**ASSIGNMENT 6**

(due Thursday November 10, 2022)

1. `(20 points)` An incident plane wave is linearly polarized along a vector lying in the $x0z$ plane and tilted at an angle of $\theta_i = 60^\circ$ with respect to the $z$-axis. The receiving antenna is also linearly polarized and its polarization vector (as determined from its field if it were to transmit) lies in the $y0z$ plane at an angle of $\theta_a = 45^\circ$ from the $z$-axis. Find the polarization vectors of the incident wave $\hat{\boldsymbol{\rho}}_w$ and the receiving antenna $\hat{\boldsymbol{\rho}}_a$ and calculate the PLF. NOTE: Express the polarization vectors $\hat{\boldsymbol{\rho}}_w$ and $\hat{\boldsymbol{\rho}}_a$ in terms of rectangular components.

2. `(20 points)` The electric field radiated by a rectangular aperture (transmitting antenna), which is mounted on an infinite ground plane with the $z$ -axis perpendicular to the aperture, is given by

$$\mathbf{E}(r,\theta,\varphi) = (\hat{\boldsymbol{\theta}}\cos\varphi - \hat{\boldsymbol{\varphi}}\sin\varphi\cdot\cos\theta)\cdot f(r,\theta,\varphi)$$

in a global coordinate system. Find the PLF as a function of $(\theta,\varphi)$ if the receiving antenna is linearly polarized (in this same coordinate system) along the $y$-axis, i.e., $\hat{\boldsymbol{\rho}}_a = \hat{\mathbf{y}}$ .

**Hint**: Covert $\hat{\boldsymbol{\rho}}_a = \hat{\mathbf{y}}$ to spherical components.

3. `(20 points)` A transmitting antenna launches a circularly polarized (CP) wave, which travels in the $+z$ direction. This wave is received by an elliptically polarized antenna with its main beam being directed along $-z$. The polarization vector of the receiving antenna is measured in transmitting mode when its main beam is along $+z$ and it is given by

$$\hat{\boldsymbol{\rho}}_{\text{Rx}} = \left(2\hat{\mathbf{x}} - j\hat{\mathbf{y}}\right)/\sqrt{5}\ .$$

a) Find the PLF when the incident wave is right-hand (or CW) CP.
b) Find the PLF when the incident wave is left-hand (or CCW) CP.

4. `(40 points)` A small loop antenna has a radius of $a = 0.02\lambda$, where $\lambda$ corresponds to frequency of 30 MHz (in vacuum). The loop is made of tubing (perfect conductor), the cross-section of which has a radius $b = 2\cdot 10^{-4}\lambda$. Simulate the loop using *FEKO*. For a faster simulation, use a wire representation of the tubing.

- What is the input impedance $Z_{in} = R_{in} + jX_{in}$ of the loop? Using Eq. (3.33) in Lecture 3, derive the formula for the input resistance $R_{in}$ of a small loop. Use this formula to calculate analytically the loop's input resistance. Does the analytical value agree with the simulation result?
- Plot the magnitude of the loop current as a function of the position along the loop. Is the current distribution uniform?
- Plot the 3-D gain pattern for the total field. [Note: Activate legends in your plot in order to observe units and scales.] How does this pattern compare to the 3-D gain pattern of the small top-hat dipole in your previous simulations?
- What is the polarization of the loop and how does it compare to that of the small top-hat dipole?

  **Help with the simulation setup**: (1) Use voltage-source excitation in your simulation. (2) Use at least 16 wire segments to represent the loop in order to obtain a good plot of the current distribution. (3) Set the segment radius to match the prescribed tubing cross-section radius.