ASSIGNMENT 3

(due Thursday Oct. 6, 2022)

1. (15 points) Prove Eq. (3.34): i.e., prove that the radiated power of a very small loop of current is

$$\Pi_{\text{rad}} = \frac{1}{12\pi}\eta\beta^4 (IA)^2 .$$

Hint: Use the field expressions in the far zone only in order to calculate the power density in the far zone and from it the total radiated power. An alternative fast way to solve the problem is to use duality and the known expression for the radiated power of an infinitesimal electrical dipole.

2. (20 points) Assume that an antenna radiates isotropically over a half-space above a perfectly conducting flat ground plane. If $E = 5$ mV/m (rms) at a distance of 1 km (far zone), find:
   a) the vector of the average power flux density $\mathbf{p}_{av}$ (give the direction and the magnitude) at 1 km;
   b) the total radiated power Π;
   c) the radiation resistance R_r, if the antenna input current is $I = 0.5$ A (rms).

3. (15 points) For an antenna with a maximum linear dimension *D*, find the inner and outer boundaries of the Fresnel region so that the maximum phase error does not exceed $\pi/16$ rad.

4. (50 points) A capacitor-plate (top-hat) antenna has the following dimensions: radius of the capacitive plates $a = 0.01\lambda$ and overall dipole length $L = 0.02\lambda$, where λ is the free-space wavelength. Calculate the radiation resistance R_{rad} assuming constant current magnitude along the dipole. Simulate the antenna with *FEKO* at $f = 30$ MHz and compare the simulated R_{rad} with your calculation. What is the antenna reactance according to the *FEKO* simulation? Plot the current magnitude distribution along the dipole – is it really uniform?

Help with the simulation setup: (1) Use voltage-source excitation in your simulation. (2) Use at least 16 segments to discretize the dipole wire in order to obtain a good plot of your current distribution. (3) To avoid meshing errors, set the segment radius to be at least 20 times smaller than the segment length. (4) The capacitive plates will be discretized into triangular patches. You can enforce the largest length of the triangular edge to be used (using the options of local meshing). Set it to no more than half the radius of the plate. (5) The software has extensive help files and manuals. Please use them.