variance in one channel that can be explained by a linear transformation of the data in another channel.
We first define the cross spectrum which is a measure of the joint spectral properties of two data channels. It is a measure of the covariance between two signals at one frequency. The cross spectrum of channels u and v at frequency f_n can be estimated from pairs of Fourier coefficients as an average over K epochs.

$$C_{uv}(f_n) = A_{uv}e^{j\phi_{uv}} = \frac{2}{K}\sum_{k=1}^{K} F_{uk}(f_n)F_{vk}^{*}(f_n) \quad n = 1,2,\ldots\ldots N/2-1$$

When $u = v$ this reduces to the power spectrum and the factor 2 reflects the fact we only consider positive frequencies. The cross spectrum is complex with an amplitude A_{uv} and phase Φ_{uv}. The phase of the cross spectrum is average relative phase. If we normalize the squared magnitude of the cross spectrum by the power spectrum of each channel we obtain the coherence

$$\Upsilon_{uv}^{2}(f_n) = \frac{|C_{uv}(f_n)|^2}{P_u(f_n)P_v(f_n)} \qquad n = 0,1,2,\ldots\ldots(N-1)/2$$

Like the r^2 statistic, coherence measures the fraction of variance of one channel at a particular frequency that can be explained by a constant linear transformation of the Fourier coefficient at that frequency of another channel. This means both constant relative amplitude and constant relative phase. Figure 9-10 shows the power and coherence spectra obtained from a pair of channels with 6 Hz sinusoid added to independent Gaussian noise with 100 times the variance. For each epoch the phase in one channel is random, and the relative phase of the other channel changes by a random amount < ± 45 degrees. As seen, the coherence estimates become more representative as the number of epochs increases, but only if the signal remains stationary over the increased number of epochs.
Coherence depends on both relative amplitude and relative phase at each frequency between the two channels. If we want to only measure phase synchronization, independent of amplitude fluctuations, we can estimate coherence by normalizing each Fourier coefficient by its amplitude. Generally speaking, given that the recorded signal also contains noise, larger amplitude components are more reliable than smaller, and therefore the estimates for coherence will be more reliable if both amplitude and phase are considered. If one uses the same signal model as before with the previous relative phase fluctuations between channels but with amplitude varying between epochs, we achieve the results shown in Figure 9-11. In this figure different amounts of Gaussian noise are added to each channel, showing that the phase only coherence estimates are less reliable in larger amounts of noise. The maximum noise added is only 4 times the average power of the sinusoid reinforcing the fact that **phase only coherence should be used with considerable caution.**

Effects of Spatial Filtering by Volume Conduction on Coherence Estimates
Two other factors in addition to the underlying signal synchronization, influence the amount of coherence estimated between channel pairs, spatial filtering by volume

conduction and choice of reference electrode location. Nunez and Srinivasan have

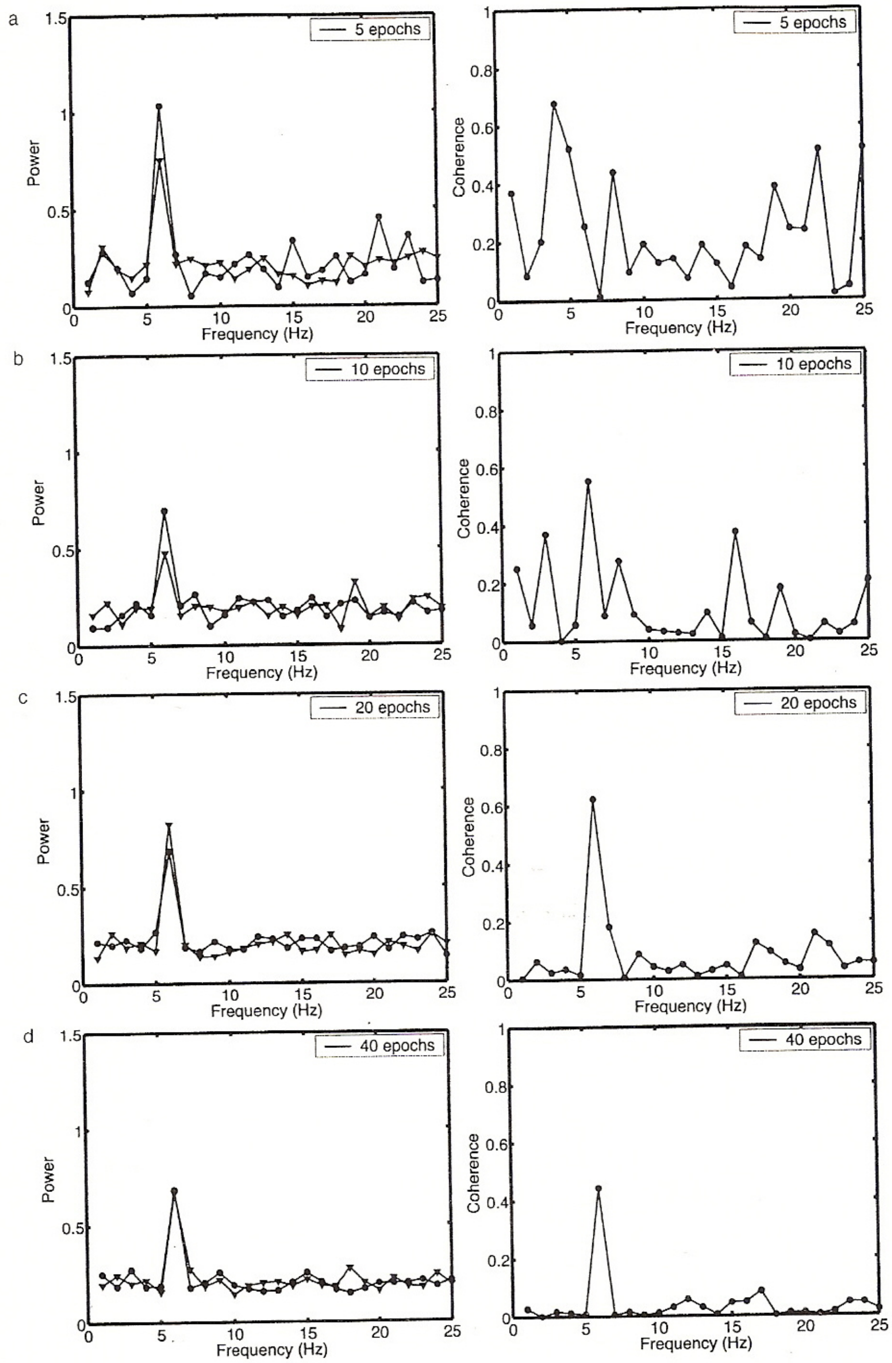

Figure 9-10 Simulated power and coherence spectra for a simulation with two channels with a 6 Hz sine wave added to Gaussian random noise with 100 times the variance of the sinusoid. In each epoch, the phase of the oscillators is random but the phase difference is less than 45°. (a) Power spectra of the channels indicated by circles and triangles, using only a 5 epoch average is shown on the left. Coherence spectrum estimate for the two channels based on the same 5 epoch average shown on the right. Same as (a) using (b) 10 epochs, (c) 20 epochs, (d) 40 epochs.

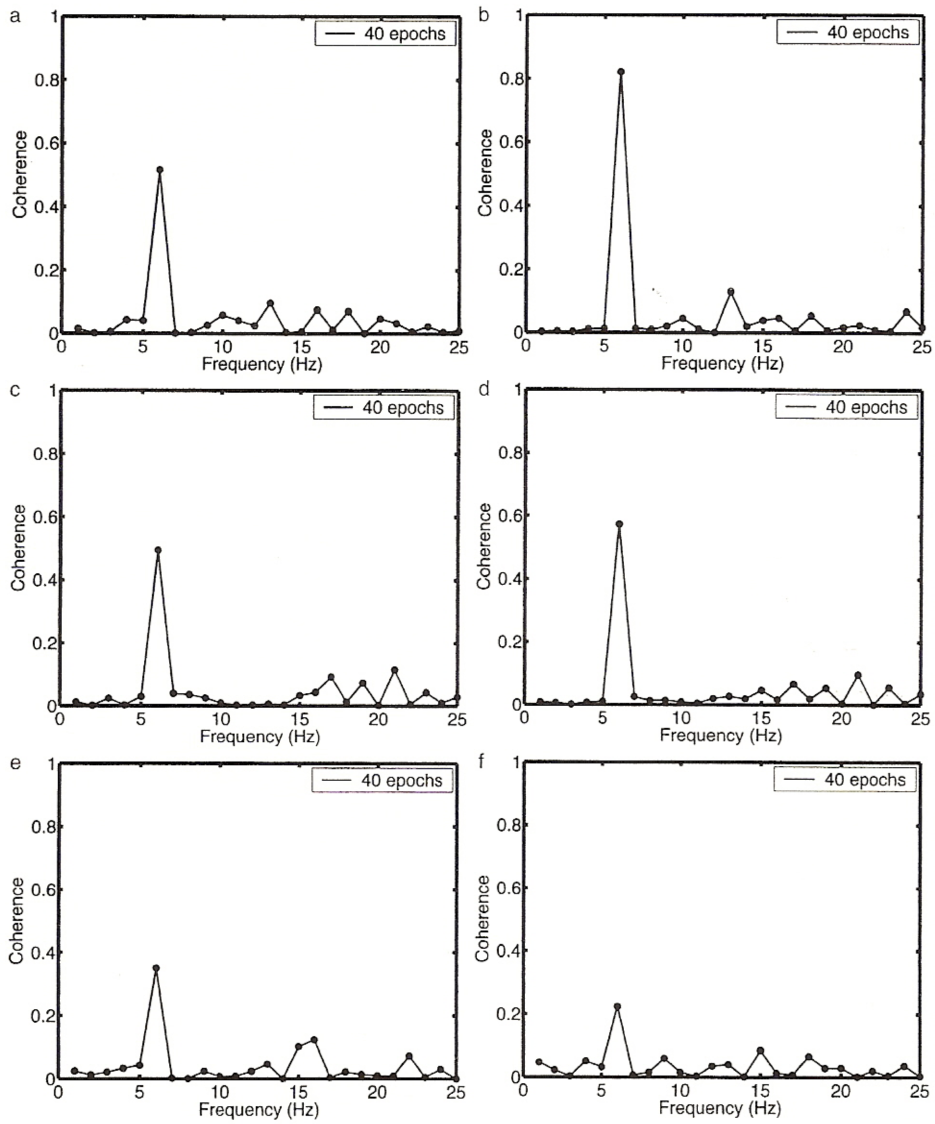

Figure 9-11 Simulated comparison of conventional coherence estimates (*left column*) calculated with (9.11) to coherence estimates obtained by first normalizing each Fourier coefficient from each epoch by its magnitude (*right column*). (a) Coherence between two channels with small variability in the phase difference between channels (less than ±45°) and random amplitudes between 0 and 100 μV for each channel and each epoch. Minimal noise is added (1% of the sinusoid power). (b) Coherence of the same channel pair in (a) calculated by first normalizing each Fourier coefficient from each epoch by its magnitude. This phase-only coherence is higher reflecting only the phase variability. (c) Same as in (a) with noise power increased to 2 times the average power of the sinusoids. (d) Same as (b) with noise power increased to 2 times the average power of the sinusoid. (e) Same as (a) with noise power increased to 4 times the average power of the sinusoid. (f) Same as (b) with noise power increased to 4 times the average power of the sinusoid. Removing amplitude information makes coherence estimates more sensitive to noise. Note that in fig. 9-10 conventional coherence estimates are robust even when noise power is 100 times the sinusoid power.

explored these factors by creating simulated sources at the cortex and using a 4-sphere head model to derive the signals recorded on the scalp at various electrode locations.

Thus if coherence in the brain sphere between all possible source locations is zero, they can predict that scalp potential coherence is due only to volume conduction. I will not go into the math of their estimation process since it is too complex for this course and concentrate only on results. Since the head remains a resistive network at these low frequencies we do not expect the effects of volume conduction to be frequency dependent. Figure 9-12 shows the coherence estimated when the sources are random white noise sources at the cortex. Part (b) shows numerical simulation with 3600 dipole sources (Gaussian random processes) and a 111 electrode array each 2.7 cm apart.

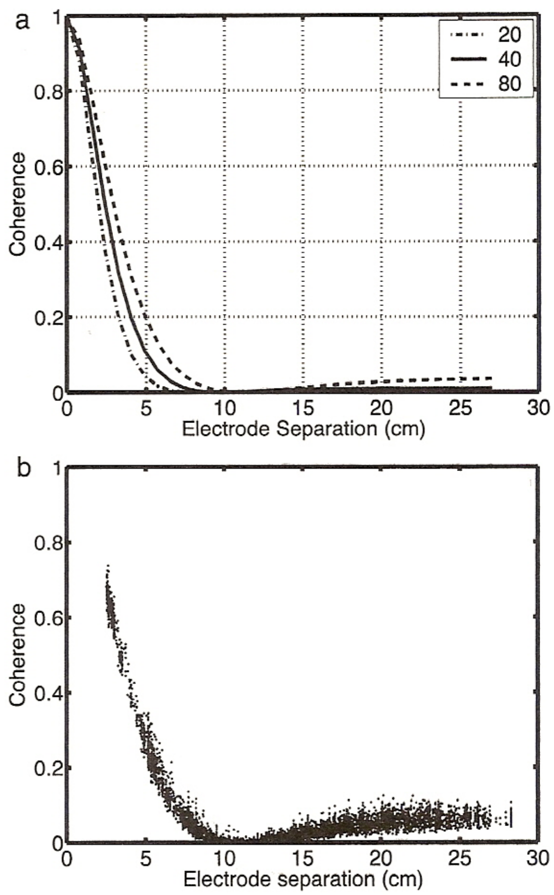

Figure 9-12 (a) Theoretical predictions of coherence between scalp potentials due to volume conduction alone, with an assumed spatial white-noise source distribution in the 4-sphere model of volume conduction in the head given by (9.15). The source is a spherical dipole layer at a radial location $r_z = 7.8$ cm. The model parameters are radii $(r_1, r_2, r_3, r_4) = (8, 8.1, 8.6, 9.2)$ and conductivity ratios $(\sigma_1/\sigma_2, \sigma_1/\sigma_3, \sigma_1/\sigma_4) = (0.2, 40, 1)$. Source coherence is zero and scalp potential coherence depends only on distance. (b) A numerical simulation in the 4-sphere model using ~3600 dipole sources distributed under an electrode array of 111 electrodes, with average electrode separation of 2.7 cm (matching the geodesic net shown in fig. 7-1). Dipole sources are independent Gaussian random processes. Again scalp potential falls off with electrode separation similar to the theoretical fall-off shown in (a).

The three curves in (a) are for different brain-to-skull conductivity. We can conclude that over inter-electrode distances < 8 – 10 cm we can expect significant contributions to coherence from volume conduction. That is when considering electrode pairs close together, coherence is inflated due to volume conduction. This is true also for real data measures. Figure 9-13 shows coherence estimates for real EEG between electrode X and the other electrodes, spaced 2.7 cm apart for eyes open case (less alpha which can be broadly coherent).

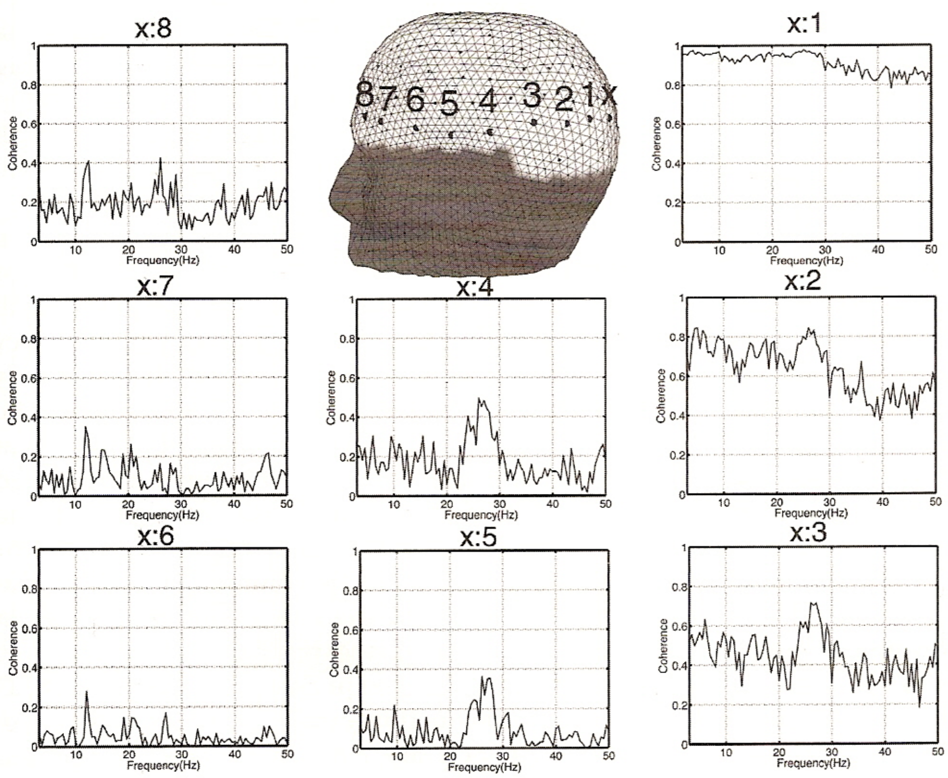

Figure 9-13 EEG coherence spectra estimates based on 100 s averages. The subject is a 36-year-old male (with *eyes open* to minimize alpha band coherence). Coherence was estimated with $T = 2$ s epoch lengths ($\Delta f = 0.5$ Hz). The head plot shows the location of nine electrodes, labeled X and 1 through 8. Coherence spectra estimates are shown between electrode X and each of the electrodes 1–8, at increasing distances along the scalp. Very close electrodes have high coherence independent of frequency as expected from the theoretical model.

This figure shows that X:1 shows general non-frequency specific coherence indicate of volume conduction, X:4 and X:5 show low general frequency coherence but true source coherence for frequencies between 22 and 32 Hz. By X:8 general frequency coherence has increased slightly because of the curvature of the head.
The choice of reference system does affect the values of coherence and in simulation studies Ninez and Srinivasan of shown that simulated sources at the cortex (9-12b) show increased coherence estimates when the reference is the vertex, left mastoid, or linked mastoids (digitally averaged). Figure 9-14 shows these simulations. The minimum coherence occurred for a simulation with the vertex as reference and then finding the average reference (9-14d). For those results the coherence estimates resembled the results of reference free estimations shown in Fig 9-12b.

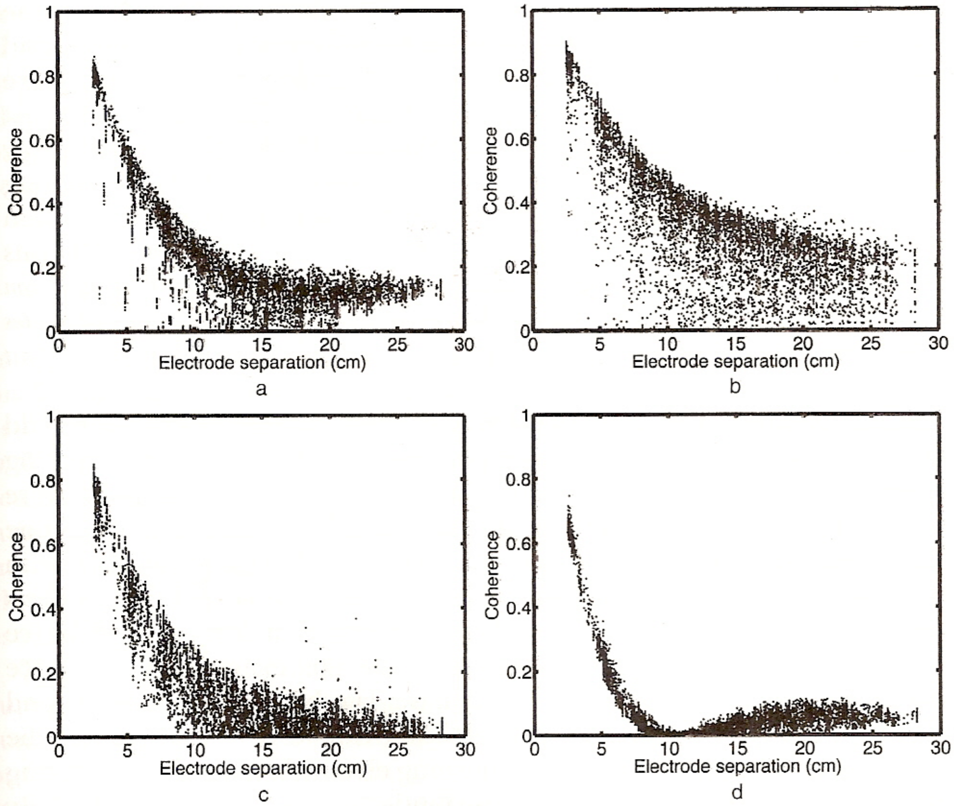

Figure 9-14 Simulations following the procedure used in fig. 9-12b with different reference electrodes. Simulated coherence between 111 electrodes using 3600 dipoles sources distributed under an array of 111 electrodes in the 4-sphere model. Dipole sources were independent Gaussian random processes. Coherence plotted as function of electrode separation excluding reference electrode. (a) Vertex reference using 110 electrodes. (b) Left mastoid reference. (c) Digitally averaged mastoids reference. (d) Coherence calculated by first referencing the potentials to the vertex and then calculating the average reference potential at 111 electrode sites. Note the average reference coherence curve closely follows reference-independent potentials.

There is a method of reference-free calculations called surface Laplacian that seems to solve the volume conduction problem. I am not sure since that technique is universally better, since the technique is valid when cortical sources have small surface areas but diminishes the scalp potentials for large cortical surfaces. As well, for the technique to be accurate, a large number of electrodes (64) are needed with relatively close spacing. However, in earlier versions based on the 10-20 system, the Laplacian could be estimated as the average potential between an electrode and its four surrounding electrodes. This got rid of volume conduction effects, and reference electrode effects. However, if the cortical source is large and contributes to the majority of the electrodes, this potential will be much smaller than the true scalp potential. Figure 9-17 and 9-18 show coherence estimates obtained from real eyes closed data (more alpha expected) using conventional reference and recording with 111 electrodes and using spline Laplacians to process the signal at each electrode respectively

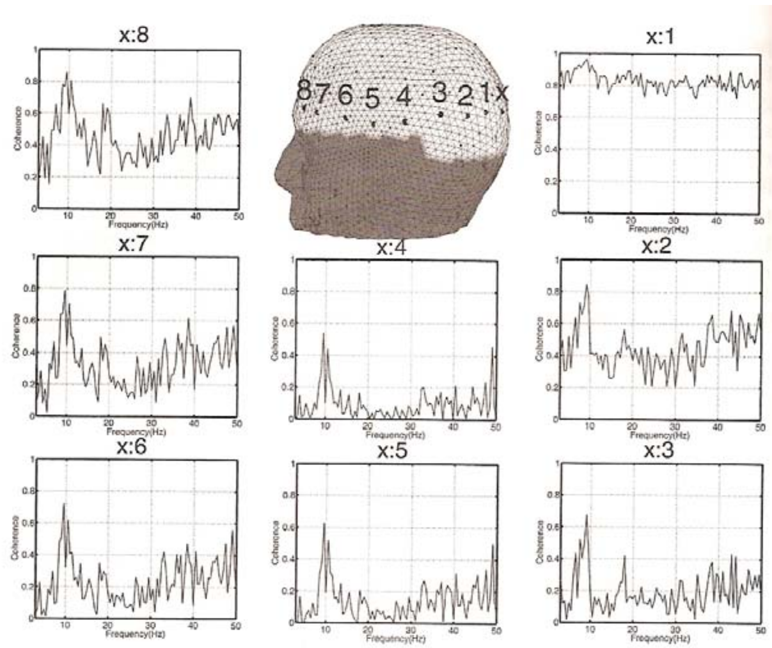

Figure 9-17 EEG (scalp potential) ordinary coherence spectra from a 22-year-old female subject at rest with eyes closed (to maximize alpha coherence). Coherence was estimated with $T = 2$ s ($\Delta f = 0.5$ Hz) in a 60 s record. The head plot shows the location of 9 electrodes, labeled x and 1 through 8. Coherence spectra between electrode X and each of the other electrodes 1–8 are shown, with increasing separations along the scalp. Note that very close electrodes have higher coherence independent of frequency as predicted by the theoretical model. Alpha band coherence is high for large electrode separations, apparently reflecting the large cortical source coherence. Power spectra for this subject are shown in fig. 9-2.

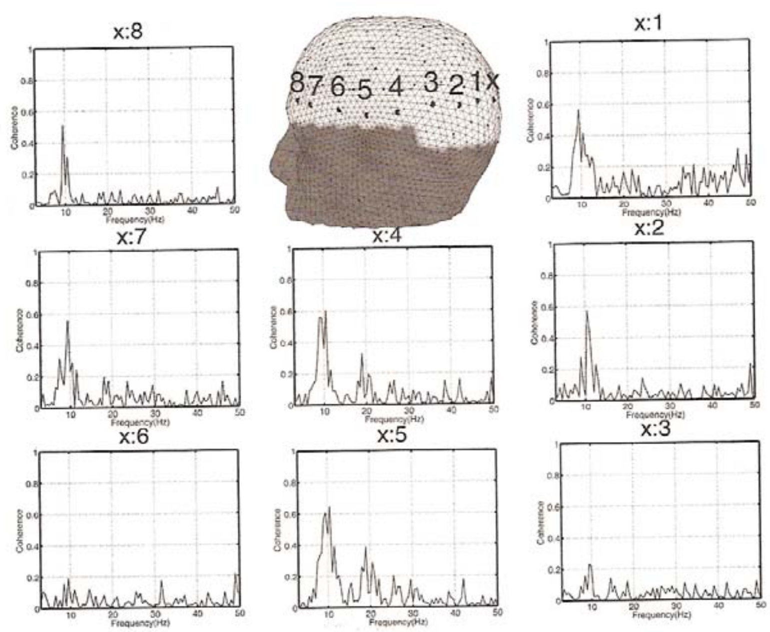

Figure 9-18 EEG (spline Laplacian) coherence spectra using the same data in fig. 9-17: the female subject at rest with eyes closed for 60 s. All 111 channels of data were submitted to the New Orleans spline Laplacian to estimate coherence spectra for these few channel pairs. The spline Laplacian coherence spectra show distinct coherence spectra between electrode X and even very closely spaced electrodes as the result of filtering out volume conduction effects.