LECTURE 4: Fundamental Antenna Parameters

(*Radiation pattern. Pattern beamwidths. Radiation intensity. Directivity. Gain. Antenna efficiency and radiation efficiency. Frequency bandwidth. Input impedance and radiation resistance. Antenna effective area. Relationship between directivity and antenna effective area. Other antenna equivalent areas.*)

The antenna parameters describe the antenna performance with respect to space distribution of the radiated energy, power efficiency, matching to the feed circuitry, etc. Many of these parameters are interrelated. There are several parameters not described here, in particular, *antenna temperature and noise characteristics*. They are discussed later in conjunction with radio-wave propagation and system performance.

1. Radiation Pattern

Definitions:

The *radiation pattern* (RP) (or *antenna pattern*) is the representation of a radiation property of the antenna as a function of the angular coordinates.

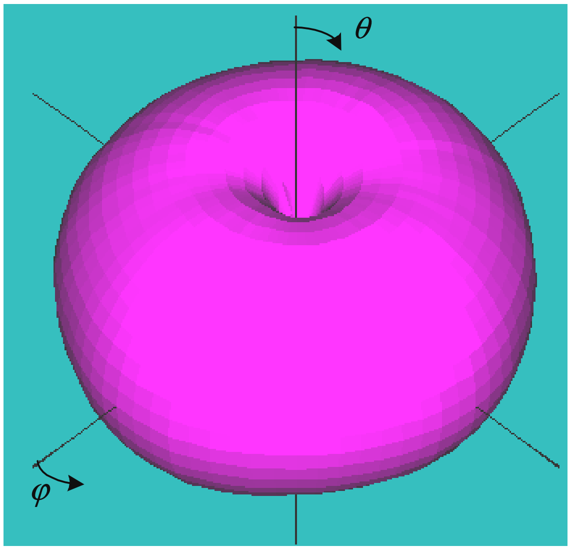
The trace of the angular variation of the received/radiated power at a constant radius from the antenna is called the *power pattern*.

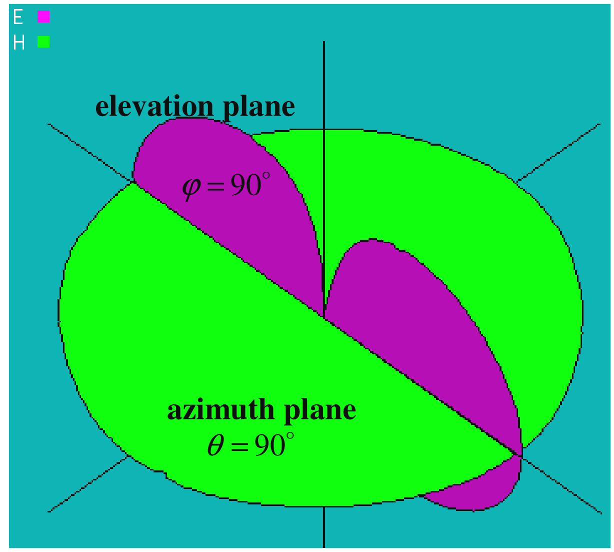
The trace of the angular variation of the magnitude of the electric (or magnetic) field at a constant radius from the antenna is called the *amplitude field pattern*.

RPs are measured in the far-field region, where the angular distribution of the radiated power does not depend on the distance. We measure and plot either the field intensity, $\sim|\mathbf{E}(\theta,\varphi)|$, or the power $\sim|\mathbf{E}(\theta,\varphi)|^2/\eta = \eta|\mathbf{H}(\theta,\varphi)|^2$. Usually, the pattern describes the *normalized* field (or power) values with respect to the maximum value.

Note: The power pattern and the amplitude field pattern are the same when computed and plotted in dB.

The pattern can be a 3-D plot (both θ and φ vary), or a 2-D plot. A 2-D plot is obtained as an intersection of the 3-D RP with a given plane, usually a $\theta = const.$ plane or a $\varphi = const.$ plane that must contain the pattern's maximum.





3-D pattern of a dipole *2-D elevation & azimuth patterns of a dipole*

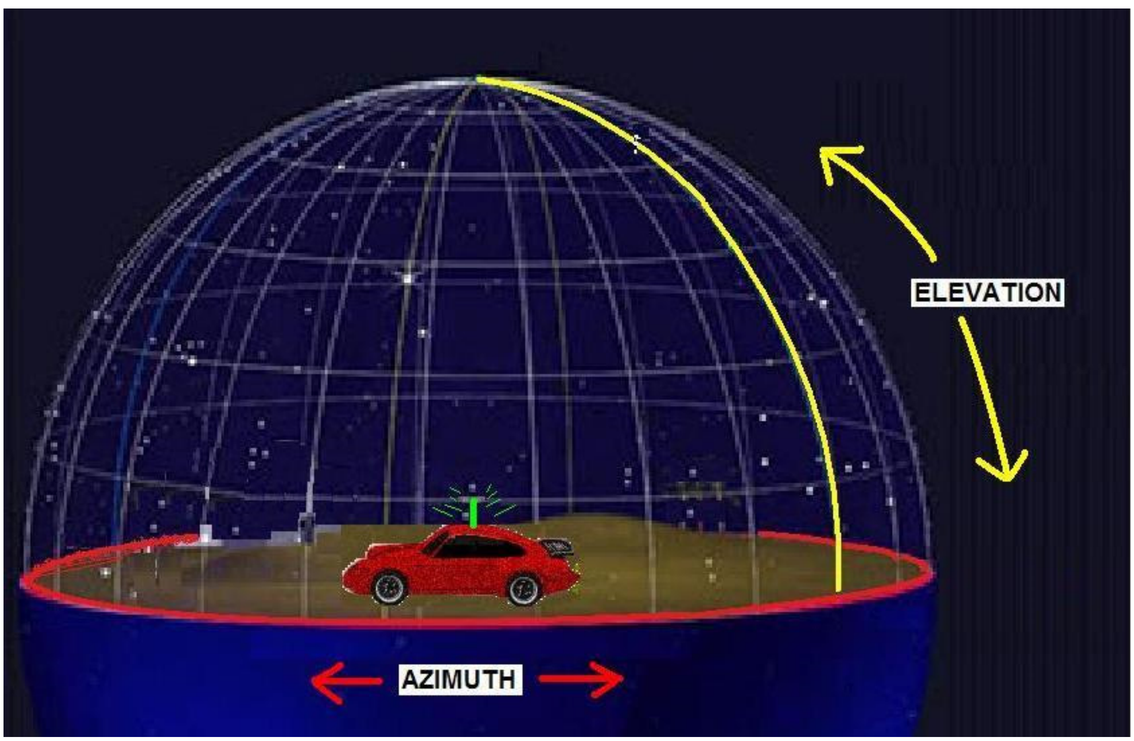
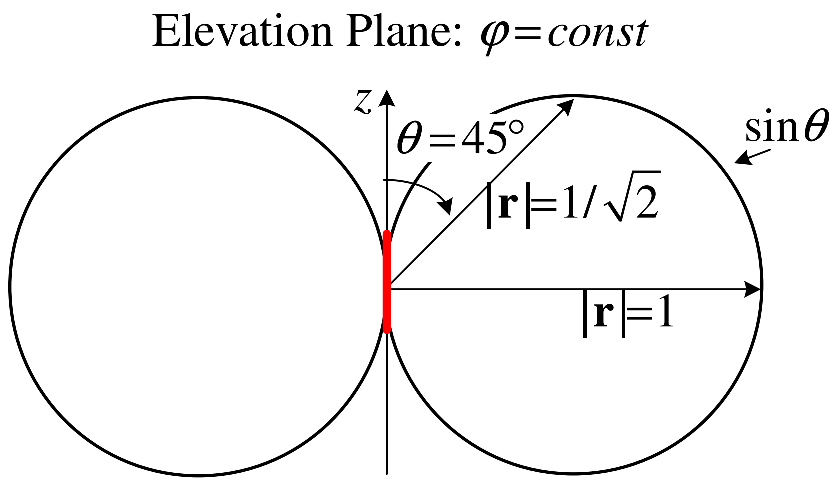


Illustration of azimuth and elevation

Plotting the pattern: the trace of the pattern is obtained by setting the distance from the origin in the direction (θ,φ) to be proportional to the strength of the field $|\mathbf{E}(\theta,\varphi)|$ (in the case of an amplitude field pattern) or proportional to the power density $|\mathbf{E}(\theta,\varphi)|^2$ (in the case of a power pattern).

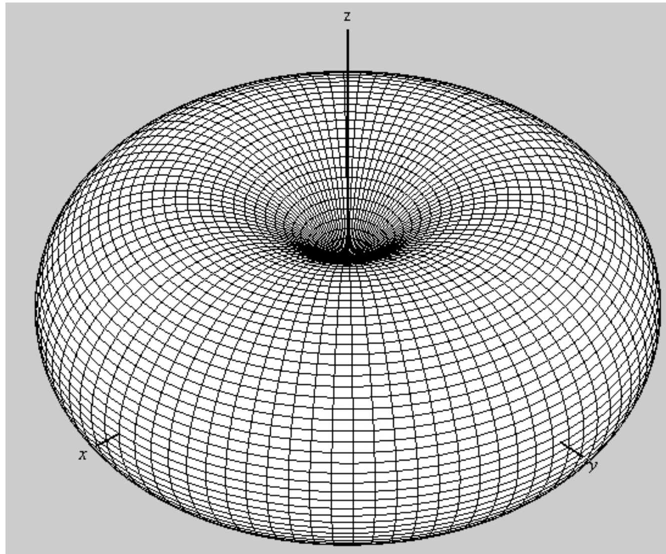


Some concepts related to the pattern terminology

a) ***Isotropic pattern*** is the pattern of an antenna having equal radiation in all directions. This is an ideal concept, which, strictly speaking, is achievable only approximately in a narrow frequency band. However, it is used to define other antenna parameters. It is represented simply by a sphere whose center coincides with the location of the isotropic radiator.

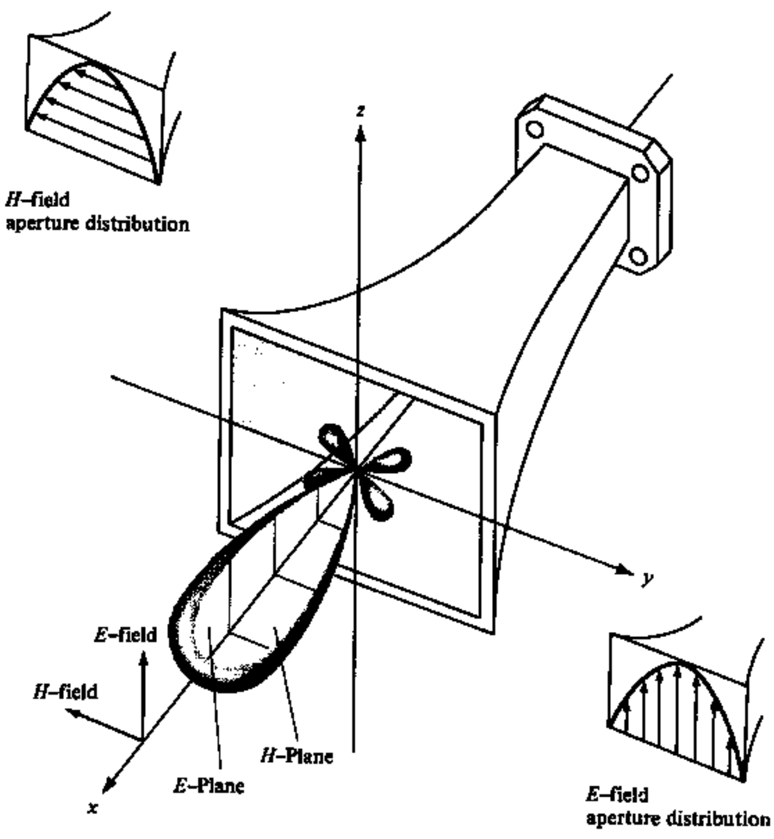
b) ***Directional antenna*** is an antenna, which radiates (receives) much more efficiently in some directions than in others. Usually, this term is applied to antennas whose directivity is much higher than that of a half-wavelength dipole.

c) ***Omnidirectional antenna*** is an antenna, which has a non-directional pattern in a given plane, and a directional pattern in any orthogonal plane (e.g. single-wire antenna). The pattern in the figure below is that of a dipole – it is omnidirectional.



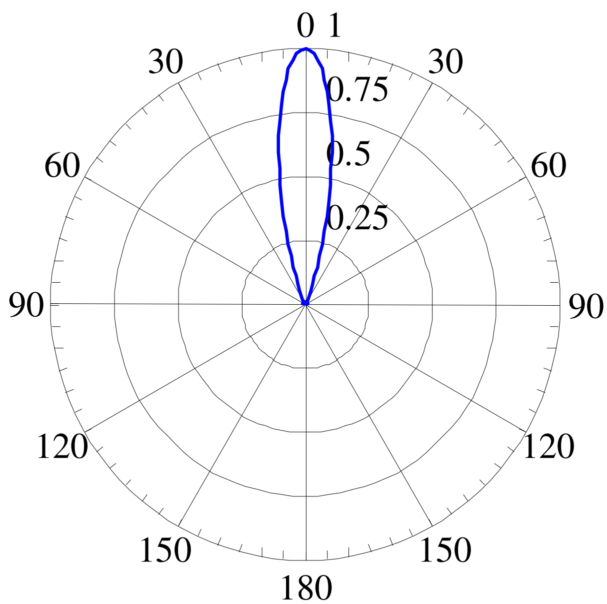
Omnidirectional 3-D pattern

d) ***Principal patterns*** are the 2-D patterns of linearly polarized antennas, measured in the ***E-plane*** (a plane parallel to the **E** vector and containing the direction of maximum radiation) and in the ***H-plane*** (a plane parallel to the **H** vector, orthogonal to the *E*-plane, and containing the direction of maximum radiation).

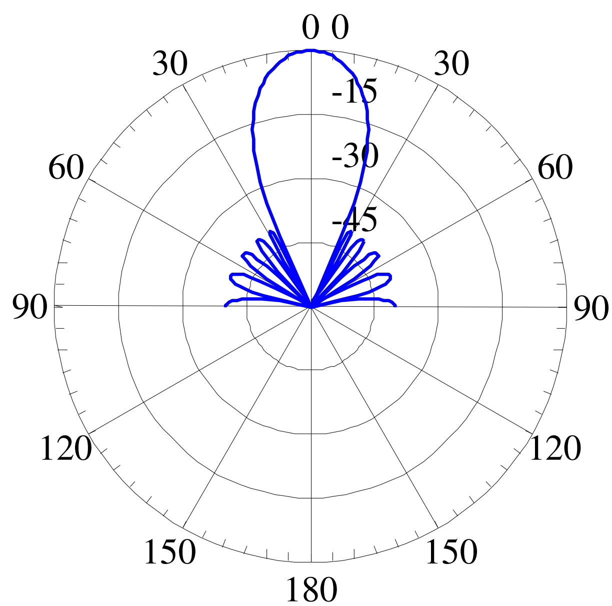


[Balanis, 3rd ed.]

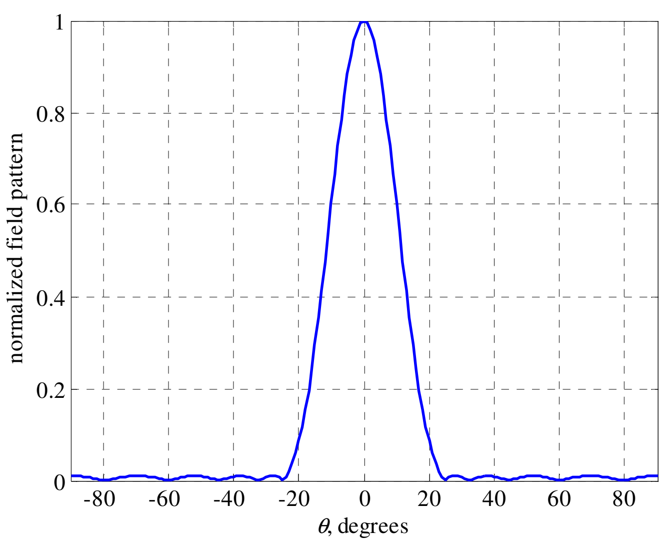
2-D patterns can be *polar* or *rectangular*, depending on the way the angle is depicted, and *linear* or *logarithmic* (in dB), depending on the chosen pattern scale. The plots below show the same 2-D pattern in 4 different formats.



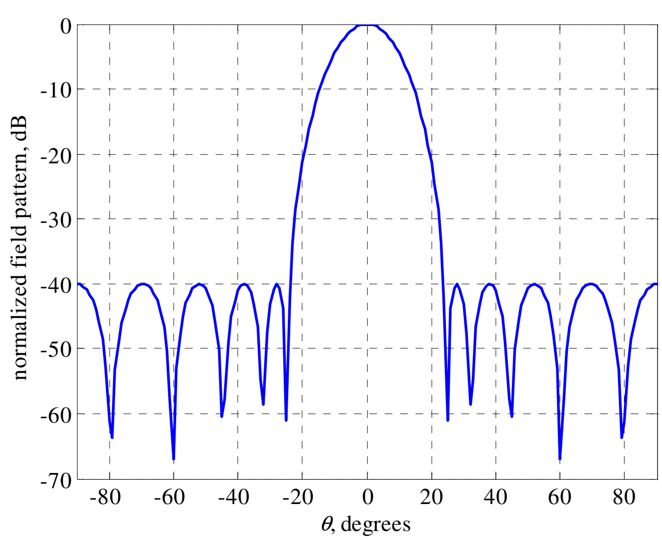
Polar Pattern (linear scale)



Polar Pattern (dB scale, min @-60 dB)

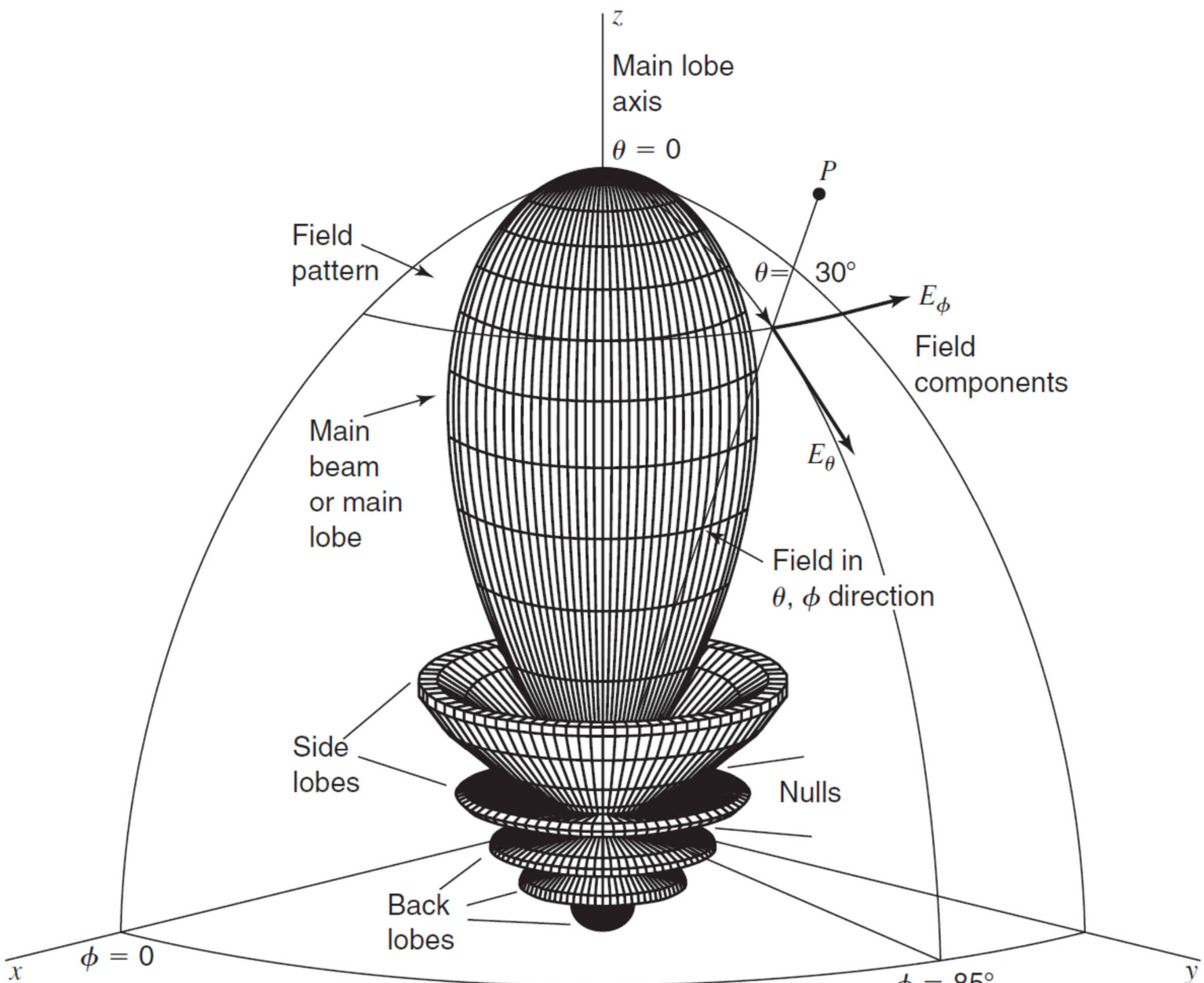


Rectangular Pattern (linear scale)



Rectangular Pattern (dB)

e) ***Pattern lobe*** is a portion of the RP with a local radiation-intensity maximum and limits defined by neighboring nulls. Lobes are classified as: major, minor, side lobes, back lobes.



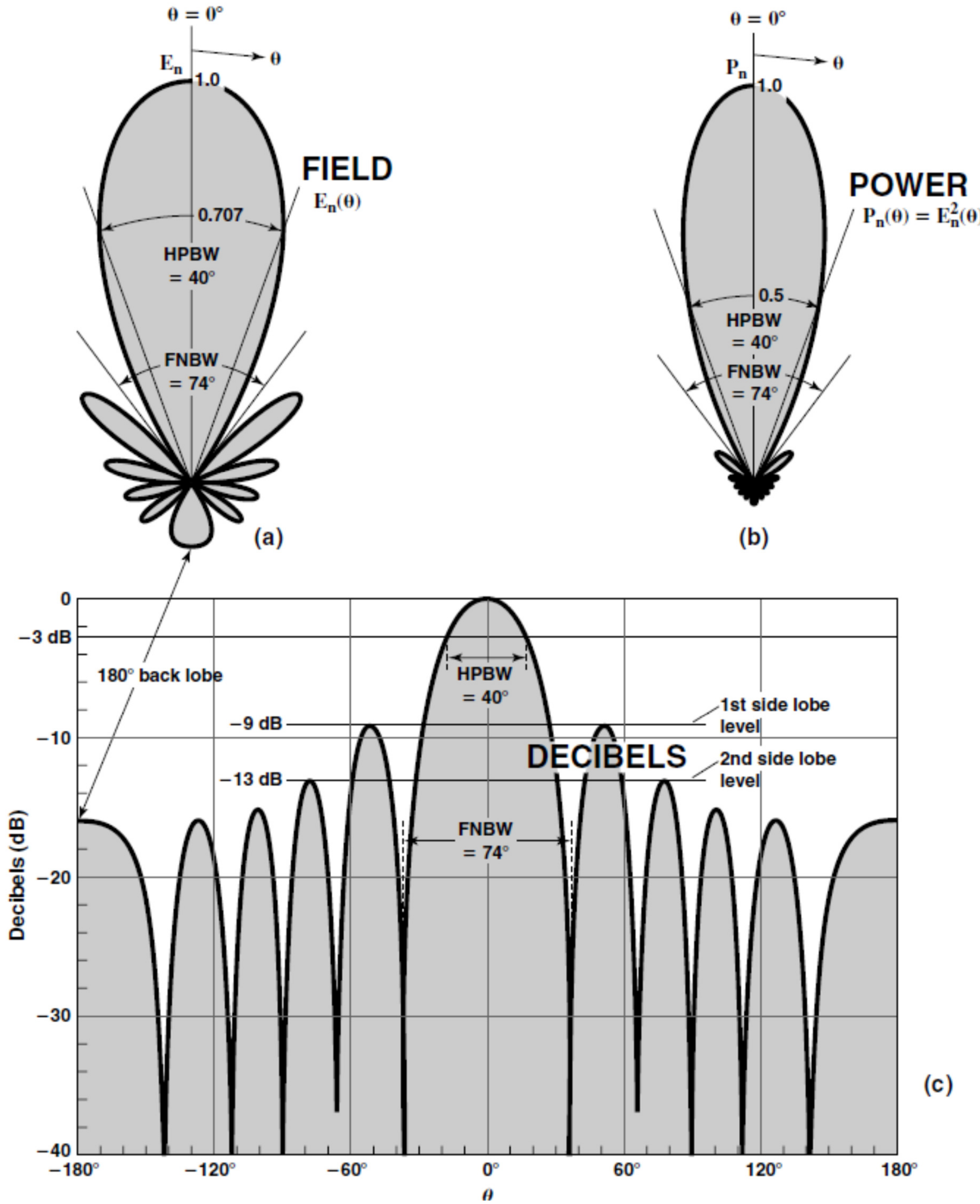
J.D. Kraus and R.J. Marhefka, *Antennas*, 3rd. ed., Fig. 2-3

2. Pattern Beamwidth

Half-power beamwidth (HPBW) is the angle between two vectors, originating at the pattern's origin and passing through these points of the major lobe where the radiation intensity is half its maximum.

First-null beamwidth (FNBW) is the angle between two vectors, originating at the pattern's origin and tangent to the main beam at its base. Often, the approximation FNBW ≈ 2·HPBW is used.

The HPBW is the best parameter to describe the antenna resolution properties. In radar technology as well as in radioastronomy, the antenna resolution capability is of primary importance.



3. Radiation Intensity

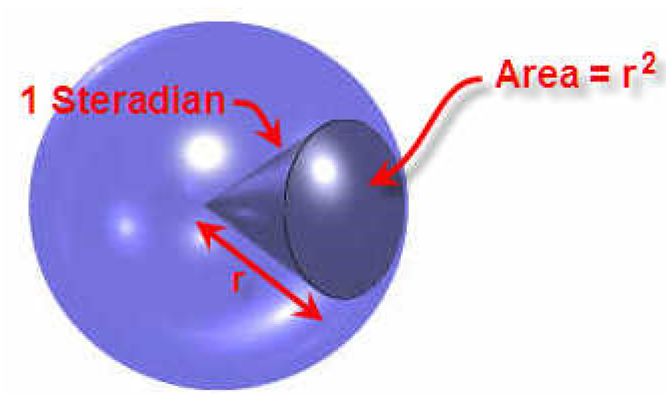
Radiation intensity in a given direction is the power per unit solid angle radiated in this direction by the antenna.

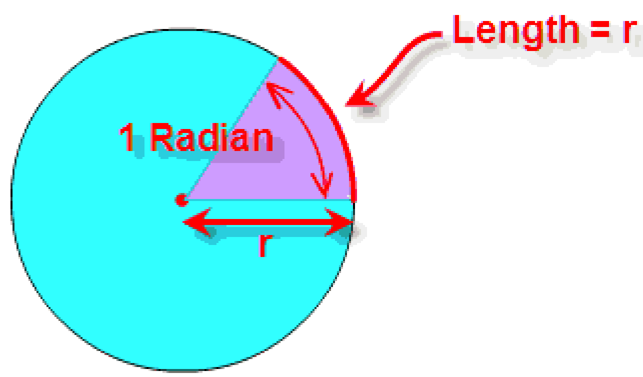
a) Solid angle

One ***steradian*** (***sr***) is the solid angle with its vertex at the center of a sphere of radius r, which is subtended by a spherical surface of area r^2. In a closed sphere, there are 4π steradians. A solid angle is defined as

$$\Omega = \frac{S_\Omega}{r^2}, \text{ sr} \tag{4.1}$$

Note: The above definition is analogous to the definition of a 2-D angle in radians, $\omega = l_\omega / \rho$, where l_ω is the length of the arc segment supported by the angle ω in a circle of radius ρ.





The infinitesimal area ds on a surface of a sphere of radius r in spherical coordinates is

$$ds = r^2 \sin\theta d\theta d\varphi, \text{ m}^2. \tag{4.2}$$

Therefore,

$$d\Omega = \sin\theta d\theta d\varphi, \text{ sr}, \tag{4.3}$$

and

$$ds = r^2 d\Omega. \tag{4.4}$$

b) Radiation intensity U

The ***radiation intensity*** is the power radiated within unit solid angle:

$$U = \lim_{\Delta\Omega \to 0} \frac{\Delta\Pi_{rad}}{\Delta\Omega} = \frac{d\Pi_{rad}}{d\Omega}, \text{ W/sr.} \tag{4.5}$$

The expression inverse to that in (4.5) is

$$\Pi_{rad} = \oiint_{4\pi} U d\Omega, \text{ W.} \tag{4.6}$$

From now on, we will denote the radiated power simply by Π. There is a direct relation between the radiation intensity U and the radiation power density P (that is the time-average Poynting vector magnitude in the far zone). Since

$$P = \frac{d\Pi}{ds} = \frac{d\Pi}{r^2 d\Omega} = \frac{1}{r^2} U, \text{ W/m}^2 \tag{4.7}$$

then

$$U = r^2 \cdot P \tag{4.8}$$

It was already shown that the power density of the far field depends on the distance from the source as $1/r^2$, since the far field magnitude depends on r as $1/r$. Thus, the radiation intensity *U depends only on the direction (θ,φ) but not on the distance r.*

The power pattern is a trace of the function $|U(\theta,\varphi)|$ usually normalized to its maximum value. The normalized pattern will be denoted as $\bar{U}(\theta,\varphi)$.

In the far-field zone, the radial field components vanish, and the remaining **E** and **H** transverse components are in phase and have magnitudes related by

$$|\mathbf{E}| = \eta |\mathbf{H}|. \tag{4.9}$$

This is why the far-field Poynting vector has only a radial component and it is a real number showing the *radiation power-flow density*:

$$P_{rad} = P = \frac{1}{2}\eta |\mathbf{H}|^2 = \frac{1}{2}\frac{|\mathbf{E}|^2}{\eta} \sim \frac{1}{r^2}. \tag{4.10}$$

Then, for the *radiation intensity*, we obtain in terms of the electric field

$$U(\theta,\varphi) = \frac{r^2}{2\eta} |\mathbf{E}|^2. \tag{4.11}$$

Equation (4.11) leads to a useful relation between the power pattern and the amplitude field pattern:

$$U(\theta,\varphi)=\frac{r^2}{2\eta}\left[E_\theta^2(r,\theta,\varphi)+E_\varphi^2(r,\theta,\varphi)\right]=\frac{1}{2\eta}\left[E_{\theta_p}^2(\theta,\varphi)+E_{\varphi_p}^2(\theta,\varphi)\right]. \quad (4.12)$$

Here, $E_{\theta_p}(\theta,\varphi)$ and $E_{\varphi_p}(\theta,\varphi)$ denote the far-zone field patterns for the two orthogonal polarizations.

Examples:

1) Radiation intensity and pattern of an isotropic radiator:

$$P(r,\theta,\varphi)=\frac{\Pi}{4\pi r^2}$$

$$U(\theta,\varphi)=r^2\cdot P=\frac{\Pi}{4\pi}=const.$$

$$\Rightarrow \underline{\underline{\bar{U}(\theta,\varphi)=1}}.$$

The normalized pattern of an isotropic radiator is simply a sphere of a unit radius.

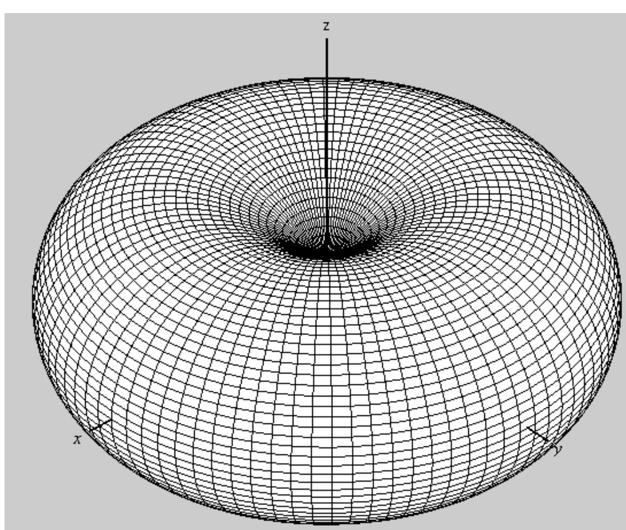
2) Radiation intensity and pattern of an infinitesimal dipole:

From Lecture 3, the far-field term of the electric field is:

$$E_\theta = j\eta\frac{\beta\cdot(I\Delta l)\cdot e^{-j\beta r}}{4\pi r}\cdot\sin\theta \Rightarrow \bar{E}(\theta,\varphi)=\sin\theta,$$

$$U=\frac{r^2}{2\eta}\cdot|\mathbf{E}|^2=\eta\frac{\beta^2\cdot(I\Delta l)^2}{32\pi^2}\cdot\sin^2\theta,$$

$$\Rightarrow \underline{\underline{\bar{U}(\theta,\varphi)=\sin^2\theta}}.$$



4. Directivity

4.1. Definitions and examples

Directivity of an antenna (in a given direction) is the ratio of the radiation intensity in this direction and the radiation intensity averaged over all directions. The radiation intensity averaged over all directions is equal to the total power radiated by the antenna divided by 4π. If a direction is not specified, then the direction of maximum radiation is implied.

It can be also defined as the ratio of the radiation intensity (RI) of the antenna in a given direction and the RI of an isotropic radiator fed by the same amount of power:

$$D(\theta,\varphi)=\frac{U(\theta,\varphi)}{U_{av}}=4\pi\frac{U(\theta,\varphi)}{\Pi}, \tag{4.13}$$

and

$$D_{\max}=D_0=4\pi\frac{U_{\max}}{\Pi}.$$

The directivity is a dimensionless quantity. The maximum directivity is always ≥ 1.

Examples:

1) Directivity of an isotropic source:

$$U\left(\theta,\varphi\right)=U_0=const.$$

$$\Rightarrow \Pi=4\pi U_0$$

$$\Rightarrow D\left(\theta,\varphi\right)=4\pi\frac{U\left(\theta,\varphi\right)}{\Pi}=1$$

$$\Rightarrow \underline{\underline{D_0=1}}.$$

2) Directivity of an infinitesimal dipole:

$$U(\theta,\varphi) = \eta \frac{\beta^2 \cdot (I\Delta l)^2}{32\pi^2} \cdot \sin^2\theta$$

$$\Rightarrow \bar{U}(\theta,\varphi) = \sin^2\theta; \;\; U(\theta,\varphi) = M \cdot \bar{U}(\theta,\varphi) = M \sin^2\theta$$

As per (4.6),

$$\Pi = \oiint_{4\pi} U d\Omega = M \cdot \int_0^{\pi}\int_0^{2\pi} \sin^2\theta \cdot \sin\theta d\varphi d\theta = M \cdot \frac{8\pi}{3}$$

$$D(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{\Pi} = 4\pi \frac{M \sin^2\theta}{M \cdot 8\pi} \cdot 3 = \frac{3}{2}\sin^2\theta$$

$$\Rightarrow \underline{\underline{D_0 = 1.5}}.$$

Exercise: Calculate the maximum directivity of an antenna with a radiation intensity $U = M \sin\theta$. (*Answer*: $D_0 = 4/\pi \approx 1.27$)

The ***partial directivity*** of an antenna is specified for a given polarization of the field. It is defined as that part of the radiation intensity, which corresponds to a given polarization, divided by the total radiation intensity averaged over all directions.

The total directivity is the sum of the partial directivities for any two orthogonal polarizations:

$$D = D_\theta + D_\varphi, \tag{4.14}$$

where:

$$D_\theta = 4\pi \frac{U_\theta}{\Pi_\theta + \Pi_\varphi},$$

$$D_\varphi = 4\pi \frac{U_\varphi}{\Pi_\theta + \Pi_\varphi}.$$

4.2. Directivity in terms of normalized radiation intensity $\bar{U}(\theta,\varphi)$

$$U(\theta,\varphi) = M \cdot \bar{U}(\theta,\varphi) \tag{4.15}$$

$$\Pi = \oiint_{4\pi} U d\Omega = M \cdot \int_0^{\pi}\int_0^{2\pi} \bar{U}(\theta,\varphi)\sin\theta d\varphi d\theta \tag{4.16}$$

$$D(\theta,\varphi) = 4\pi \frac{\bar{U}(\theta,\varphi)}{\int_0^{\pi}\int_0^{2\pi} \bar{U}(\theta',\varphi')\sin\theta' d\varphi' d\theta'} \tag{4.17}$$

For the maximum directivity D_0, we have $\bar{U}(\theta_0,\varphi_0) = 1$; therefore,

$$\underline{\underline{D_0 = 4\pi \frac{1}{\int_0^{\pi}\int_0^{2\pi} \bar{U}(\theta,\varphi)\sin\theta d\varphi d\theta}}}\,. \tag{4.18}$$

This expression is used to compute the directivity of an antenna from its measured and normalized power pattern. In this computation, the integral in the denominator is represented as a discrete sum.

4.3. Beam solid angle Ω_A

The ***beam solid angle*** Ω_A of an antenna is the solid angle through which all the power of the antenna would flow if its radiation intensity were constant and equal to the maximum radiation intensity U_0 for all angles within Ω_A.

$$\Omega_A = \int_0^{\pi}\int_0^{2\pi} \bar{U}(\theta,\varphi)\sin\theta d\varphi d\theta \tag{4.19}$$

The relation between the maximum directivity and the beam solid angle is obvious from (4.18) and (4.19):

$$D_0 = 4\pi / \Omega_A\,. \tag{4.20}$$

In order to understand how (4.19) is obtained, follow the derivations below (they reflect the mathematical meaning of the definition above):

$$\Pi = \oiint_{4\pi} U d\Omega = \underbrace{\iint_{\Omega_A} U_0 d\Omega = U_0\Omega_A}_{\text{assumed constant radiation intensity}}$$

$$\Rightarrow \Omega_A = \frac{\oiint\limits_{4\pi} U d\Omega}{U_0} = \oiint\limits_{4\pi} \bar{U} d\Omega = \int_0^{\pi}\int_0^{2\pi} \bar{U}(\theta,\varphi)\sin\theta d\varphi d\theta .$$

4.4. Approximate expressions for directivity

The complexity of the calculation of the antenna directivity D_0 depends on the power pattern $\bar{U}(\theta,\varphi)$, which has to be integrated over a spherical surface. In most practical cases, this function is not available in closed analytical form (e.g., it might be a data set). Even if it is available in closed analytical form, the integral in (4.18) may not have a closed analytical solution. In practice, simpler although not exact expressions are often used for approximate and fast calculations. These formulas are based on the two orthogonal-plane half-power beamwidths (HPBW) of the pattern. The approximations for the directivity are usually valid for highly directive (pencil-beam) antennas such as large reflectors and horns.

a) ***Kraus' formula***

For antennas with narrow major lobe and with negligible minor lobes, the beam solid angle Ω_A is approximately equal to the product of the HPBWs in two orthogonal planes:

$$\Omega_A = \Theta_1\Theta_2 , \tag{4.21}$$

where the HPBW *angles are in radians*. Another variation of (4.21) is

$$D_0 \simeq \frac{41000}{\Theta_1^{\circ}\Theta_2^{\circ}}, \tag{4.22}$$

where Θ_1° and Θ_1° *are in degrees*.

b) ***Formula of Tai and Pereira***

$$D_0 \simeq \frac{32\ln 2}{\Theta_1^2 + \Theta_2^2} \tag{4.23}$$

The angles in (4.23) are in radians. For details see: C. Tai and C. Pereira, "An approximate formula for calculating the directivity of an antenna," *IEEE Trans. Antennas Propagat.*, vol. AP-24, No. 2, March 1976, pp. 235-236.

5. Antenna Gain

The gain G of an antenna is the ratio of the radiation intensity U in a given direction and the radiation intensity that would be obtained, if the power fed to the antenna were radiated isotropically.

$$G(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{\Pi_{\text{in}}} \tag{4.24}$$

The gain is a dimensionless quantity, which is very similar to the directivity D. When the antenna has no losses, i.e. when $\Pi_{\text{in}} = \Pi$, then $G(\theta,\varphi) = D(\theta,\varphi)$. Thus, the gain of the antenna takes into account the losses in the antenna system. It is calculated using the *input power* Π_{in}, which can be measured directly. In contrast, the directivity is calculated via the radiated power Π.

There are many factors that can worsen the transfer of energy from the transmitter to the antenna (or from the antenna to the receiver):

- mismatch losses,
- losses in the transmission line,
- losses in the antenna: dielectric losses, conduction losses, polarization losses.

The power radiated by the antenna is always less than the power fed to it, i.e., $\Pi \le \Pi_{\text{in}}$, unless the antenna has integrated active devices. That is why, usually, $G \le D$.

! **According to the IEEE Standards, the gain does not include losses arising from impedance mismatch and from polarization mismatch.**

Therefore, the gain takes into account only the dielectric and conduction losses of the antenna itself.

The radiated power Π is related to the input power Π_{in} through a coefficient called the ***radiation efficiency*** e:

$$\Pi = e \cdot \Pi_{\text{in}}, \quad e \le 1, \tag{4.25}$$

$$\Rightarrow G(\theta,\varphi) = e \cdot D(\theta,\varphi). \tag{4.26}$$

Partial gains with respect to a given field polarization are defined in the same way as it is done with the partial directivities; see equation (4.14).

6. Antenna Efficiency

The total efficiency of the antenna e_t is used to estimate the total loss of energy at the input terminals of the antenna and within the antenna structure. It includes all mismatch losses and the dielectric/conduction losses (described by the ***radiation efficiency*** e as defined by the IEEE Standards):

$$e_t = e_p e_r \underbrace{e_c e_d}_{e} = e_p e_r \cdot e . \tag{4.27}$$

Here: e_r is the reflection (impedance mismatch) efficiency,
e_p is the polarization mismatch efficiency,
e_c is the conduction efficiency,
e_d is the dielectric efficiency.

The reflection efficiency can be calculated through the reflection coefficient Γ at the antenna terminals:

$$e_r = 1 - |\Gamma|^2 . \tag{4.28}$$

Γ can be either measured or calculated, provided the antenna impedance is known:

$$\Gamma = \frac{Z_{in} - Z_c}{Z_{in} + Z_c} . \tag{4.29}$$

Z_{in} is the antenna input impedance and Z_c is the characteristic impedance of the feed line. If there are no polarization losses, then the total efficiency is related to the radiation efficiency e as

$$e_t = e \cdot \left(1 - |\Gamma|^2\right) . \tag{4.30}$$

7. Beam Efficiency

The ***beam efficiency*** is the ratio of the power radiated in a cone of angle $2\Theta_1$ and the total radiated power. The angle $2\Theta_1$ can be generally any angle, but usually this is the first-null beam width (the FNBW of the main lobe).

$$BE = \frac{\int_0^{2\pi}\int_0^{\Theta_1} U(\theta,\varphi)\sin\theta d\theta d\varphi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\sin\theta d\theta d\varphi} = \frac{\int_0^{2\pi}\int_0^{\Theta_1} \bar{U}(\theta,\varphi)\sin\theta d\theta d\varphi}{\Omega_A} = \frac{\Omega_{\text{beam}}}{\Omega_A} \tag{4.31}$$

If the antenna has its main beam directed along the z-axis ($\theta = 0$) and if Θ_1 is the angle where the first null occurs in two principal planes, formula (4.31) defines the main-beam efficiency and the BE will show what part of the total radiated power is channeled through the main beam.

Very high beam-efficiency antennas are needed in radars, radiometry and radioastronomy.

8. Frequency Bandwidth (FBW)

This is the range of frequencies, within which the antenna characteristics (input impedance, pattern) conform to certain specifications.

Antenna characteristics, which should conform to certain requirements, might be: input impedance, radiation pattern, beamwidth, polarization, side-lobe level, gain, beam direction, beamwidth, radiation efficiency. Separate bandwidths may be introduced: impedance bandwidth, pattern bandwidth, etc.

The FBW of broadband antennas is expressed as the ratio of the upper to the lower frequencies, where the antenna performance is acceptable:

$$\text{FBW} = f_{\max} / f_{\min} . \tag{4.32}$$

Broadband antennas with FBW as large as 40:1 have been designed. Such antennas are referred to as *frequency independent antennas*.

For narrowband antennas, the FBW is expressed as a percentage of the maximum and minimum frequency difference over the center frequency:

$$\text{FBW} = \frac{f_{\max} - f_{\min}}{f_0} \cdot 100 \ \%. \tag{4.33}$$

Usually, $f_0 = \left(f_{\max} + f_{\min} \right) / 2$ or $f_0 = \sqrt{f_{\max} f_{\min}}$.

9. Input Impedance

$$Z_A = R_A + jX_A \tag{4.34}$$

Here, R_A is the antenna resistance and X_A is its reactance. Generally, the antenna resistance has two terms:

$$R_A = R_{rad} + R_{loss}, \tag{4.35}$$

where R_{rad} is the radiation resistance and R_{loss} is the loss resistance.

The antenna impedance is related to the radiated power $\Pi \equiv \Pi_{rad}$, the dissipated (loss) power Π_{loss}, and the stored reactive energy as:

$$Z_A = \frac{\Pi_{rad} + \Pi_{loss} + 2j\omega(W_m - W_e)}{0.5 I_0 I_0^*}. \tag{4.36}$$

Here, I_0 is the current phasor at the antenna terminals; W_m is the time-average (stored) magnetic energy, and W_e is the time-average electric energy, both stored in the near-field region. When the stored magnetic and electric energy values are equal, a condition of resonance occurs and the reactive part of Z_A vanishes. For a thin dipole antenna, this occurs when the antenna length is close to a multiple of a half wavelength.

9.1. <u>Radiation resistance</u>

The radiation resistance relates the radiated power to the voltage (or current) at the antenna terminals. For example, in the Thevenin equivalent of the antenna, the following holds:

$$R_{rad} = 2\Pi / |I|^2, \ \ \Omega. \tag{4.37}$$

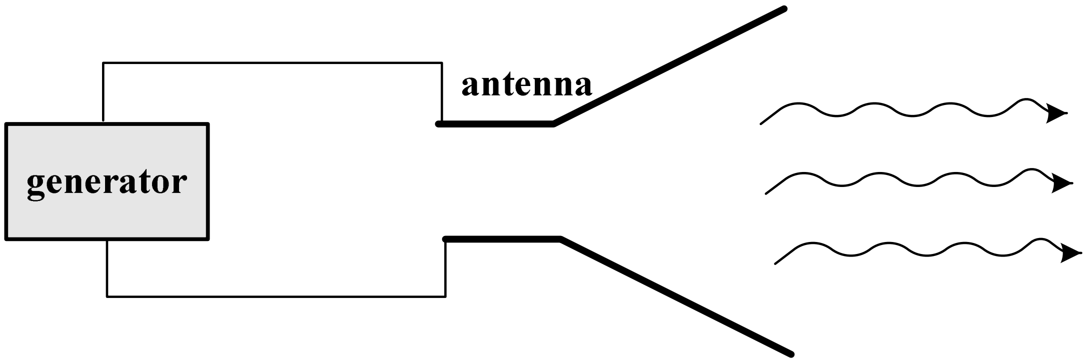
Example: Find the radiation resistance of an infinitesimal dipole in terms of the ratio $(\Delta l / \lambda)$.

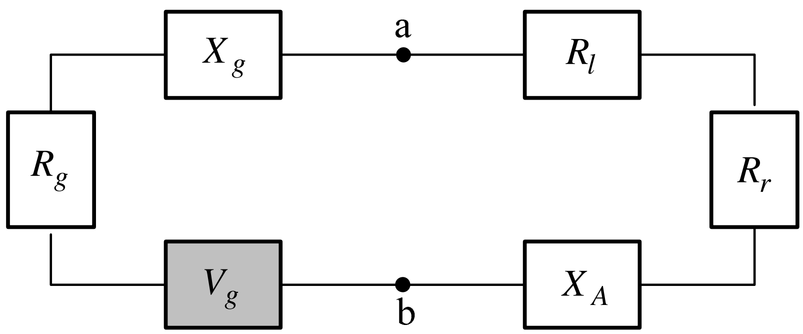
We have already derived the radiated power of an infinitesimal dipole in Lecture 3, as:

$$\Pi^{id} = \eta \frac{\pi}{3}\left(\frac{I\Delta l}{\lambda}\right)^2 \tag{4.38}$$

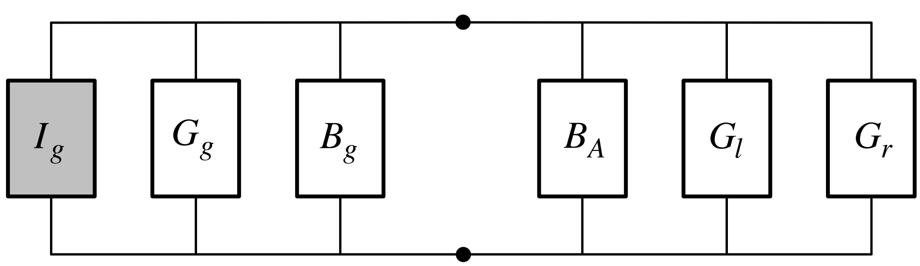
$$\Rightarrow R_{rad}^{id} = \eta \frac{2\pi}{3}\left(\frac{\Delta l}{\lambda}\right)^2. \tag{4.39}$$

9.2. Equivalent circuits of the transmitting antenna





(a) Thevenin equivalent



(b) Norton equivalent

In the above model, it is assumed that the generator is connected to the antenna directly. If there is a transmission line between the generator and the antenna, which is usually the case, then $Z_g = R_g + jX_g$ represents the equivalent impedance of the generator transferred to the input terminals of the antenna. Transmission lines themselves often have significant losses.

Reminder: The impedance transformation by a long transmission line is given by

$$Z_{\text{in}} = Z_0 \left[\frac{Z_{\text{L}} + Z_0 \tanh(\gamma L)}{Z_0 + Z_{\text{L}} \tanh(\gamma L)} \right]. \tag{4.40}$$

Here, Z_0 is the characteristic impedance of the line, γ is its propagation constant, Z_{L} is the load impedance, and Z_{in} is the input impedance. In the case of a loss-free line,

$$Z_{\text{in}} = Z_0 \frac{Z_{\text{L}} + jZ_0 \tan(\beta L)}{Z_0 + jZ_{\text{L}} \tan(\beta L)}, \tag{4.41}$$

since $\gamma = j\beta$. To avoid infinite values of the tangent function in case $\beta L = \pi / 2 + n\pi$, the input-impedance formula is often used in the form

$$Z_{\text{in}} = Z_0 \frac{Z_{\text{L}} \cos(\beta L) + jZ_0 \sin(\beta L)}{Z_0 \cos(\beta L) + jZ_{\text{L}} \sin(\beta L)}. \tag{4.42}$$

Maximum power is delivered to the antenna when conjugate matching of impedances is achieved:

$$\left| \begin{array}{l} R_{\text{A}} = R_{\text{loss}} + R_{\text{rad}} = R_{\text{g}}, \\ X_{\text{A}} = -X_{\text{g}}. \end{array} \right. \tag{4.43}$$

Using circuit theory, we can derive the following formulas in the case of matched impedances:

a) power delivered to the antenna

$$P_{\text{A}} = \frac{|V_{\text{g}}|^2}{8(R_{\text{rad}} + R_{\text{loss}})} \tag{4.44}$$

b) power dissipated as heat in the generator

$$P_{\text{g}} = P_{\text{A}} = \frac{|V_{\text{g}}|^2}{8R_{\text{g}}} = \frac{|V_{\text{g}}|^2}{8(R_{\text{rad}} + R_{\text{loss}})} \tag{4.45}$$

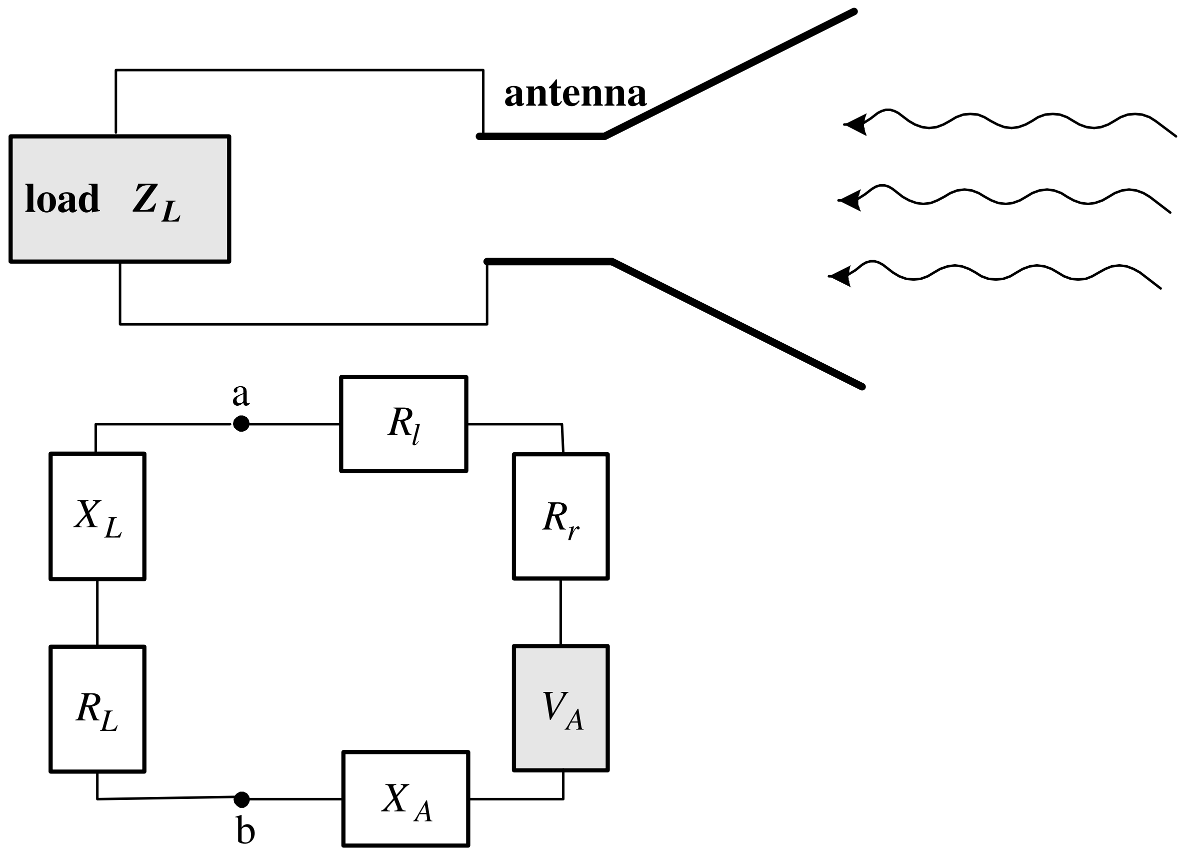
c) radiated power

$$\Pi = P_{\text{rad}} = \frac{|V_{\text{g}}|^2}{8} \frac{R_{\text{rad}}}{(R_{\text{rad}} + R_{\text{loss}})^2} \tag{4.46}$$

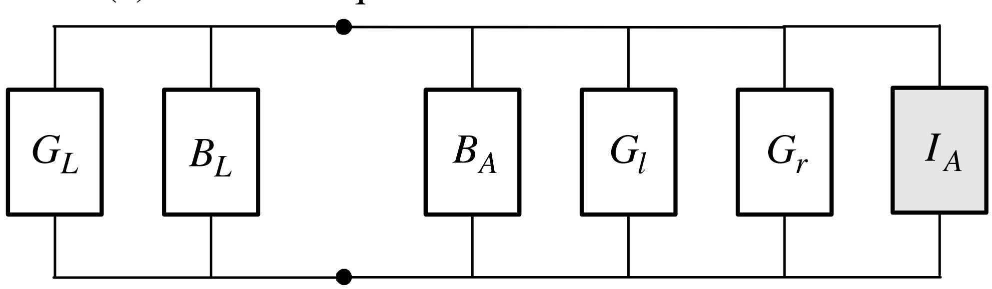
d) power dissipated as heat in the antenna

$$P_{\text{loss}} = \frac{|V_{\text{g}}|^2}{8} \frac{R_{\text{loss}}}{(R_{\text{rad}} + R_{\text{loss}})^2}. \tag{4.47}$$

9.3. Equivalent circuits of the receiving antenna



(a) Thevenin equivalent



(b) Norton equivalent

The incident wave induces voltage V_{A} at the antenna terminals (measured when the antenna is open circuited). Conjugate impedance matching is required between the antenna and the load (the receiver) to achieve maximum power delivery:

$$\left| \begin{aligned} & R_{\mathrm{L}} = R_{\mathrm{A}} = R_{\mathrm{loss}} + R_{\mathrm{rad}} \, , \\ & X_{\mathrm{L}} = -X_{\mathrm{A}} \, . \end{aligned} \right. \tag{4.48}$$

For the case of conjugate matching, the following power expressions hold:

a) power delivered to the load

$$P_{\mathrm{L}} = \frac{|V_{\mathrm{A}}|^2}{8R_{\mathrm{L}}} = \frac{|V_{\mathrm{A}}|^2}{8R_{\mathrm{A}}} \tag{4.49}$$

b) power dissipated as heat in the antenna

$$P_{\mathrm{loss}} = \frac{|V_{\mathrm{A}}|^2}{8} \frac{R_{\mathrm{loss}}}{R_{\mathrm{A}}^2} \tag{4.50}$$

c) scattered (re-radiated) power

$$P_{\mathrm{rad}} = \frac{|V_{\mathrm{A}}|^2}{8} \frac{R_{\mathrm{rad}}}{R_{\mathrm{A}}^2} \tag{4.51}$$

d) total captured power

$$P_{\mathrm{c}} = \frac{|V_{\mathrm{A}}|^2}{4(R_{\mathrm{rad}} + R_{\mathrm{loss}})} = \frac{|V_{\mathrm{A}}|^2}{4R_{\mathrm{A}}} \tag{4.52}$$

When conjugate matching is achieved, half of the captured power P_{c} is delivered to the load (the receiver) and half is antenna loss. The antenna losses are heat dissipation P_{loss} and reradiated (scattered) power P_{rad}. When the antenna is non-dissipative (loss-free), half of the power is delivered to the load and the other half is scattered back into space. Thus, a receiving antenna is also a scatterer.

! **The antenna input impedance is frequency dependent. Thus, it is matched to its load in a certain frequency band. It can be influenced by the proximity of objects, too.**

9.4. Radiation efficiency and antenna losses

The radiation efficiency e takes into account the conductor and dielectric dissipative losses of the antenna. It is the ratio of the power radiated by the antenna and the total power delivered to the antenna terminals (in transmitting mode). In terms of equivalent circuit parameters,

$$\boxed{e = \frac{R_{\mathrm{rad}}}{R_{\mathrm{rad}} + R_{\mathrm{loss}}}}. \tag{4.53}$$

Some useful formulas to calculate conduction losses are given below:

a) dc resistance per unit length

$$R'_{\text{dc}} = \frac{1}{\sigma A}, \quad \Omega/\text{m} \tag{4.54}$$

σ - specific conductivity, S/m

A – conductor's cross-section, m^2.

b) <u>high-frequency surface resistance</u>

At high frequencies, the current is confined in a thin layer at the conductor's surface (skin effect). This thin layer, called the *skin layer*, has much smaller cross-section than that of the conductor itself. Its effective thickness, known as the *skin depth* or *penetration depth*, is calculated as

$$\delta \approx \frac{1}{\sqrt{\pi f \sigma \mu}}, \text{ m}, \tag{4.55}$$

in the case of very good conductors, where f is the frequency in Hz, and μ is the magnetic permeability in H/m. Remember that (4.55) holds for very good conductors only ($\sigma / \omega\varepsilon >> 1$). The exact definition of the skin depth is $\delta = 1/\alpha$, where $\alpha = \text{Re}(\gamma)$, i.e., it is inverse proportional to the attenuation constant of the conducting medium. Here, $\gamma = j\omega\sqrt{\mu\varepsilon}$, $\varepsilon = \varepsilon' - j(\varepsilon'' + \sigma/\omega)$. Due to the exponential decay of the current density in the conductor as $\sim e^{-\alpha x}$, where x denotes the distance from the surface, it can be shown that the total current I flowing along the conductor (along z) is

$$I = \iint_S \mathbf{J} \cdot d\mathbf{s} = \int_C \int_0^\infty J_0 e^{-\alpha x} dx dc = \frac{1}{\alpha} \int_C J_0 dc = \delta \int_C J_0 dc = \int_C J_s dc \tag{4.56}$$

where J_0 is the current density at the conductor surface (in A/m^2), $J_s = J_0\delta$ is the equivalent surface current density (in A/m), and C is the contour of the conductor's cross-section. If the equivalent surface current density J_s is distributed uniformly on the contour of the conductor's cross-section, then $I = J_s p$, where p is the perimeter of the conductor (or the length of its cross-sectional contour).

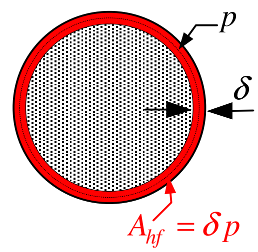
The surface resistance R_s (in Ω) is defined as the real part of the intrinsic impedance of the conductor η_c, which in the case of very good conductors can be found to be

$$R_s = \text{Re}\,\eta_c \approx \sqrt{\frac{\mu\omega}{2\sigma}} = \sqrt{\frac{\mu\pi f}{\sigma}} = \frac{1}{\sigma\delta}, \ \Omega\ . \tag{4.57}$$

For the case where the current density is uniformly distributed on the conductor's cross-sectional contour, we can find a simple relation between the *high-frequency resistance per unit length* R'_{hf} of a conducting rod, its cross-sectional perimeter p and its surface resistance R_{s}:

$$R'_{\text{hf}} = \frac{1}{\sigma A_{\text{hf}}} = \frac{1}{\sigma\delta p} = \frac{R_{\text{s}}}{p}, \ \Omega/\text{m}. \tag{4.58}$$

Here the area $A_{\text{hf}} = \delta p$ is not the actual cross-sectional area of the conducting rod but the effective area through which the high-frequency current flows.



If the surface current distribution is not uniform over the contour of the conductor's cross-section, R'_{hf} appears as a function of R_{s} and this distribution. The surface density of the loss power in a good conductor is

$$p_\ell = \frac{1}{2} R_{\text{s}} |\mathbf{J}_{\text{s}}|^2 \ \text{W/m}^2. \tag{4.59}$$

Then, the power loss per unit length is

$$P'_\ell = \int_C p_\ell dc = \frac{R_{\text{s}}}{2}\int_C |\mathbf{J}_{\text{s}}|^2\, dc = \frac{1}{2} R'_{\text{hf}} I^2 = \frac{1}{2} R'_{\text{hf}} \left(\int_C |\mathbf{J}_{\text{s}}|\, dc \right)^2 \ \text{W/m}. \tag{4.60}$$

It then follows that

$$R'_{\text{hf}} = R_{\text{s}} \frac{\int_C |\mathbf{J}_{\text{s}}|^2\, dc}{\left(\int_C |\mathbf{J}_{\text{s}}|\, dc \right)^2} \ \Omega/\text{m}. \tag{4.61}$$

The above expression reduces to (4.58) if $\mathbf{J}_{\text{s}}$ is constant over C.

Example: A half-wavelength dipole (fed at its center) is made of copper ($\sigma = 5.7\times 10^7$ S/m). Determine the radiation efficiency e, if the operating frequency is $f = 100$ MHz, the radius of the wire is $b = 10^{-4}\lambda$, and the radiation resistance is $R_{\text{rad}} = 73\ \Omega$.

$$f = 10^8 \text{ Hz} \Rightarrow \lambda = \frac{c}{f} \approx 3 \text{ m} \Rightarrow l = \frac{\lambda}{2} \approx 1.5 \text{ m}$$

$$p = 2\pi b = 2\pi 10^{-4}\lambda = 2\pi 10^{-4} \times 3 = 6\pi \times 10^{-4}, \text{ m.}$$

If the current along the dipole were uniform, the high-frequency loss power would be uniformly distributed along the dipole. However, the current has approximately a sine distribution along a dipole as we will discuss in Lecture 9:

$$I(z) = I_0 \sin\left[\beta\left(\frac{l}{2} - |z|\right)\right], \quad -\frac{l}{2} \le z \le \frac{l}{2}. \tag{4.62}$$

Equation (4.58) can be now used to express the high-frequency loss resistance per wire differential element of infinitesimal length dz:

$$dR_{\text{hf}} = R'_{\text{hf}} dz = \frac{dz}{p} \underbrace{\sqrt{\frac{\mu_0 \pi f}{\sigma}}}_{R_s}.$$

The high-frequency loss power per wire element of infinitesimal length dz is then obtained as

$$dP_{\text{hf}}(z) = \frac{1}{2} I^2(z) \cdot \underbrace{\frac{dz}{p}\sqrt{\frac{\mu_0 \pi f}{\sigma}}}_{dR_{\text{hf}}}.$$

The total loss power is obtained by integrating along the dipole's length. The symmetry in the current distribution along z means that the two arms of the dipole dissipate the same amount of power. Thus,

$$P_{\text{hf}} = 2\int_0^{l/2} \frac{I_0^2}{2} \sin^2\left[\beta\left(\frac{l}{2} - z\right)\right] \cdot \frac{1}{p}\sqrt{\frac{\mu_0 \pi f}{\sigma}} dz,$$

$$\Rightarrow P_{\text{hf}} = \frac{I_0^2}{p}\sqrt{\frac{\mu_0 \pi f}{\sigma}} \cdot \int_0^{l/2} \sin^2\left[\beta\left(\frac{l}{2} - z\right)\right] dz.$$

Changing variable as

$$x = \beta\left(\frac{l}{2} - z\right)$$

results in

$$P_{\text{hf}} = I_0^2 \underbrace{\left(\frac{l}{p}\sqrt{\frac{\mu_0 \pi f}{\sigma}}\right)}_{\bar{R}_{\text{hf}}} \cdot \frac{1}{l} \cdot \frac{1}{\beta} \int_0^{\beta l/2} \frac{1 - \cos 2x}{2} dx,$$

$$\Rightarrow P_{\text{hf}} = I_0^2 \bar{R}_{\text{hf}} \cdot \frac{1}{l} \cdot \frac{l}{4}\left[1 - \frac{\sin(\beta l)}{\beta l}\right] = \underline{\underline{\frac{I_0^2 \bar{R}_{\text{hf}}}{4}\left[1 - \frac{\sin(\beta l)}{\beta l}\right]}}. \tag{4.63}$$

Since $P_{\text{hf}} = 0.5 I_0^2 R_{\text{loss}}$ (R_{loss} being the loss resistance of the dipole), we obtain

$$R_{\text{loss}} = \frac{1}{2}\bar{R}_{\text{hf}}\left[1 - \frac{\sin(\beta l)}{\beta l}\right] = \frac{1}{2}\frac{l}{p}\sqrt{\frac{\mu_0 \pi f}{\sigma}}\left[1 - \frac{\sin(\beta l)}{\beta l}\right]. \tag{4.64}$$

In the case of $l = \lambda/2$, $\beta l = \pi$ and $\sin(\beta l) = 0$, which leads to

$$R_{\text{loss}} = 0.5\bar{R}_{\text{hf}} = 0.5\frac{l}{p}\sqrt{\frac{\pi f \mu_0}{\sigma}} = 0.349 \;\; \Omega.$$

The antenna efficiency is:

$$e = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{73}{73 + 0.349} = 0.9952 \;\; (99.52\%)$$

$$e_{[\text{dB}]} = 10\log_{10} 0.9952 = -0.02.$$

The formula (4.64) that we derived in the example above assumed that the loss power depends on the current as $P_{\text{hf}} = 0.5 I_0^2 R_{\text{loss}}$, which in turn implies that the current at the center-feed point has a value of I_0. This may not always be the case. Note that I_0 is the magnitude of the sinusoidal current distribution in (4.62) assumed for the dipole. What if the dipole is shorter than half-wavelength, $l < \lambda/2$? The current at the center of the dipole (the feed point) will be smaller than I_0. From (4.62), it follows that this feed-point current is

$$I(z=0) = I_{\text{c}} = I_0 \sin(0.5\beta l). \tag{4.65}$$

Note that in the case of a half-wavelength dipole, $\beta l = \pi$ and, indeed, $I_{\text{c}} = I_0$. For the case where $l \neq \lambda/2$, the case of $l < \lambda/2$ included, we can employ the expression (4.63) for P_{hf}, where we replace I_0 with the expression for I_{c} in (4.65):

$$P_{\text{hf}} = \frac{1}{2} I_{\text{c}}^2 R_{\text{loss}} = \frac{1}{2} I_0^2 \sin^2(0.5\beta l) R_{\text{loss}} = \frac{I_0^2 \bar{R}_{\text{hf}}}{4}\left[1 - \frac{\sin(\beta l)}{\beta l}\right], \tag{4.66}$$

where

$$\bar{R}_{\text{hf}} = \frac{l}{p}\sqrt{\frac{\mu_0 \pi f}{\sigma}}. \tag{4.67}$$

It follows that the loss resistance is now

$$R_{\text{loss}} = \frac{\bar{R}_{\text{hf}}}{2\sin^2(0.5\beta l)}\left[1 - \frac{\sin(\beta l)}{\beta l}\right]. \tag{4.68}$$

The above formula is general as it applies for any length l of a dipole as long as the dipole is center-fed.

10. Effective Area (Effective Aperture) A_e

The ***effective antenna aperture*** is the ratio of the available power at the terminals of the antenna (operating in receiving mode) to the power flux density of a plane wave incident upon the antenna, where the plane wave is matched to the antenna polarization. If no direction of incidence is specified, the direction of the antenna's maximum radiation is implied, which is also this antenna's direction of best reception.

$$A_e = P_A / W_i, \tag{4.69}$$

where

A_e is the effective aperture, m^2,

P_A is the power delivered from the antenna to a matched load, W,

W_i is the power flux density (Poynting vector) of the incident wave, W/m^2.

Using the Thevenin equivalent of a receiving antenna, we can show that equation (4.69) relates the antenna impedance and its effective aperture as

$$A_e = \frac{|I_A|^2 R_L / 2}{W_i} = \frac{|V_A|^2}{2W_i} \cdot \frac{R_L}{\left[(R_r + R_l + R_L)^2 + (X_A + X_L)^2\right]}. \tag{4.70}$$

Under conditions of conjugate matching ($R_A = R_r + R_l = R_L,\ X_A = -X_L$),

$$A_e = \frac{|V_A|^2}{8W_i} \frac{1}{\underbrace{(R_r + R_l)}_{R_A = R_L}}. \tag{4.71}$$

For aperture type antennas, the effective area is smaller than the physical area of their aperture. Antennas with constant field amplitude and phase distribution across their aperture have the maximum possible effective area, which, in the case of aperture antennas, is practically equal to their physical aperture area. The effective aperture of wire antennas is much larger than the surface of the wire itself. Sometimes, the ***aperture efficiency*** of an antenna is provided as the ratio of the effective antenna aperture and its physical area:

$$\varepsilon_{ap} = \frac{A_e}{A_p}. \tag{4.72}$$

Example: A uniform plane wave is incident upon a very short dipole. Find the effective area A_e assuming that the radiation resistance is $R_r = 80(\pi l / \lambda)^2$ Ω and that the field is linearly polarized along the axis of the dipole. Compare A_e with the physical surface of the wire if $l = \lambda / 50$ and $d = \lambda / 300$, where d is the wire's diameter.

Since the dipole is very short, we can neglect the conduction losses. Wire antennas do not have dielectric losses. Therefore, we assume that $R_l = 0$. Under conjugate matching (which is implied unless specified otherwise),

$$A_e = \frac{|V_A|^2}{8W_i R_r}.$$

The dipole is very short and we can assume that the **E**-field intensity is the same along the whole wire. Then, the voltage created by the induced electromotive force of the incident wave is

$$V_A = |\mathbf{E}| \cdot l.$$

The Poynting vector has a magnitude of $W_i = |\mathbf{E}|^2 /(2\eta)$. Then, under conditions of conjugate matching, see (4.71),

$$A_e = \frac{|\mathbf{E}|^2 \cdot l^2 \cdot 2\eta}{8 \cdot |\mathbf{E}|^2 \cdot R_r} = \frac{3\lambda^2}{8\pi} = 0.119 \cdot \lambda^2.$$

The physical surface of the dipole is

$$A_p = \pi dl = \pi \frac{\lambda}{300} \frac{\lambda}{50} = \frac{\pi}{15} 10^{-3} \lambda^2 = 2.1 \times 10^{-4} \cdot \lambda^2.$$

The aperture efficiency of this dipole is then

$$\varepsilon_{ap} = \frac{A_e}{A_p} = \frac{0.119}{2.1 \times 10^{-4}} = 568.2.$$

It is evident from the above example, that the aperture efficiency is not a suitable parameter for wire antennas, which have very small surface area. However, the effective area is still a useful parameter for wire antennas as it has direct relation with the directivity, as discussed next.

11. Relation Between Directivity D_0 and Effective Aperture A_e

The simplest derivation of this relation goes through two stages.

Stage 1: Using reciprocity, prove that the ratio $D_0 / A_e = \gamma$ is the same for any antenna.

Consider two antennas: A1 and A2. Let A1 be the transmitting antenna, and A2 be the receiving one. Let the distance between the two antennas be R. The power flux density generated by A1 at A2 is

$$W_1 = \frac{D_1 \Pi_1}{4\pi R^2}.$$

Here, Π_1 is the total power radiated by A1 and D_1 is the directivity of A1. The above follows directly from the definition of directivity:

$$D(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{\Pi} = \frac{4\pi R^2 W(\theta,\varphi)}{\Pi} \Rightarrow W(\theta,\varphi) = \frac{\Pi D(\theta,\varphi)}{4\pi R^2}.$$

The power received by A2 and delivered to its load is

$$P_{1\to 2} = A_{e_2} W_1 = A_{e_2} \frac{D_1 \Pi_1}{4\pi R^2},$$

where A_{e_2} is the effective area of A2.

$$\Rightarrow D_1 A_{e_2} = 4\pi R^2 \frac{P_{1\to 2}}{\Pi_1}.$$

Now, let A1 be the receiving antenna and A2 be the transmitting one. We can derive the following:

$$D_2 A_{e_1} = 4\pi R^2 \frac{P_{2\to 1}}{\Pi_2}.$$

If $\Pi_1 = \Pi_2$, then, according to the reciprocity principle in electromagnetics♣, $P_{1\to 2} = P_{2\to 1}$. Therefore,

$$D_1 A_{e_2} = D_2 A_{e_1} \Rightarrow \underline{\underline{\frac{D_1}{A_{e_1}} = \frac{D_2}{A_{e_2}} = \gamma}}\,.$$

♣ Reciprocity in antenna theory states that if antenna #1 is a transmitting antenna and antenna #2 is a receiving antenna, then the ratio of transmitted to received power $P_{\text{Tx}} / P_{\text{Rx}}$ will not change if antenna #1 becomes the receiving antenna and antenna #2 becomes the transmitting one.

We thus proved that γ is the same for every antenna.

Stage 2: Find the ratio $\gamma = D_0 / A_e$ for an infinitesimal dipole.

The directivity of a very short dipole (infinitesimal dipole) is $D_0^{\text{id}} = 1.5$ (see **Examples** of **Section 4**, this Lecture). The effective aperture of an infinitesimal dipole is $A_e^{\text{id}} = 3\lambda^2 / (8\pi)$ (see the **Example** of **Section 10**, this Lecture). Then,

$$\gamma = \frac{D_0}{A_e} = \frac{1.5}{3\lambda^2} \cdot 8\pi ,$$

$$\boxed{\gamma = \frac{D_0}{A_e} = \frac{4\pi}{\lambda^2}}. \tag{4.73}$$

Equation (4.73) is true if there are no dissipation, polarization mismatch, and impedance mismatch in the antenna system. If these are present, then

$$A_e = (1 - |\Gamma|^2)\, |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 \left(\frac{\lambda^2}{4\pi} \right) \underbrace{eD_0}_{G_0} . \tag{4.74}$$

From (4.20) and (4.73), we can obtain a simple relation between the antenna beam solid angle Ω_A and A_e:

$$\boxed{A_e = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{\Omega_A}}. \tag{4.75}$$

12. Other Antenna Equivalent Areas

Before, we have defined the antenna effective area (or effective aperture) A_e as the area, which when multiplied by the incident wave power density W_i, produces the power delivered to the load (the terminals of the antenna) P_A. In a similar manner, we define the antenna ***scattering area*** A_s. It is the area, which when multiplied with the incident wave power density, produces the re-radiated (scattered) power:

$$A_s = \frac{P_s}{W_i} = \frac{|I_A|^2 R_r}{2W_i} ,\ \text{m}^2. \tag{4.76}$$

In the case of conjugate matching,

$$A_s = \frac{|V_A|^2}{2W_i} \frac{R_r}{4(\underbrace{R_r + R_l}_{R_A})^2} = \frac{|V_A|^2}{8W_i} \frac{R_r}{R_A^2} ,\ \text{m}^2. \tag{4.77}$$

The ***loss area*** is the area, which, when multiplied by the incident wave power density, produces the dissipated power of the antenna.

$$A_l = \frac{P_l}{W_i} = \frac{|I_A|^2 R_l}{2W_i}, \text{ m}^2. \tag{4.78}$$

In the case of conjugate matching,

$$A_l = \frac{|V_A|^2}{2W_i} \frac{R_l}{4(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \frac{R_l}{R_A^2}, \text{ m}^2. \tag{4.79}$$

The ***capture area*** is the area, which when multiplied with the incident wave power density, produces the total power intercepted by the antenna:

$$A_c = \frac{P_t}{W_i} = \frac{|I_A|^2 (R_r + R_l + R_L)}{2W_i}. \tag{4.80}$$

In the case of conjugate matching,

$$A_c = \frac{|V_A|^2}{8W_i} \frac{(R_r + R_l + R_L)}{(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \frac{(R_A + R_L)}{R_A^2} = \frac{|V_A|^2}{4W_i} \frac{1}{R_A}. \tag{4.81}$$

The capture area is the sum of the effective area, the loss area and the scattering area:

$$A_c = A_e + A_l + A_s = \frac{|V_A|^2}{8W_i} \left(\frac{1}{R_A} + \frac{R_l}{R_A^2} + \frac{R_r}{R_A^2} \right). \tag{4.82}$$

When conjugate matching is achieved, we have from (4.71) that

$$A_e = \frac{|V_A|^2}{8W_i} \frac{1}{R_A}. \tag{4.83}$$

Comparing (4.83) with (4.77), (4.79) and (4.82), we see that

$$A_e = A_l + A_s = 0.5 A_c. \tag{4.84}$$

If conjugate matching is achieved for a loss-free antenna, then

$$A_e = A_s = 0.5 A_c. \tag{4.85}$$

The results in (4.84) and (4.85) suggest that even under optimal conditions for delivering power to the receiver (conjugate match), only one-half of the power captured by the antenna is delivered. The other half is simply scattered back into space if the antenna is loss-free. If the antenna has loss, a portion of this other half of the captured power (corresponding to $0.5A_c$) is scattered back into space and the other portion is dissipated.