EE 791 EEG-5
Measures of EEG Dynamic Properties

Computer analysis of EEG

EEG scientists must be especially wary of mathematics in search of applications – after all the number of ways to transform data is infinite. In evaluating new methods, the central question is *not what EEG can do for mathematics, but rather what mathematics can do for EEG.* Avoid the approach of some signal processors – *have Matlab toolbox, will travel to any problem.*
We may wish to simply reduce the data to a more manageable form. Looking at EEG in its raw form is tedious and inter-patient comparisons impossible. The spatial and temporal properties of EEG can vary widely, and consequently no single method is universally applicable. However, since frequency features have been used over the years to identify EEG under different brain states, spectral analysis and spectral coefficients are a logical start to compressing the data and identifying sensitive features. One error in the past has been to think of each frequency or band originating from a particular source. Rather, the brain is a dynamic complex process with regions constantly changing in both function and size. This suggests that analyses of spatial properties of the EEG spectrum are required – measures of phase synchronization between different brain regions appear especially promising. If you think of the concept of brain cells with synchronous (in phase dipoles) activity growing and diminishing as the brain changes state, this approach would be consistent with some modern ways of viewing brain activity. Coherence, a method of determining phase synchronization using Fourier analysis, is one such method but others such as principal components analysis, and propagation velocities determination across the scalp are also applicable here. The methods chosen should be based on the theoretical model of EEG and brain behaviour (inexact as that may be) since we do not want to fall into the trap that our results are determined by the assumption inherent to and limitations of the analysis method chosen. In other words we should have some idea why a particular approach may give useful results rather than embark on fishing expeditions. Here a theoretical approach may guide us in determining the limitations of different methods and guide us in our instrumentation and experimental design.

Synaptic Action Generates EEG

Our theoretical model could be the cell assemblies discussed in EEG-1, which can range from small local networks to sizes up to the entire brain, all connected in a dynamic communication network. These can develop, disconnect and reform on roughly 10 to 100 ms timescales. EEG is a very large-scale measure of brain source activity, apparently recording some mixture of cell assembly and global activity organized over macroscopic (cm) and even whole brain spatial scales. This approach can even account for very localized sources such focal epilepsy or short latency evoked potentials which could be the result of isolated cell assemblies. One can think of large-scale cortical and scalp potentials being generated by millisecond scale modulations of synaptic current sources at the surfaces of neocortical neurons about longer time-scale background neuromodulator activity all caused by neurotransmitter release and absorption.

Although cortical and scalp potentials are generated by microsources $s(r_n, t)$ associated with synaptic action, it is more convenient to think of the intermediate mesosource function **P**(r,t) or dipole moment per unit volume. The potential anywhere in the brain can then be thought of the weighted integral of all active mesosources.

$$\Phi_S(t) \;=\; g_1 p_1(t) \;+\; g_2 p_2(t) + \;.....+ g_N p_N(t)$$

Where there are N volumes (voxels) in the brain and each g_n is determined by the distance from the n^{th} voxel to the measurement site and the intervening volume conductor. To be measurable at the scalp there must be sufficient mesosources (several cm) synchronously active (dipoles lined up) and these must be within cm of the recording electrode.

Fourier Analysis

The earliest form of spectral analysis was zero-crossing detection but modern FFT software has made this redundant. In Fig 9.1 the raw EEG is shown as example 2 sec windows while the frequency power spectra are shown as the averages of 31 windows. Averaging the results of 1 min of EEG is more representative of the ongoing EEG. The power spectra (amplitude squared) is a measure of the power at each frequency. Fourier analysis is concerned with expressing an arbitrary time series such as EEG as a sum of sine waves with different frequencies.

$$\sum_n A_n \sin(2\pi f_n t + \theta_n)$$

To avoid aliasing (higher frequencies being misrepresented as lower frequencies, the time sampling rate must be at least twice the highest frequency component of the EEG signal. To be safer we should use he engineer's sampling rate where

$$f_{samp} = \frac{5}{2} f_{\max} \;\; f_{samp} = \frac{5}{2} f_{\max}$$

Figure 9.2 shows the effects of undersampling.

If there are N data samples in your time series and the sampling interval is Δt, then the total window length in time is T. The FFT algorithm also returns N frequency coefficients with a frequency separation of

$$\Delta f = \frac{1}{T} = \frac{1}{N\Delta t}$$

and the total frequency covered is $F = N\Delta f = f_{samp}$ $F = N\Delta f = f_{samp}$

Only the first N/2 coefficients can be considered and their values doubled since the other N/2 are the mirror image of them (they really represent the negative frequency coefficients in the Fourier series). That is you obtain frequency coefficients over the range $f_{samp}/2$. The first coefficient represents the average or DC value and should be ignored since the EEG signal is AC coupled. In commercial machines the sampling rate is set to 200 Hz for spontaneous EEG and no more than 500 Hz for longer latency evoked potentials. The signal is lowpass filtered at 70 Hz to avoid aliasing.

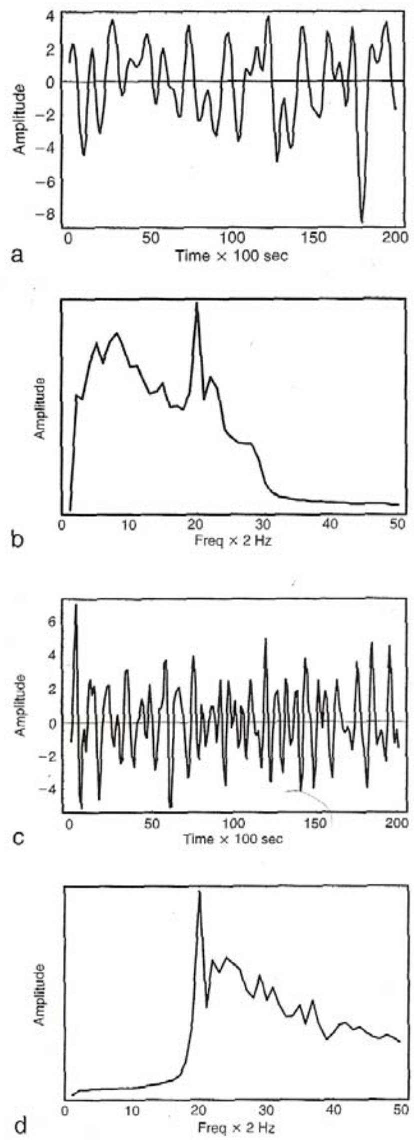

Figure 9-1 (a) A two-second simulated waveform composed of many frequency components producing an average of 14.2 zero crossings per second (7.1 Hz). (b) The amplitude spectrum of waveform (a) obtained by averaging the spectra of 31 two-second epochs. (c) A two-second simulated waveform producing an average of 30.2 zero crossings per second (15.1 Hz). (d) The amplitude spectrum of waveform (c) obtained by averaging the spectra of 31 two-second epochs.

To get accurate frequency coefficients it is assumed that each frequency in the signal has an integer number of periods. This is not the case in practice especially when the time data window is short compared to the periods of the frequency components. The result is that some of the power in each frequency component “bleeds” into the neighbouring coefficients, an effect known as “fence picketing”. Figure 9.3 shows this effect. For the mathematically inclined selecting a data window (collecting a fixed time duration) is equivalent to multiplying the infinite EEG signal by a rectangular data window function. This multiplication in the time domain is equivalent to convolution in the frequency domain resulting in a sinc function being convolved with the spectrum of the signal. One

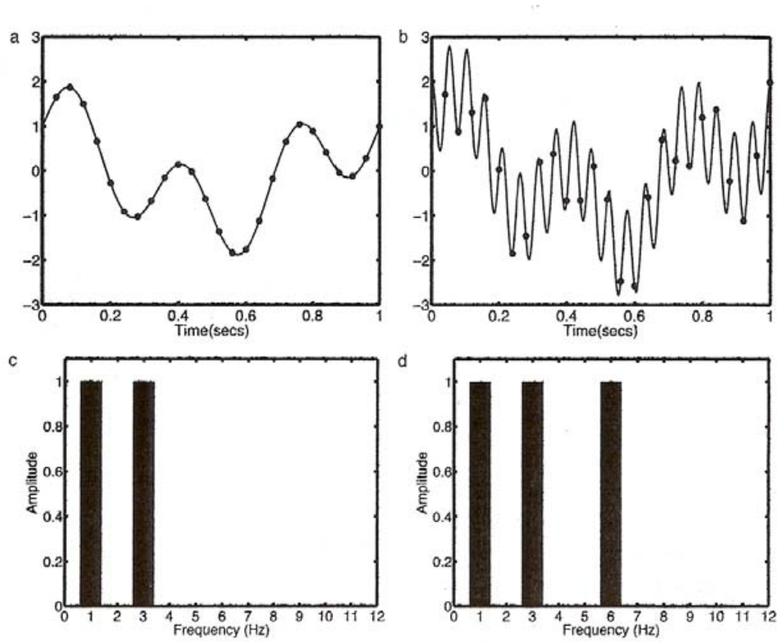

Figure 9-2 (a) A 1 s simulated waveform composed of 1 and 3 Hz sine waves of equal amplitude. The composite waveform is sampled every 40 ms as indicated by the gray dots. (b) A 1 s simulated waveform composed of a 1 Hz, 3 Hz, and 19 Hz sine waves of equal amplitude. Sampling every 40 ms (gray dots) aliases the signal because peaks and troughs of the 19 Hz oscillation are missed. (c) Amplitude spectrum obtained by applying the FFT to the signal (a) sampled every 40 ms. The 1 Hz and 3 Hz components have amplitude 1. (d) Amplitude spectrum obtained by applying the FFT to the signal of example (b) sampled every 40 ms. This aliased signal has an additional component at 6 Hz due to aliasing of the 19 Hz component.

way of overcoming this is to multiply the data window by a function that begins and ends in zero, typically some function including a raised cosine such as the Hamming or Hanning window.

Time Domain Spectral Analysis

Spectral analysis allows us to assess statistical properties of the oscillations in different frequency bands. Each EEG signal recorded is one realization of a stochastic process. The amplitude spectrum of one epoch of EEG is an exact representation of the frequency content of that particular signal epoch, but only provides one observation about the stochastic process. An ensemble of *K* observations $V_k(t)$ must be used to make a statistical estimate of the power spectrum (auto spectral density function) of an EEG signal. The power spectrum provides a decomposition of the variance of the signal as a function of frequency. The ensemble mean of the observations

$$\mu(t) = \frac{1}{K}\sum_{k=1}^{K} V_k(t)$$

and the power spectrum provide all that is required to estimate the signal probability density function. It is assumed in spectral analysis that the signal is stationary (it is weakly stationary if the mean and power spectrum are invariant with shifts in time. This can generally be assumed true for spontaneous EEG over a period of 10 to 20 sec or even longer if the subject doesn't change the brain's state but is definitely not true for evoked potential data. One can still perform Fourier analysis but the results will have to be interpreted differently.

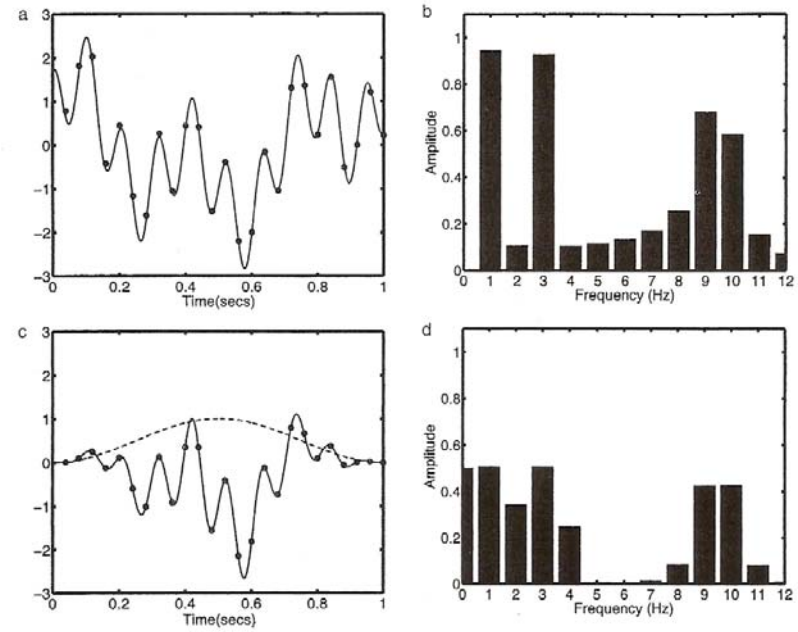

Figure 9-3 (a) A 1 s simulated waveform composed of 1 Hz, 3 Hz, and 9.5 Hz sine waves of equal amplitude sampled every 40 ms as indicated by the gray dots. (b) Amplitude spectrum of the signal shown in (a). The 1 Hz and 3 Hz sine waves are clearly identified, but the 9.5 Hz signal appears mainly at 9 and 10 Hz and is smeared throughout the spectrum. (c) A Hanning window function, shown by the dashed lines, is applied to the data of (a). The windowed data are shown by the solid lines. (d) The amplitude spectrum after windowing shows that the 9.5 Hz oscillation appears mainly in the 8–11 Hz bins. However, the 1 Hz and 3 Hz oscillations are now smeared due to loss of frequency resolution.

As is the case of any other statistical measure, we can never know the actual power spectrum of a stochastic process. Rather we can find an estimate of the power spectrum if we obtain the spectra for *K* epochs. That is

$$P(f_n) = \frac{2}{K}\sum_{k=1}^{K} F_k(f_n)F_k(f_n)^* = \frac{2}{K}\sum_{k=1}^{K} |F_k(f_n)|^2 \qquad n = 1,2,\ldots\ldots N/2-1$$

For each sample $V_k(t_n)$ Fourier coefficients $F_k(f_n)$ can be obtained using the FFT procedure. The factor 2 is included because we use only half the derived coefficients and the DC term f_0 is not included but should be in the above equation. The power spectrum summed over all positive and negative frequencies (including f_0 and $f_{N/2}$ the folding frequency) is equal to the variance of the signal (Parseval's theorem). The power spectrum is given as μV^2 and the power is $\mu V^2/Hz$ if normalized with respect to Δf. Of course using this approach includes certain tradeoffs. If one calculates the power spectrum for a 60 sec epoch of EEG the frequency resolution would be Δf = 0.017 Hz with no ensemble averaging as shown in Fig 9-4. The exact spectrum is obtained but not estimates of the statistical properties of the underlying stochastic process. A choice must be made of how long an epoch should be analysed. This depends on the frequency resolution one would like to have. The shorter the epoch length, the more epochs to average but the coarser the resolution. In Figure 9-4, the sampling rate looks like 50 Hz, giving us 25 coefficients if the epoch length is 1 sec (Δf = 1 Hz), where if 2 sec windows are used Δf = 0.5 Hz.

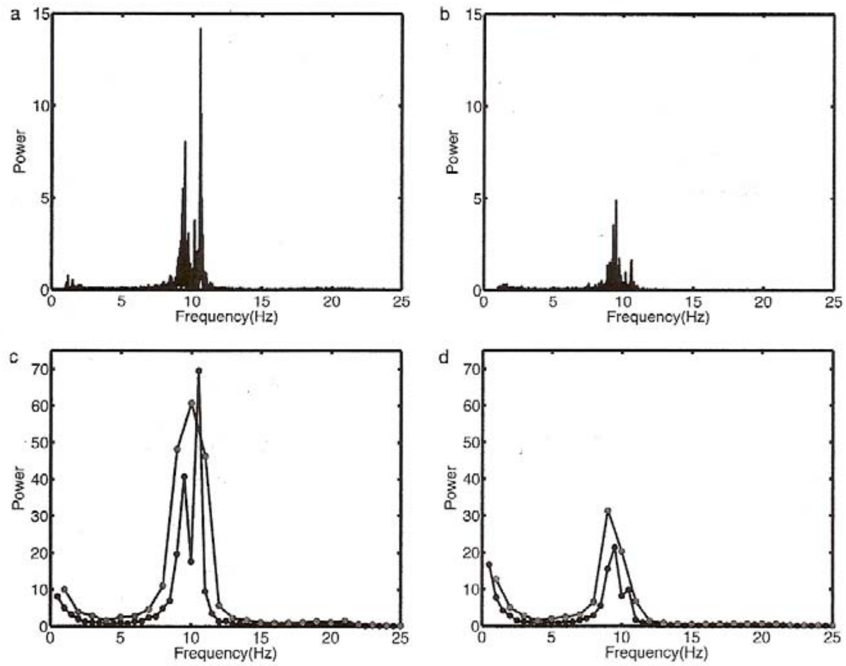

Figure 9-4 Example EEG power spectra from a single subject (female, 22 years). The subject is at rest with *eyes closed*. (a) Power spectrum of a midline occipital channel with epoch length $T=60$ s and $K=1$ epochs. The power spectrum appears to have two distinct peaks, one below 10 Hz and one above 10 Hz. (b) Power spectrum at a midline frontal channel with epoch length $T=60$ s and $K=1$ epochs. Here only the peak below 10 Hz is prominent. (c) Power spectra of a midline occipital channel calculated with two different choices of epoch length T and number of epochs K. The gray circles indicate the power spectrum with $T=1$ s and $K=60$ epochs. The black circles indicate the power spectrum with $T=2$ s and $K=30$ epochs. (d) Power spectra of a midline frontal channel calculated as in (c).

The Impact of Source Synchrony and Spatial Filtering on EEG Power Spectra

Scalp potential at any frequency can change for several reasons related to "synchrony". EEG scientists and clinicians have adopted the label desynchronization to indicate large amplitude reductions, particularly in the alpha band. We can divide the brain into several thousand voxels (mesosource elements) each having a dipole moment **P**(r,t). Microscale changes in synaptic source synchrony across cortical columns can change mesosource strength **P**(r,t). As mesosource strength increases we expect scalp potential to increase proportionally if there are no other changes. As the dipole layer changes in diameter from 1 to 3 cm large increases in scalp amplitudes are expected based on the exponential curves of scalp potential for dipole diameter. Finally even larger increases in dipole layer diameter from 3 to 10 cm result in modest increases in scalp potential (curve flattens out) Potential magnitudes can even decrease as the diameter gets even larger because of the cancelling effects of the curved scalp.

Figure 9-6 shows simulated scalp potentials due to a single dipole and dipole layers of diameter ranging from 3 to 5 cm, composed only of radial dipoles source. Each dipole source has a time series composed of a 6 Hz sinusoid of fixed amplitude A = 15 added to a Gaussian random process with mean $\mu = 0$ and standard deviation $\sigma = 150$ µV. The 6 Hz components are phase locked across the dipole layers, where all other components are random phase.

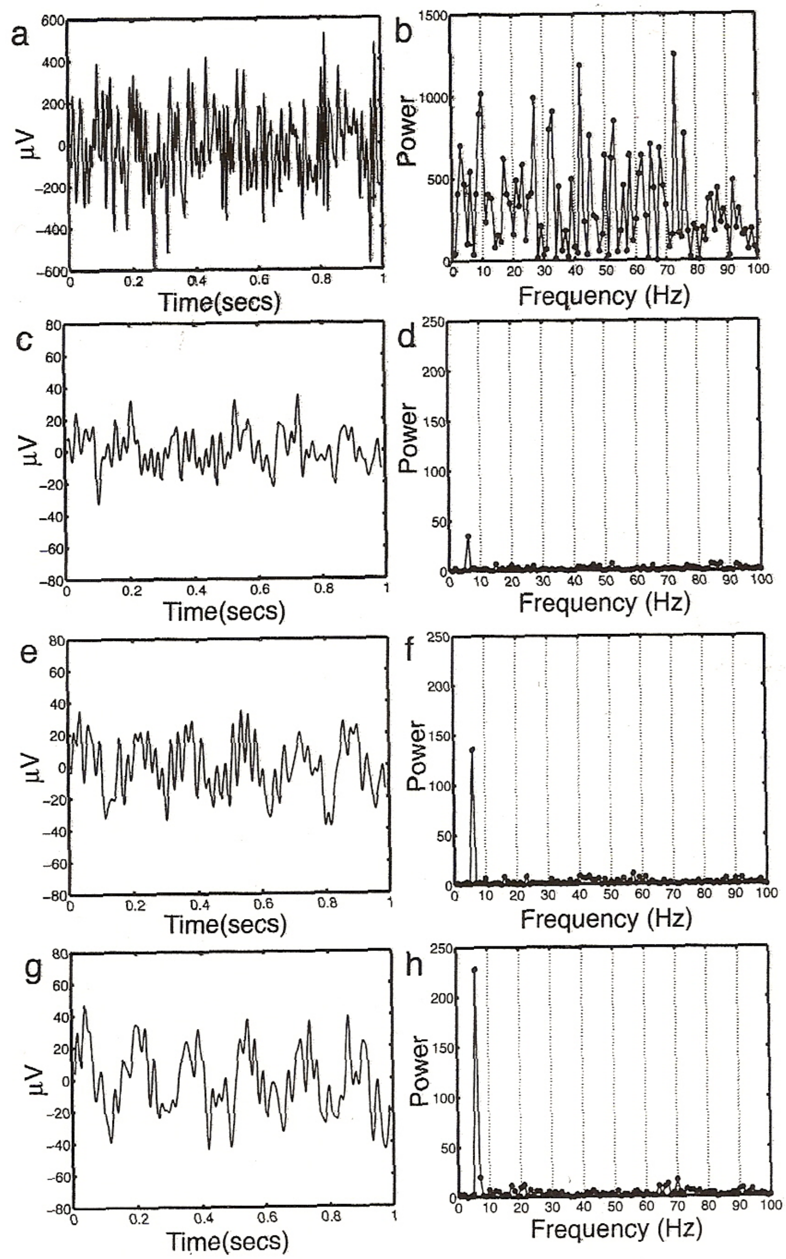

Figure 9-6 Simulated data. (a) time series of a dipole mesosource $\mathbf{P}(\mathbf{r}, t)$ composed of a 15 μV sine wave added to Gaussian random noise with standard deviation 150 μV. The Gaussian random noise was low-pass filtered at 100 Hz. The sine wave has variance (power) equal 1% of the noise. (b) Power spectrum of the time series shown in (a). The power spectrum has substantial power at frequencies other than 6 Hz. (c) Time series recorded by an electrode on the outer sphere (scalp) of a 4-sphere model above the center of a dipole layer of diameter 3 cm. The dipole layer is composed of 32 dipole sources $\mathbf{P}(\mathbf{r}, t)$ with time series constructed similar to (a) with independent Gaussian noise (uncorrelated) at each dipole source. Scalp potential was calculated for a dipole layer at a radius $r_z = 7.8$ cm in a 4-sphere model. (d) is power spectrum of surface potential in (c). (e) time series similar to (c) but due to a dipole layer of 4 cm with 68 dipole sources and power spectrum in (f). (g) similar to (c) but with layer 5 cm in diameter and 112 dipoles with power spectrum in

(h). Figure 9-7 summarizes the idea of dipole layer sources of different strength and size. The 6 Hz source could be any frequency. Each set of symbols represents a different mesosource strength of the 6 Hz oscillation as a percentage of the total generated mesosource strength (variance). The power has been normalized with respect to the power generated by a dipole layer of 1.5 cm diameter with a very weak (0.5%) relative mesosource strength. In this figure you can see that one strong dipole layer source (20%0 with 1.5 cm diameter under the electrode produces the same scalp power at 6 Hz as a weak source (1%) with 4.2 cm diameter.

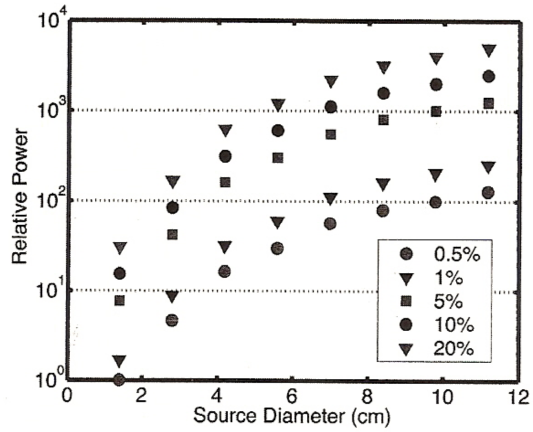

Figure 9-7 A simulated summary of the dependence of scalp power on the size and strength of a synchronous dipole layer, based on the simulation of fig. 9-6. Both source strength and source size independently contribute to the power at a scalp electrode. Source strength is expressed as the ratio of the power of sinusoid to the power of noise in the source activity and labeled with different symbols as indicated by the legend. Source diameter varies from 1.5 to 11.4 cm. Power is expressed relative to the power of the smallest dipole layer with weakest strength.