ASSIGNMENT 2

(due Thursday Sep. 22, 2022)

1. (25 points) (a) Prove that if

$$\left| \begin{aligned} &\mathbf{e}(\mathbf{r},t) = \mathrm{Re}\left\{ \mathbf{E}(\mathbf{r}) e^{j\omega t} \right\} \\ &\mathbf{h}(\mathbf{r},t) = \mathrm{Re}\left\{ \mathbf{H}(\mathbf{r}) e^{j\omega t} \right\} \end{aligned} \right. ,$$

then the time-varying Poynting vector $\mathbf{p}$ can be represented as

$$\mathbf{p}(\mathbf{r},t) = \frac{1}{2}\mathrm{Re}\left\{ \mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r}) \right\} + \frac{1}{2}\mathrm{Re}\left\{ \mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r}) \cdot e^{j2\omega t} \right\}.$$

Here, $\mathbf{r}$ denotes position in space, t is time and ω is radian frequency.

2. (25 points) At position $\mathbf{r}_0$, the time-dependent field vectors are given by

$$\mathbf{e}(\mathbf{r}_0,t) = \hat{\mathbf{e}} m_{\mathrm{E}} \cos(\omega t + \Delta\phi),$$
$$\mathbf{h}(\mathbf{r}_0,t) = \hat{\mathbf{h}} m_{\mathrm{H}} \cos(\omega t),$$

where $\hat{\mathbf{e}}$ and $\hat{\mathbf{h}}$ are unit vectors describing the polarizations of the electric and magnetic field. Using trigonometric identities, prove that the time-dependent Poynting vector can be expressed as

$$\mathbf{p}(\mathbf{r}_0,t) = 0.5(\hat{\mathbf{e}} \times \hat{\mathbf{h}}) m_{\mathrm{E}} m_{\mathrm{H}} \left\{ \cos\Delta\varphi \cdot [1 + \cos(2\omega t)] - \sin\Delta\varphi \cdot \sin(2\omega t) \right\} \text{ W/m}^2 .$$

Comment on the meaning of the two terms whose sum form the Poynting's vector value.

3. (10 points) Prove that Eq. (2.3), Gauss law of electrostatics, follows from Eq. (2.2), Ampere's law, and the continuity of current law, Eq. (2.4).

4. (20 points) The far-zone **E**-field radiated by an antenna is given in a phasor form as

(a) $\tilde{\mathbf{E}}(z) = \dfrac{5\exp(-jkz)}{z}(\hat{\mathbf{x}} + j\hat{\mathbf{y}})$ V/m for radiation along the $+z$ axis,

(b) $\tilde{\mathbf{E}}(z) = \dfrac{5\exp(-jkr)}{r}(\hat{\boldsymbol{\theta}} + j\hat{\boldsymbol{\phi}})$ V/m for radiation in the radial direction $\hat{\mathbf{r}}$.

The antenna is at the origin of the coordinate system. Find the phasor of the far-zone magnetic field $\tilde{\mathbf{H}}(z)$ in both cases (a) and (b), if the medium is air.

5. (20 points) The measured complex relative permittivity of sea water is given as $\tilde{\varepsilon}_r = 77 - j12$ at 2 GHz. Find the penetration (or skin) depth δ_{s}.