Introduction to Microwave Imaging Part I: Forward Models

Natalia K. Nikolova
nikolova@ieee.org

McMaster University
Department of Electrical and Computer Engineering
Electromagnetic Vision (EMVi) Research Laboratory
Hamilton, ON CANADA
I COME FROM …

- World University Ranking 77 (Times Higher Education)
- Canada’s most research intensive university ($/faculty member)
COURSE OVERVIEW

Day 1: Introduction & Forward Models of Microwave Imaging
• Field-based Integral Solutions of the Scattering Problem in Time and Frequency
• Born and Rytov Approximations of the Forward Model of Scattering
• Scattering Parameters and Integral Solutions in Terms of S-parameters
• 2D Model of Tomography in Microwave Scattering

Day 2: Linear Inversion Methods
• Deconvolution Methods
  Microwave Holography (MH)
  Scattered Power Mapping (SPM) Image
• Reconstruction of Pulsed-radar Data
  Synthetic Focusing: Delay and Sum (DAS)

Day 3: Performance Metrics & Hardware
• Spatial Resolution
• Dynamic Range
• Data Signal-to-noise Ratio

Select Topics
• Overview of Nonlinear Inversion Methods
  Direct Iterative Methods
  Model-based Optimization Methods
• Tissue Imaging – Challenges and Advancements
INTRODUCTION INTO THE SUBJECT
WHY DO WE CARE ABOUT MICROWAVE IMAGING

- penetration into optically obscured objects (fog/clouds, foliage, soil, brick, concrete, clothing, walls, luggage, living tissue…)
  - the lower the frequency the better the penetration
  - frequency bands from 500 MHz well into the mm-wave bands ($\leq 300$ GHz)
- long-range radar – weather radar, airport and marine radars, automotive radars
- compact relatively cheap electronics esp. in the low-GHz range
- diverse suite of image reconstruction methods

SHORT-RANGE RADAR: NUMEROUS APPLICATIONS

- whole body scanners
- nondestructive testing
- through-wall imaging
- medical imaging
- underground radar
RECENT APPLICATIONS: LUGGAGE INSPECTION, NDT


20 GHz to 30 GHz frequency range

Prof. Zoughi’s team at Missouri University of Science & Technology

[https://youtu.be/RE-PPXmtTeA]

Fig. 15. Example of video camera utility for imaging a box cutter and a pair of scissors inside a laptop bag. (a) Picture of laptop bag in front of the camera aperture with inset showing the objects inside the bag. (b) 3-D view. (c) 2-D image slice focused on the box cutter. (d) 2-D image slice focused on the pair of scissors.
APPLICATIONS: WHOLE BODY SCANNERS

[Sheen et al., “Near-field three-dimensional radar imaging techniques and applications,” Applied Optics 2010]

Pacific Northwest National Laboratory, Washington, USA

40 GHz to 60 GHz (U band) cylindrical scan

40 GHz to 60 GHz (U band) cylindrical scan

10 GHz to 20 GHz polarimetric cylindrical scan
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APPLICATIONS: THROUGH-WALL IMAGING


Prof. Mostofi’s team at the University of California Santa Barbara

https://www.youtube.com/watch?v=THu3ZvAHl9A

Area of interest – top view

3D binary ground-truth image of the unknown area to be imaged (2.96 m x 2.96 m x 0.4 m)

Our 3D image of the area, based on 3.84 % measurements
APPLICATIONS: MEDICAL IMAGING


Prof. Kikkawa’s team at Hiroshima University, Japan

Figure 3. Dome antenna array design. (a) The top view of the antenna in x-y plane. (b) The side view of the antenna in x-z plane. (c) Top view photograph. (d) Bottom view photograph.

Fig. 7 in Song 2017
MICROWAVE NEAR-FIELD IMAGING: FAST GROWTH COMMERCIALLY

• mm-wave whole-body imagers for airport security inspection (> 30 GHz)

• through-wall and through-floor infrastructure inspection for contractors and home inspectors (UWB, 3 GHz to 8 GHz)

• numerous underground radar applications: detection of pipes, cables, tunnels, etc. (< 3 GHz)
MAIN PRINCIPLE OF ACTIVE MICROWAVE IMAGING

• microwave radiation penetrates and interacts with the imaged object
• the wave is modified (amplitude decay, phase delay, etc.) by the object’s EM properties and geometry
• scattered wave samples are collected and processed to deduce the object’s EM properties and geometry
**COMPONENTS OF THE IMAGING PROCESS**

**MEASURED DATA:** $d$
- UWB frequency sweep
- or UWB pulsed radar

**FORWARD MODELS:**
\[
\begin{align*}
F(x) &= d \\
\mathcal{L}_{\text{ME}} \{ x, E \} &= E
\end{align*}
\]
- data equation
- state equations
- analytical EM models
- EM simulators

**INVERSION STRATEGY:**
\[
x = F^{-1} \{ d \} \quad \text{subject to} \quad \mathcal{L}_{\text{ME}} \{ x, E \} = E
\]
- linear and nonlinear solvers
- deconvolution and optimization methods
- sensitivity analysis
- noise analysis & suppression
- data filtering
- image post-processing

**Imaging research is an intersection of engineering, math and physics**
Main Principle: imaging needs abundant and diverse data

- spatial data abundance
  - illuminate target from various angles
  - collect scattered signals at various angles/distances
  - scanning is required (acquisition surfaces – planar, cylindrical, spherical)
  - scanning approaches

<table>
<thead>
<tr>
<th></th>
<th>mechanical scanning</th>
<th>electronically switched arrays</th>
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<tbody>
<tr>
<td>speed</td>
<td>low</td>
<td>HIGH</td>
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<tr>
<td>complexity</td>
<td>LOW</td>
<td>high</td>
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<tr>
<td>flexibility in adjusting scan parameters</td>
<td>GREAT</td>
<td>limited</td>
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\[ k_b = \omega \sqrt{\mu_0 \varepsilon_b} \]

\[ k_s = \sqrt{\mu_0 \varepsilon_s} \]
DATA ACQUISITION: SPATIAL SAMPLING

- **over-sampling** does not ensure diversity but increases acquisition time
- over-sampling counteracts noise effectively in cross-correlation reconstruction methods
- each sample must add independent information
- linearly dependent data may lead to ill-posed inversion problems

> stay below but close to the **maximum spatial sampling step** – ensures diversity

\[
\Delta \zeta \leq \Delta \zeta_{\text{max}} \approx \frac{\lambda_{\text{min}}}{4 \sin \alpha}, \ \zeta \equiv x, y
\]

> effective near-field wavelengths may be shorter than

\[
\lambda = \frac{v_b}{f} = \frac{2\pi}{k_b}
\]

\[
\lambda_{\text{eff,min}} = \frac{2\pi}{k_{x,y}^{\text{max}}}
\]

\[
\tilde{S}(k_x, k_y) = \mathcal{F}_{2D}\{S(x, y)\}
\]
• frequency data diversity in frequency-domain measurements

- stay below but close to the *maximum frequency sampling step*
- it ensures that back-scattered signals from all targets \( \leq R_{\text{max}} \) do not overlap

\[
\Delta f \leq \Delta f_{\text{max}} = \frac{1}{2T_{\text{max}}} \approx \frac{v_b}{4R_{\text{max}}}
\]

*maximum range*

*maximum observation period*
• temporal data diversity in time-domain (pulsed-radar) measurements

  ➢ stay below but close to the maximum time sampling step
  ➢ it ensures that all frequency components of the pulsed signals are fully used (Nyquist)

\[ \Delta t \leq \Delta t_{\text{max}} \approx \frac{T_{\text{min}}}{2} = \frac{1}{2 f_{\text{max}}} \]
any one of the conditions below implies near-field imaging

\[ r \leq \frac{2D_{A,\text{max}}^2}{\lambda}, \quad r \leq D_{A,\text{max}} \]
\[ r \leq \frac{2D_{\text{OUT,\text{max}}}^2}{\lambda}, \quad r \leq D_{\text{OUT,\text{max}}} \]
\[ r \leq \lambda \]

- OUT is in Tx/Rx antennas’ near field
- antennas are in the OUT’s near field
- implication \( A \): multiple scattering & coupling between antennas and OUT

\[ S_{ik} \neq S_{ik}^{\text{inc}} + S_{ik}^{\text{sc}} \]

- implication \( B \): incident antenna fields do not conform to free-space far-zone model

\[ E^{\text{inc}}(r') \sim \hat{p}G(\theta, \varphi) e^{-ik_b r} \]

\[ \text{not valid!} \]
FORWARD MODELS OF ELECTROMAGNETIC SCATTERING
FORWARD vs. INVERSE PROBLEM

forward problem

- from cause toward effect
- unique solution

inverse problem

- from effect toward cause
- not unique
FORWARD vs. INVERSE PROBLEM IN EM SCATTERING

EM ANALYSIS

• cause (known)
  excitation
  boundary conditions
  medium properties

• effect (unknown)
  scattering parameters
  radar cross-section
  antenna far-field pattern
  etc.

Example: EM simulators –
general-purpose forward solvers

INVERSE SCATTERING

• cause (known)
  excitation
  boundary conditions

• cause (unknown)
  medium properties

• effect (somewhat known)
  scattering parameters
  radar return

General-purpose inverse solvers
  do not exist
FORWARD MODELS: DATA and STATE EQUATIONS

**data equation:** maps contrast to the data (field measured outside OUT) \( r \not\in V_s \)

\[
\mathbf{E}^{\text{sc}}(r \in S_a) = \left[ \mathbf{E} - \mathbf{E}^{\text{inc}} \right]_{r \in S_a} = \iiint_{V_s} K(r') \mathbf{G}_b(r, r') \cdot \mathbf{E}(r') \, dr' \quad \mathbf{F}_D : K \rightarrow d
\]

\[ K(r') = k_s^2(r') - k_b^2(r') \]

- ensures contrast produces result matching measurements
- **contrast source** concept: \( S(r') = K(r') \cdot \mathbf{E}(r') \)

**state equation:** maps contrast to field inside OUT (state variables) \( r \in V_s \)

\[
\mathbf{E}(r \in V_s) = \iiint_{V_s} K(r') \mathbf{G}_b(r, r') \cdot \mathbf{E}(r') \, dr' \quad \mathbf{F}_E : K \rightarrow \mathbf{E}(r \in V_s)
\]

- ensures contrast source satisfies Maxwell’s equations
ROLE OF DATA AND STATE EQUATIONS IN IMAGE RECONSTRUCTION

FORWARD MODELS:

\[ F(x) = d \quad \mathcal{L}_{\text{ME}} \{x, E\} = E \]

data equation \quad state equations

INVERSION:

\[ x = F^{-1}\{d\} \quad \text{subject to} \quad \mathcal{L}_{\text{ME}} \{x, E\} = E \]

solving data equation

---

data equation:

\[ E^{\text{sc}}(r \in S_a) = \iiint_{V_s} K(r') G_b(r, r') \cdot E(r') dr' \]

- the unknown is the contrast
- ensures that for a given internal field the forward model matches the data

reconstruction is an interplay of the two equations

state equation:

\[ E(r \in V_s) = \iiint_{V_s} K(r') G_b(r, r') \cdot E(r') dr' \]

- the unknown is the internal field
- ensures that for a given contrast the internal field satisfies Maxwell’s eqns.
Q1: Can we measure the scattered field? 
*No, we measure S-parameters or voltage waveforms*

Q2: Do we know Green’s dyadic $G_b(r, r')$? 
*No, unless the medium is uniform (or layered) and unbounded*

Q3: Do we know the total internal field $E(K, r')$? 
*No, unless we employ Born’s approximation $\rightarrow$ BA: $E(r') \approx E^{\text{inc}}(r')*$

Q4: Do we know the incident internal field $E^{\text{inc}}(r')$? 
*No, unless the medium is uniform (or layered) and unbounded*
DATA EQUATION FOR SCATTERING (S) PARAMETERS

[Nikolova et al., APS-URSI 2016][Beaverstone et al., IEEE Trans. MTT, 2017]

- scattering from penetrable objects (isotropic scattering is assumed)

\[
S_{ik}^{sc} = \frac{i \omega \varepsilon_0}{2a_i a_k} \int \int \int_{V_s} \Delta \varepsilon_i(r') \mathbf{E}_i^{inc}(r') \cdot \mathbf{E}_k(r') dr'
\]

\[
\Delta \varepsilon_i(r') = \varepsilon_{r,s}(r') - \varepsilon_{r,b}(r')
\]

data complex permittivity contrast Green's vector function

\[ \mathbf{E}_i^{inc}(r') : \text{incident internal field due to Rx antenna if it were to transmit} \]

\[ \mathbf{E}_k(r') : \text{total internal field due to Tx antenna} \]

\[ i, k = 1, \ldots, N_p \]

total number of experiments: \( N_p^2 \)

reciprocity: \( \frac{N_p^2 + N_p}{2} \)
DATA EQUATION: BORN’S APPROXIMATION OF TOTAL INTERNAL FIELD

\[
S_{ik}^{sc} = \frac{i \omega \varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta \varepsilon_r(r') E_i^{inc}(r') \cdot E_k^{inc}(r'; \Delta \varepsilon_r(r')) \, dr'
\]

- the total field \( E_k(r') \) is generally unknown AND it depends on the contrast: 
  data equation is nonlinear in the unknown contrast

- Born’s approximation linearizes the data equation by replacing the unknown total internal field with the known incident internal field (Max Born, 1926)

\[
E_k(r') \approx E_k^{inc}(r')
\]

Born’s approximation linearizes the data equation by replacing the unknown total internal field with the known incident internal field. The approximation simplifies the data equation by assuming that the total field can be approximated by the incident field. This is particularly useful in scenarios where the total field is generally unknown and depends on the contrast, making the data equation nonlinear in the unknown contrast. By approximating the total field, Born’s approximation allows for a linearized data equation, which can then be solved using simpler methods.
LIMITATIONS OF BORN’S APPROXIMATION OF THE TOTAL INTERNAL FIELD

• limits both the size and the contrast of the scatterer

\[ a^2 \left| k_s^2(r) - k_b^2 \right| \ll 1, \ r \in V_s \]

[Nikolova, Introduction to Microwave Imaging, 2017]

• if OUT violates the limits: images contain artifacts which reflect differences between \( E_{\text{inc}}^{\text{Tx}}(r') \) and \( E_{\text{Tx}}(r') \) rather than contrast

• Born’s approximation is underlying all direct inversion methods (real-time imaging)
1-D EXAMPLE: BORN’S APPROXIMATION OF TOTAL INTERNAL FIELD

- Gaussian pulse bandwidth: 5 GHz at 3-dB level
- 1-D incident wave coming from left
- Internal field recorded inside dielectric slab of length $L = 6$ cm
comparison of incident wave in air ($\varepsilon_r = 1.0$) with actual internal field in dielectric slabs
1-D EXAMPLE: BORN’S APPROXIMATION OF THE TOTAL INTERNAL FIELD – 3

• let us evaluate Born’s limit in this example – dielectric slab

\[ a^2 \left| k_s^2 (r) - k_b^2 \right| \ll 1 \]

\[ a^2 k_b^2 \left( \frac{k_s^2}{k_b^2} - 1 \right) = \left( \frac{2 \pi a}{\lambda_b} \right)^2 \left( \frac{\varepsilon_{r,s}}{\varepsilon_{r,b}} - 1 \right) \ll 1 \]

\[ \left( \varepsilon_{r,s}^{1\text{GHz}} \right)_{\text{max}} < 3.53 \]

\[ \left( \varepsilon_{r,s}^{5\text{GHz}} \right)_{\text{max}} < 1.10 \]

• BA holds marginally for slab of \( \varepsilon_{r,s} = 1.1 \) but error is very large at \( \varepsilon_{r,s} = 4.0 \)

• BA in the magnitude field distribution is more sensitive (than the phase) to permittivity contrast because reflections at interfaces are not taken into account – even for \( \varepsilon_{r,s} = 1.1 \) magnitude errors are appreciable (esp. at 5 GHz)

• error of the internal-field BA grows with frequency due to increase in scatterer’s electrical size \( a/\lambda \)
WHAT IS THE BASIS OF BORN’S APPROXIMATION IN WAVE THEORY?

• Born’s approximation is based on a simple linear combination of incident-field and total-field wave equations (Helmholtz equations in the frequency domain)

\[
\begin{align*}
\nabla^2 U + k_s^2 U &= 0 \\
\nabla^2 U^{\text{inc}} + k_b^2 U^{\text{inc}} &= 0 \\

k_b &= \omega \sqrt{\mu_b \varepsilon_b} \\
k_s &= \omega \sqrt{\mu_s \varepsilon_s}
\end{align*}
\]

\[
\nabla^2 \left( U - U^{\text{inc}} \right) + k_s^2 U - k_b^2 U^{\text{inc}} = 0
\]

\[
\nabla^2 U^{\text{sc}} + k_b^2 U^{\text{sc}} = -K \cdot U,
\text{ where } K = k_s^2 - k_b^2
\]

• in scattering from dielectric bodies: \( K = \omega^2 \mu_0 \varepsilon_0 (\varepsilon_{r,s} - \varepsilon_{r,b}) = k_0^2 \Delta \varepsilon_r \)

• some terminology

\[\Delta \varepsilon_r \text{ – dielectric contrast} \]
\[K \text{ – contrast function} \]
\[K \cdot U \text{ – contrast source} \]
BORN’S APPROXIMATION IN WAVE THEORY – 2

\[ \nabla^2 U^{sc} + k_b^2 U^{sc} = -K \cdot U, \text{ where } K = k_s^2 - k_b^2 \]

\[ \Rightarrow U(\mathbf{r}) = U^{inc}(\mathbf{r}) + U^{sc}(\mathbf{r}) = U^{inc}(\mathbf{r}) + \iiint_{V_s} G_b(\mathbf{r}, \mathbf{r}') \cdot K(\mathbf{r}')U(\mathbf{r}')d\mathbf{v}' \]

- Green’s function gives the solution at \( \mathbf{r} \) upon point excitation (\( \delta \)-source) at \( \mathbf{r}' \leftarrow \delta(\mathbf{r} - \mathbf{r}') \)
- examples of analytical Green’s functions for open (unbounded) uniform background medium

\[ 3D: G_b(\mathbf{r}, \mathbf{r}') = \frac{e^{-ik_b|r-r'|}}{|\mathbf{r} - \mathbf{r}'|} \]
\[ 2D: G_b(\rho, \rho') = \frac{i}{4} H_0^{(2)}(k_b|\rho - \rho'|) \]
• 0th order Born approximation of the total field

\[ U_B^{(0)}(\mathbf{r}) = U_{\text{inc}}(\mathbf{r}) \]

this is what we use to linearize the data equations by approximating the total \textit{internal} field

\[
S_{ik}^{sc} = \frac{i\omega\varepsilon_0}{2a_ia_k} \iiint_{V_s} \Delta\varepsilon_r(\mathbf{r}') E_{i\text{inc}}^{(r')}(\mathbf{r}') \cdot E_k(\mathbf{r}'; \Delta\varepsilon_r(\mathbf{r}')) d\mathbf{r}'
\]

\[ \Rightarrow (S_{ik}^{sc})_B = \frac{i\omega\varepsilon_0}{2a_ia_k} \iiint_{V_s} \Delta\varepsilon_r(\mathbf{r}') E_{i\text{inc}}^{(r')}(\mathbf{r}') \cdot E_{k\text{inc}}^{(r')} d\mathbf{r}' \]
• 1st order Born approximation of the total field

\[ U_B^{(1)}(r) = U_B^{(0)}(r) + \mathcal{L}(KU_B^{(0)}) = U^{\text{inc}}(r) + \iiint_{V_s} G_b(r, r') \cdot K(r') U^{\text{inc}}(r') dv' \]

\[ \approx U^{\text{sc}}(r) \]

➢ this is what we use to approximate the total and scattered external fields (or data)

\[ U^{\text{OUT}}(r) = U^{\text{inc}}(r) + U^{\text{sc}}(r) \Rightarrow U^{\text{sc}}(r) = U^{\text{OUT}}(r) - U^{\text{inc}}(r) \]

• nth order Born approximation of the total field – can be used to obtain iteratively the total internal field

\[ U_B^{(n)}(r) = U_B^{(n-1)}(r) + \mathcal{L}(KU_B^{(n-1)}) \]

➢ for known contrast \( K(r) \), Born’s expansion series converges to the true total field

\[ U_B^{(n)}(r) \to U(r) \text{ if } a^2 \left| k_s^2 - k_b^2 \right|_{\text{max}} < 1.58 \]

[Nikolova, *Introduction to Microwave Imaging*, 2017]
Born’s approximation of the total response is an additive correction to the incident one.

\[
S_{ik} \approx S_{ik}^{\text{inc}} + \left( S_{ik}^{\text{sc}} \right)_B = S_{ik}^{\text{inc}} + \frac{i\omega \varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta \varepsilon_r (\mathbf{r}') E_i^{\text{inc}} (\mathbf{r}') \cdot E_k^{\text{inc}} (\mathbf{r}') d\mathbf{r}'
\]

• acquisition of the incident \((aka\ baseline)\ data: the reference object (RO)\n  \(\text{RO}\ is simply the measurement setup in the absence of an OUT}
1) Born’s superposition model allows to extract the scattered portion of a response

\[ S_{ik}^{sc} = S_{ik}^{OUT} - S_{ik}^{RO} \]  

⇒ requires 2 measurements: RO and OUT

Note: RO is **not** a uniform medium – it includes all complexities of the measurement setup

2) 1st order BA supplies a linearized (but approximate) model of scattering

\[
\left( S_{ik}^{sc} \right)_B \approx S_{ik}^{sc} = S_{ik}^{OUT} - S_{ik}^{RO} \approx \frac{i\omega \varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta \varepsilon_r (r') E_i^{inc} (r') \cdot E_k^{inc} (r') dr' 
\]

Note: in reality
LIMITATIONS OF BORN’S APPROXIMATION OF THE DATA

• How accurate is the data approximation with Born’s model?

\[
\left( S_{ik}^{sc} \right)_B \approx S_{ik}^{sc} = S_{ik}^{OUT} - S_{ik}^{RO} \approx \frac{i \omega \varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta \varepsilon_r (r') E_i^{inc} (r') \cdot E_k^{inc} (r') dr' 
\]

in reality:

\[
S_{ik}^{sc} = \frac{i \omega \varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta \varepsilon_r (r') E_i^{inc} (r') \cdot E_k (r'; \Delta \varepsilon_r (r')) dr'
\]

• limit on BA data approximations – less strict compared to that for internal field

\[
2a \left| k_s (r) - k_b \right|_{\text{max}} < \pi 
\]

[Slaney et al., IEEE Trans. MTT, 1984]

compare with

\[
a^2 \left| k_s^2 (r) - k_b^2 \right| \ll 1
\]
1-D EXAMPLE: LIMITATIONS OF BORN’S APPROXIMATION OF THE DATA

- re-visiting the dielectric-slab example for scattered field at external observation points (ports 1 and 2)

- What is the contrast limit now?

\[
2a \left| k_s(r) - k_b \right|_{\text{max}} < \pi \quad \Rightarrow \quad \frac{2a k_b}{L} \left( \frac{k_s}{k_b} - 1 \right) = \left( \frac{2\pi L}{\lambda_b} \right)^2 \left( \sqrt{\frac{\varepsilon_{r,s}}{\varepsilon_{r,b}}} - 1 \right) < \pi
\]

\[
\left( \varepsilon_{r,s}^{1\text{GHz}} \right)_{\text{max}} < 12.25 \quad \text{and} \quad \left( \varepsilon_{r,s}^{5\text{GHz}} \right)_{\text{max}} < 2.25
\]

- Be aware! Slaney’s limit is derived with transmission measurements in mind!

\[a = \frac{L}{2} = 3 \text{ cm} \quad \varepsilon_{r,b}^{1\text{GHz}} = 1 \quad \lambda_b^{1\text{GHz}} \approx 30 \text{ cm} \quad \lambda_b^{5\text{GHz}} \approx 6 \text{ cm}\]

\[
\text{compare with internal-field BA limits}
\]

[Nikolova, Introduction to Microwave Imaging, 2017]
1-D EXAMPLE: LIMITATIONS OF BORN’S APPROXIMATION OF THE DATA

slab relative permittivity = 1.2

$\varepsilon_r = 1.2$, Port 1 (back-scatter)

$\varepsilon_r = 1.2$, Port 2 (forward-scatter)

60 MM THICK DIELECTRIC SLAB IN AIR
1-D EXAMPLE: LIMITATIONS OF BORN’S APPROXIMATION OF THE DATA

slab relative permittivity = 2

• errors at Port 1 (reflection measurement) are unacceptable, esp. magnitude
RYTOV’S APPROXIMATION OF THE TOTAL FIELD

• Rylov’s approximation of the total field is an exponential correction to the incident field
  \[ U(\mathbf{r}) \approx U_R(\mathbf{r}) = \exp\left[\psi_0(\mathbf{r}) + \psi_1(\mathbf{r}) + \psi_2(\mathbf{r}) + \cdots\right] \]

• the total field is represented as a complex exponent

• 0\textsuperscript{th} order Rylov approximation of the total field – used to approximate total \textit{internal} field
  \[ U_R^{(0)}(\mathbf{r}) = \exp[\psi_0(\mathbf{r})] = U^{\text{inc}}(\mathbf{r}) \quad \text{\textit{same as 0\textsuperscript{th} order Born approximation}} \]

• 1\textsuperscript{st} order Rylov approximation of the total field – used to approximate total \textit{external} field (data)
  \[ U_R^{(1)}(\mathbf{r}) = \exp[\psi_0(\mathbf{r}) + \psi_1(\mathbf{r})] = U^{\text{inc}}(\mathbf{r}) \cdot \exp\left\{ \left[ U^{\text{inc}}(\mathbf{r}) \right]^{-1} \int\int\int_{\mathcal{V}_b} G_b(\mathbf{r}, \mathbf{r}') \cdot K(\mathbf{r}') U^{\text{inc}}(\mathbf{r}') d\mathbf{v}' \right\} \]
  or
  \[ U_R^{(1)}(\mathbf{r}) = U^{\text{inc}}(\mathbf{r}) \cdot \exp\left[ U_B^{(1)}(\mathbf{r}) / U^{\text{inc}}(\mathbf{r}) \right] \]
RYTOV’S APPROXIMATION OF THE SCATTERED-FIELD DATA

• the case of S-parameters

\[ S_{ik}^{\text{OUT}} = S_{ik}^{\text{RO}} \cdot \exp \left( \frac{(S_{ik}^{\text{sc}})^{(1)}}{S_{ik}^{\text{RO}}} \right) \Rightarrow S_{ik}^{\text{sc}} \approx (S_{ik}^{\text{sc}})_R = S_{ik}^{\text{RO}} \ln \left( \frac{S_{ik}^{\text{OUT}}}{S_{ik}^{\text{RO}}} \right) \]

➢ compare with the BA data approximation

\[ (S_{ik}^{\text{sc}})_B = S_{ik}^{\text{OUT}} - S_{ik}^{\text{RO}} \]

• limitation of the Rylov’s approximation of the data

\[ \left( k_s^2 - k_b^2 \right) / k_b^2 < 1 \quad \text{or} \quad \left( \varepsilon_{r,s} - \varepsilon_{r,b} \right) / \varepsilon_{r,b} < 1 \]

➢ no limitation on the size of the scattering object – advantage over Born’s approximation

➢ strict limitation on the relative contrast – disadvantage to Born’s approximation
EXAMPLE: BORN vs. RYTOV APPROXIMATION OF THE DATA

- re-visiting the dielectric-slab example for scattered field at external observation points (ports 1 and 2)

- What is Rytov’s contrast limit?

\[
\frac{(\varepsilon_{r,s} - \varepsilon_{r,b})}{\varepsilon_{r,b}} < 1 \quad \Rightarrow \quad \varepsilon_{r,s} < 2\varepsilon_{r,b} = 2
\]

- notice independence of electrical size

- compare with BA limits

\[
\left( \varepsilon_{r,s}^{1\text{GHz}} \right)_{\text{max}} < 12.25
\]
\[
\left( \varepsilon_{r,s}^{5\text{GHz}} \right)_{\text{max}} < 2.25
\]
EXAMPLE: BORN vs. RYTOV APPROXIMATION OF THE DATA – 2

slab relative permittivity = 1.2

- both approximations perform very well: BA slightly better on back-scatter, RA slightly better on forward-scatter
slab relative permittivity = 2.0

- both approximation show errors in magnitude of back-scatter
- RA better on forward-scatter and in the phase of back-scatter
EXAMPLE: BORN vs. RYTOV APPROXIMATION OF THE DATA – 4

slab relative permittivity = 4.0

• both approximation show large errors in magnitude of back-scatter
• RA much better on forward-scatter and the phase of both back-scatter
1) Rylov’s approximation is prone to errors in unwrapping the phase of the $S$-parameters

\[
(S_{ik}^{sc})_R = S_{ik}^{RO} \ln \left( \frac{S_{ik}^{OUT}}{S_{ik}^{RO}} \right) \Rightarrow (S_{ik}^{sc})_R = \left\{ \ln \left| \frac{S_{ik}^{OUT}}{S_{ik}^{RO}} \right| + i \left( \angle S_{ik}^{OUT} - \angle S_{ik}^{RO} \right) \right\}
\]

- unwrapped data in frequency does not ensure continuity in space (over the acquisition surface) → spurious large differences between OUT and RO phases corrupt inversion!
- safe to use for object thickness $D$ such that $|k_s - k_b|D \ll 2\pi \Rightarrow |\sqrt{\varepsilon_{r,s}} - \sqrt{\varepsilon_{r,b}}|D / \lambda_0 \ll 2\pi$
2) Rytov’s approximation is prone to errors when incident field (RO data) is weak

\[
(S_{ik}^{\text{sc}})_R = S_{ik}^{\text{RO}} \ln \left( \frac{S_{ik}^{\text{OUT}}}{S_{ik}^{\text{RO}}} \right) \Rightarrow \frac{(S_{ik}^{\text{sc}})_R}{S_{ik}^{\text{RO}}} = \left\{ \ln \left| \frac{S_{ik}^{\text{OUT}}}{S_{ik}} \right| + i \left( \angle S_{ik}^{\text{OUT}} - \angle S_{ik}^{\text{RO}} \right) \right\}
\]

• best use in transmission measurement with significant RO signal strength

division by zero or noisy signal!
THE LINEARIZED DATA EQUATION: A CLOSER LOOK AGAIN!

Do we really know the incident internal field distributions?

- *NO*, unless the RO is uniform (or layered) and unbounded
- ...and unless $V_s$ is in the far-zone of the antennas
THE LINEARIZED DATA EQUATION: ANALYTICAL INCIDENT FIELD MODELS

- IF $V_s$ is in the far-zone of the antennas and the RO can be assumed uniform & unbounded
  THEN analytical incident-field models exist

- **plane waves:** $E_{\text{inc}}^{\text{Tx}}(r, r_{\text{Tx}}) \sim \hat{p} e^{-ik_b|r-r_{\text{Tx}}|}$

- **spherical waves:** $E_{\text{inc}}^{\text{Tx}}(r, r_{\text{Tx}}) \sim \hat{p} \frac{e^{-ik_b|r-r_{\text{Tx}}|}}{|r-r_{\text{Tx}}|}$

- **cylindrical waves:** $E_{\text{inc}}^{\text{Tx}}(r, r_{\text{Tx}}) \sim \hat{p} H_0^{(2)}(k_b\rho), \quad \rho = \sqrt{(x-x_{\text{Tx}})^2 + (y-y_{\text{Tx}})^2}$

- antenna far-field pattern $F(\theta,\phi)$ improves incident-field model, for example

  $E_{\text{inc}}^{\text{Tx}}(r, \theta, \phi) \sim \hat{p} F(\theta, \phi) \frac{e^{-ik_br}}{r}$

[Amineh et al., IEEE AWPL, 2012]
simulated incident fields are often used in near-field imaging where the analytical far-zone models do not apply

incident-field distributions often suffer from modeling errors

modeling errors increase with: (i) decreasing the stand-off distance between the antennas and the OUT, (ii) the complexity of the measurement setup

errors in incident fields corrupt the \textit{resolvent kernel} of the data equation

\[
S_{ik}^{sc} \approx \frac{i \omega \varepsilon_0}{2a_i a_k} \iint_{\Omega} \Delta \varepsilon_r (r') \underbrace{E_i^{inc} (r'} \cdot \underbrace{E_k^{inc} (r')}_{\text{resolvent kernel}} dr'
\]
EXAMPLE: SIMULATED vs. MEASURED INCIDENT FIELD MODELS

[Amineh et al., Trans. IM, 2015]

$\Delta x = \Delta y = 5 \text{ mm}$
$\Delta f = 250 \text{ MHz}$

<table>
<thead>
<tr>
<th>$f$ (GHz)</th>
<th>$\lambda$ (mm)</th>
<th>$D_{\text{far}}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>100</td>
<td>12.5</td>
</tr>
<tr>
<td>8.2</td>
<td>37</td>
<td>34</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>83</td>
</tr>
</tbody>
</table>

X-band (WR90) open-end waveguides ($f_c \approx 6.56 \text{ GHz}$)

[photo credit: Justin McCombe]
METALLIC TARGETS IN AIR – RESULTS WITH SIMULATED KERNEL

[Amines et al., Trans. IM, 2015]
METALLIC TARGETS IN AIR – RESULTS WITH MEASURED KERNEL

[Amineh et al., Trans. IM, 2015]
• analytical and simulated kernels of the data equation are often inadequate for measurements in the antenna near zone
• measured kernels provide accurate system specific data equation for the reconstruction process
• we discuss how to obtain the data-equation kernel through measurements in our next lecture
OTHER APPROXIMATIONS IN THE FORWARD MODELS OF IMAGING

• the forward-model choice is a compromise between two opposing requirements: speed and accuracy

Example 1: whole-body imagers for concealed weapon detection

➢ we can get away with analytical kernels in the data equation

plane-wave kernel approximation for reflection data:

\[
\mathcal{K}(x', y', z'; x, y, z; \omega) = \mathbf{E}_{\text{inc}}^{\text{Rx}} \cdot \mathbf{E}_{\text{inc}}^{\text{Rx}} \sim e^{-i2k_b\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}
\]

➢ we assume that Born’s approximation of the total internal field satisfies the state equation – no need to solve the state equation

\[
\mathbf{E}_{\text{inc}}^{\text{Tx}} \approx \mathbf{E}_{\text{Rx}} \approx \mathbf{E}_{\text{Tx}}
\]

[video credit: https://www.youtube.com/watch?v=_YZEa1hiGO0]
• tomography is an imaging procedure where a 3-D image of an object is obtained by a series of 2-D image reconstructions (one slice at a time)
microwave tomography is common in tissue imaging where Born’s approximation of
the internal field is inaccurate and the state equation must be solved
it achieves better reconstruction speed by solving many 2-D inversions instead of one
3-D inversion
it solves iteratively both the data and state equations (using EM simulations) where
hundreds of simulations may be required – EM simulation speed is critical!
  ➢ arithmetic-operation count for linear systems of equations: $O(N^3)$, sparse $O(N \log_{10} N)$
microwave tomography assumes a plane of field symmetry at the imaged slice
  ➢ this amounts to TM$_z$ field

$$E_x = E_y = 0, \quad E_z \neq 0$$
$$i \omega \mu_b H_x = -\frac{\partial E_z}{\partial y}$$
$$i \omega \mu_b H_y = \frac{\partial E_z}{\partial x}$$

2-D scalar problem!

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \omega^2 \mu_b \varepsilon_s E_z = 0, \quad z = \text{const.}$$
• to achieve the desired symmetry, careful design of the antennas and the imaging setup is required
• symmetry condition holds strictly only in incident-field measurements (RO data)
• sources of error: OUT corrupts the symmetry assumption
SUMMARY OF DAY ONE

- Forward models are an essential component of the imaging process – they reflect our understanding of the relationship between measured data and reconstructed EM properties.

- We need 2 forward models:
  - **Data equations**: relate measured data to contrast.
  - **State equations**: relate total internal field to contrast.

- There are 2 approximations that we can use to linearize the data equation: Born’s and Rytov’s approximations.

- For best accuracy, the data equation must be expressed in terms of the measured data (e.g., $S$-parameters) instead of the $E$-field.

- Analytical representations of the total internal field are strictly limited to far-zone measurements in uniform reflection-free environment.
THANK YOU!