Lecture 7: Antenna Noise Temperature and System Signal-to-Noise Ratio

(Noise temperature. Antenna noise temperature. System noise temperature. Minimum detectable temperature. System signal-to-noise ratio.)

1. Noise Temperature of Bright Bodies

The performance of a telecommunication system depends on the signal-to-noise ratio (SNR) at the receiver's input. The electronic circuitry of the RF front end (amplifiers, mixers, etc.) has a significant contribution to the system noise. However, the antenna itself is sometimes a significant source of noise, too. The antenna noise can be divided into two types according to its physical source: noise due to the loss resistance of the antenna and noise, which the antenna picks up from the surrounding environment.

Any object whose temperature is above the absolute zero radiates EM energy. Thus, an antenna is surrounded by noise sources, which create noise power at the antenna terminals. Here, we are not concerned with man-made sources of noise, which are the subject of the EM interference (EMI) science. We are also not concerned with intentional sources of EM interference (jamming). We are concerned with natural sources of EM noise, which is thermal in nature, such as *sky noise* and *ground noise*.

The concept of antenna noise temperature is critical in understanding how the antenna contributes to the system noise in low-noise receiving systems such as radio-astronomy and radiometry. It is also important in understanding the relation between an object's temperature and the power generates at the receiving antenna terminals. This thermal power is the signal used in passive remote sensing (radiometry) and imaging. A radiometer can create temperature images of objects. Typically, the remote object's temperature is measured by comparison with the noise due to background sources and the receiver itself.

Every object (e.g., a resistor R) with a physical temperature above zero (0° K = –273° C) possesses heat energy. The ***noise power per unit bandwidth*** p_h is proportional to the object's temperature and is given by Nyquist's relation:

$$p_h = kT_P \text{, W/Hz} \tag{7.1}$$

where T_P is the physical temperature of the object in K (Kelvin degrees) and k

is Boltzmann's constant ($\approx 1.38\times10^{-23}$ J/K).

In the case of a resistor, this is the noise power, which can be measured at the resistor's terminals with a matched load. Thus, a resistor can serve as a noise generator. Often, we assume that heat energy is evenly distributed in the frequency band Δf . Then, the associated heat power in Δf is

$$P_h = kT_P \Delta f \text{ , W.} \tag{7.2}$$

The noise power radiated by the object depends not only on its physical temperature but also on the ability of its surface to let the heat leak out. This radiated heat power (or brightness power P_B) is associated with the so-called ***equivalent temperature*** or ***brightness temperature*** T_B of the body via the power-temperature relation in (7.2):

$$P_B = kT_B \Delta f \text{ , W.} \tag{7.3}$$

In general, the brightness temperature T_B is not the same as the physical temperature of the body T_P. The two temperatures are proportional:

$$T_B = (1-|\Gamma_s|^2)\cdot T_P = \varepsilon T_P \text{, K} \tag{7.4}$$

where

Γ_s is the reflection coefficient of the surface of the body; and

ε is what is called the ***emissivity*** of the body.

The brightness power P_B relates to the heat power P_h the same way as T_B relates to T_P, i.e., $P_B = \varepsilon P_h$.

2. Antenna Noise Temperature

The power radiated by the body P_B, when intercepted by an antenna, generates noise power P_A at its terminals. The equivalent temperature associated with the received power P_A at the antenna terminals is called the ***antenna temperature*** T_A of the object, where, again, $P_A = kT_A \Delta f$. Here, Δf is a bandwidth, which falls within the antenna bandwidth and is sufficiently narrow to ensure constant noise-power spectral density.

2.1. Antenna noise from bright background

Let us first assume that the entire antenna pattern (beam) "sees" a uniformly "bright" or "warm" object surrounding the antenna from all directions (see the figure below). To simplify matters, we also assume that the antenna is lossless, i.e., it has no loss resistance, and, therefore, it does not generate noise itself. Then, certain noise power can be measured at its terminals, which can be expressed as

$$P_A = kT_B\Delta f \text{ , W.} \tag{7.5}$$

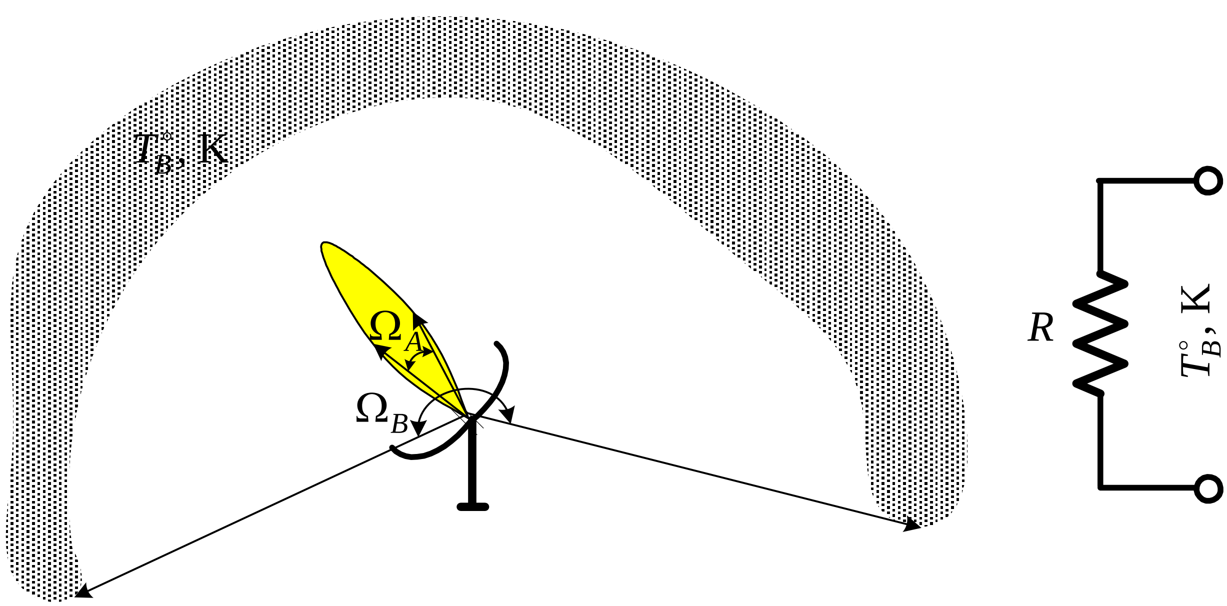
T_B is the ***brightness temperature of the object***, see (7.4), as observed at the antenna.

On the other hand, the antenna temperature is related to the measured noise power as

$$P_A = kT_A\Delta f \text{ .} \tag{7.6}$$

Thus, in this case (when the solid angle subtended by the noise source Ω_B is much larger than the antenna solid angle Ω_A, the object envelops the antenna from all directions), the antenna temperature T_A is equal to the object's temperature T_B (if the antenna is loss-free):

$$T_A = T_B, \ \text{ if } \Omega_A \ll \Omega_B. \tag{7.7}$$



2.2. Detecting large bright bodies (antenna incremental temperature)

The situation described above is of practical importance. When an antenna is pointed right at the night sky, its noise temperature is very low: $T_A = 3^\circ$ to 5° K at frequencies between 1 and 10 GHz. This is the microwave noise temperature of the night sky. The higher the elevation angle, the lower the night-sky temperature because of the lower physical temperature of the atmosphere toward zenith. The sky noise depends on the frequency. It depends on the time of the day, too. Closer to the horizon, it is mostly due to the thermal radiation from the Earth's surface and the atmosphere. Closer to the zenith, it is mostly due to cosmic rays from the sun, the moon and other bright sky objects, as well as the deep-space background temperature commonly referred to as the *cosmic microwave background* ($T_{\text{CMB}} \approx 2.725°$ K).[1] The latter is a left-over thermal effect from the very origin of the universe (the *big bang*).

An antenna may also be pointed toward the ground, e.g., when it is mounted on an airplane or a satellite. The noise temperature of the ground is much higher than that of the night sky because of its much higher physical temperature. The ground noise temperature is about 300° K and it varies during the day. The noise temperature at approximately zero elevation angle (horizon) is about 100° to 150° K.

When a single large bright body is in the antenna beam, (7.7) holds. In practice, however, the antenna temperature may include contributions from several large sources. The source under observation, although large relative to the antenna beam cross-section, may be superimposed on a background of certain temperature as well as the noise temperature due to the antenna losses, which we initially assumed zero. In order the antenna and its receiver to be able to discern a bright body while "sweeping" the background, this source has to put out more power than the noise power of its background, i.e., it has to be "brighter" than the background noise. Thus, in practice, to obtain the brightness temperature of a large object at the antenna terminals, the antenna temperature is measured with the beam on and off the target. The difference is the ***antenna incremental temperature*** ΔT_A. If the bright body is large enough to "fill in" the antenna beam completely, the difference between the background-noise antenna temperature and the temperature when the antenna solid angle is on the

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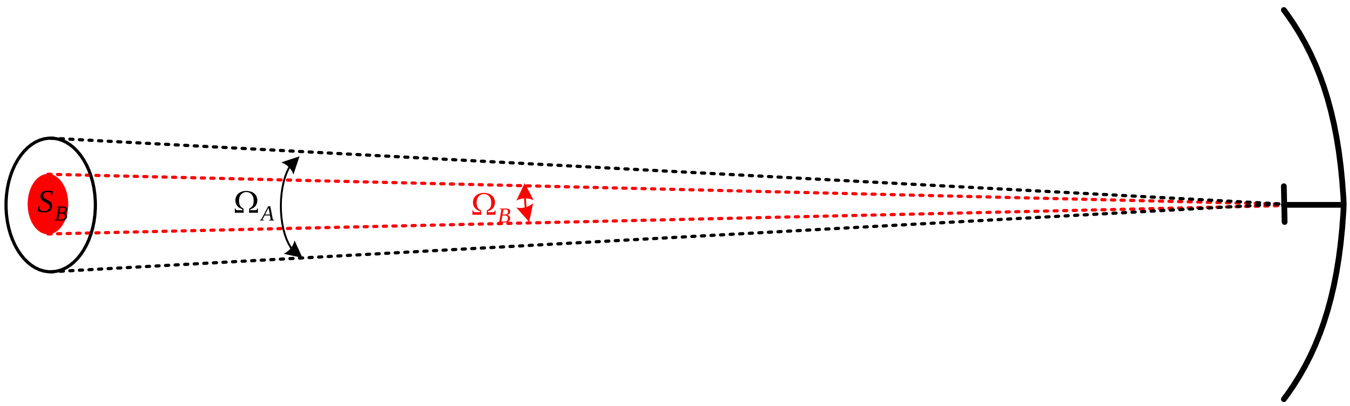
[1] C.T. Stelzried, A.J. Freiley, and M.S. Reid, *Low-noise Receiving Systems*. Artech, 2010.

object is equal to the object's brightness temperature,

$$\Delta T_A = T_B. \tag{7.8}$$

2.3. Antenna noise from small bright bodies

A different case arises in radiometry and radio astronomy. The bright object subtends such a small solid angle that it is well inside the antenna solid angle when the antenna is pointed at it: $\Omega_B \ll \Omega_A$.



To separate the power received from the bright body from the background noise, the difference in the antenna temperature ΔT_A is measured with the beam on and off the object. This time, ΔT_A is ***not equal*** to the bright body temperature T_B, as was the case of a large object. However, both temperatures are proportional. The relation is derived below.

The noise power intercepted by the antenna depends on the antenna effective aperture A_e and on the power density at the antenna's location created by the noise source W_B:

$$P_A = A_e \cdot W_B \text{, W.} \tag{7.9}$$

Assuming that the bright body radiates isotropically and expressing the effective area by the antenna solid angle, we obtain

$$P_A = \frac{\lambda^2}{\Omega_A} \cdot \frac{P_B}{4\pi R^2} \text{, W.} \tag{7.10}$$

The distance R between the noise source and the antenna is related to the effective area of the body S_B and the solid angle Ω_B it subtends as

$$R^2 = \frac{S_B}{\Omega_B}, \text{ m}^2 \tag{7.11}$$

Substituting (7.11) in (7.10) yields

$$\Rightarrow P_A\Omega_A = \frac{\lambda^2}{4\pi S_B} P_B\Omega_B. \tag{7.12}$$

Next, we notice that

$$\frac{\lambda^2}{4\pi S_B} = \frac{1}{G_B} = 1. \tag{7.13}$$

Here, G_B is the gain of the bright body (viewed as an antenna), which is unity because we assumed in (7.10) that the body radiates isotropically. In (7.13), we have used the relationship between gain and effective area; see (4.65) in Lecture 4); the effective area of the bright body being simply its cross-section S_B. Finally, substituting (7.13) in (7.12) leads to

$$P_A\Omega_A = P_B\Omega_B, \text{ if } \Omega_B \ll \Omega_A. \tag{7.14}$$

Equation (7.14) leads to the relation between the brightness temperature T_B of the bright object and the measured antenna incremental temperature ΔT_A:

$$\Delta T_A = \frac{\Omega_B}{\Omega_A} T_B \text{ K}, \ \Omega_B \ll \Omega_A. \tag{7.15}$$

For a large bright body, where $\Omega_B = \Omega_A$, (7.15) reduces to (7.8).

2.4. Source flux density from noise sources and noise PLF

The power at the antenna terminals P_A, which corresponds to the antenna incremental temperature ΔT_A, is defined by (7.6). In radio-astronomy and remote sensing, we use the ***flux density*** S of the noise source at the antenna (the effective area of which is A_e):

$$S = \frac{p_h}{A_e} = \frac{k\Delta T_A}{A_e}, \text{ Wm}^{-2}\text{Hz}^{-1}. \tag{7.16}$$

This way, the source flux density is a characteristic, which is independent of the antenna used to measure it. Notice that S is not the Poynting vector (power flux per unit area) but rather the spectral density of the Poynting vector (power flux per unit area per *hertz*). In radio-astronomy, the unit for flux density is *jansky*, 1 Jy = 10^{-26} $\mathrm{Wm^{-2}Hz^{-1}}$.[2]

From (7.16), we conclude that the measured incremental antenna temperature ΔT_A relates to the source flux density as

$$\Delta T_A = \frac{1}{k} A_e \cdot S \,. \tag{7.17}$$

This would be the case indeed if the antenna and the bright-body source were polarization matched. Since the bright-body source is a natural noise source, we cannot expect perfect match. In fact, an astronomical object is typically *unpolarized*, i.e., its polarization is random. Thus, about half of the bright-body flux density cannot be picked up by the receiving antenna, the polarization of which is fixed. For this reason, the relation in (7.17) is modified as

$$\Delta T_A = \frac{1}{2} \cdot \frac{A_e \cdot S}{k} . \tag{7.18}$$

The same correction factor should be inserted in (7.15), where the measured ΔT_A would actually correspond only to one-half of the noise temperature of the bright body:

$$\Delta T_A = \frac{1}{2} \frac{\Omega_B}{\Omega_A} T_B . \tag{7.19}$$

2.5. Antenna noise from a nonuniform noisy background

In the case of a small bright body (see previous subsection), we have tacitly assumed that the gain of the antenna is constant within the solid angle Ω_B subtended by the bright body. This is in accordance with the definition of the antenna solid angle Ω_A, which was used to obtain the ratio between ΔT_A and T_B. The solid-angle representation of the directivity of an antenna is actually quite accurate for high-directivity antennas, e.g., reflector antennas.

In general, however, the antenna gain may be strongly dependent on the observation angle (θ, φ). In this case, the noise signals arriving from different sectors of space have different contributions to the total antenna temperature. Those arriving from the direction of the maximum directivity contribute the most whereas those arriving from the direction of zero directivity will not

[2] Karl G. Jansky was the first one to use radio waves for astronomical observations.

contribute at all. The differential contribution from a sector of space of solid angle $d\Omega$ should, therefore, be weighed by the antenna normalized power pattern $\bar{F}(\theta,\varphi)$ in the respective direction:

$$dT_A = \bar{F}(\theta,\varphi)\cdot\frac{T_B(\theta,\varphi)d\Omega}{\Omega_A}. \quad (7.20)$$

The above expression can be understood by considering (7.15) where ΔT_A is replaced by a differential contribution dT_A to the antenna temperature from a bright body subtending a differential solid angle $\Omega_B \to d\Omega$. The total antenna noise power is then obtained as

$$T_A = \frac{1}{\Omega_A}\oiint_{4\pi}\bar{F}(\theta,\varphi)\cdot T_B(\theta,\varphi)d\Omega. \quad (7.21)$$

The expression in (7.21) is general and the previously discussed special cases are easily derived from it. For example, assume that the brightness temperature surrounding the antenna is the same in all directions, i.e., $T_B(\theta,\varphi) = T_{B0} = const$. Then,

$$T_A = \frac{T_{B0}}{\Omega_A}\cdot\underbrace{\oiint_{4\pi}\bar{F}(\theta,\varphi)d\Omega}_{\Omega_A} = T_{B0}. \quad (7.22)$$

The above situation was already addressed in equation (7.7).

Assume now that $T_B(\theta,\varphi) = const = T_{B0}$ but only inside a solid angle Ω_B, which is much smaller than the antenna solid angle Ω_A. Outside Ω_B, $T_B(\theta,\varphi)=0$. Since $\Omega_B \ll \Omega_A$, when the antenna is pointed at the noise source, its normalized power pattern within Ω_B is $\bar{F}(\theta,\varphi)\approx 1$. Then,

$$T_A = \frac{1}{\Omega_A}\oiint_{4\pi}\bar{F}(\theta,\varphi)\cdot T_B(\theta,\varphi)d\Omega = \frac{1}{\Omega_A}\iint_{\Omega_B} 1\cdot T_{B0}\cdot d\Omega = T_{B0}\frac{\Omega_B}{\Omega_A}. \quad (7.23)$$

This case was addressed in (7.15).

The antenna pattern strongly influences the antenna temperature. High-gain antennas (such as reflector systems), when pointed at elevation angles close to the zenith at night, have negligible noise level. However, if an antenna has significant side and back lobes, which are pointed toward the ground or the horizon, its noise power is much higher. The worst case for an antenna is when its main beam points towards the ground or the horizon, as is often the case with satellite or airborne antennas that are pointed toward the earth.

Example (modified from Kraus, p. 406): A circular reflector antenna of 500 m^2 effective aperture operating at $\lambda = 20$ cm is directed at the zenith. What is the total antenna temperature assuming the sky temperature close to zenith is equal to 10° K, while at the horizon it is 150° K? Take the ground temperature equal to 300° K and assume that one-half of the minor-lobe beam is in the back direction (toward the ground) and one-half is toward the horizon. The main beam efficiency (BE = Ω_M / Ω_A) is 0.7.

Such a large reflector antenna is highly directive and, therefore, its main beam "sees" only the sky around the zenith. The main beam efficiency is 70%. Thus, substituting in (7.23) where Ω_B is replaced by Ω_M, the noise contribution of the main beam is

$$T_A^{MB} = \frac{1}{\Omega_A}(10 \times \underbrace{0.7 \times \Omega_A}_{\Omega_M}) = 7 \text{, K.} \tag{7.24}$$

The contribution from the half back-lobe (which is a half of 30% of the antenna solid angle) directed toward ground is

$$T_A^{GBL} = \frac{1}{\Omega_A}\left(300 \times 0.15 \times \Omega_A\right) = 45 \text{, K.} \tag{7.25}$$

The contribution from the half back-lobe directed toward the horizon is

$$T_A^{HBL} = \frac{1}{\Omega_A}\left(150 \times 0.15 \times \Omega_A\right) = 22.5 \text{, K.} \tag{7.26}$$

The total antenna noise temperature is

$$T_A = T_A^{MB} + T_A^{GBL} + T_A^{HBL} = 74.5 \text{ K.} \tag{7.27}$$

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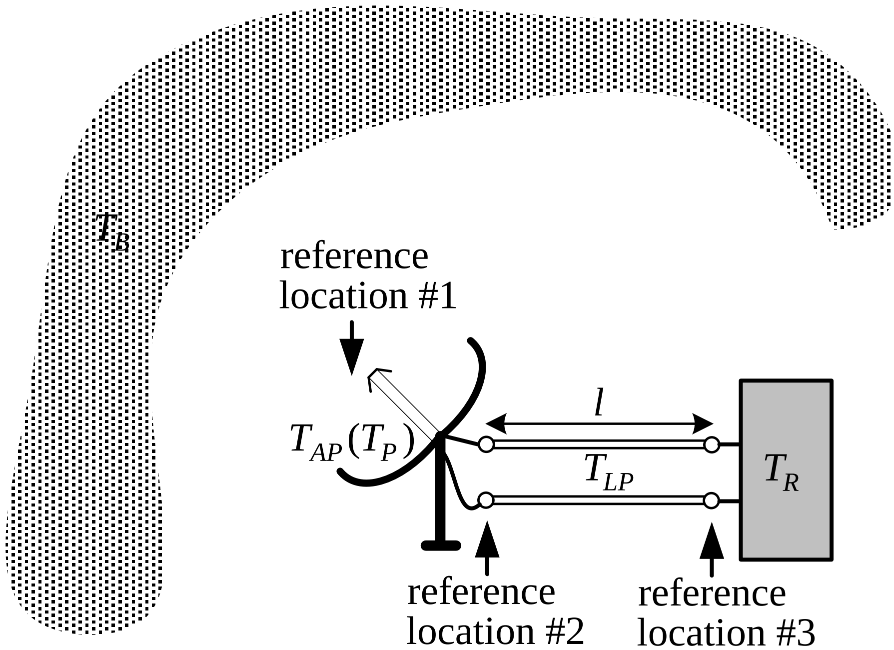
3. System Noise Temperature

The antenna is a part of a receiving system, which consists of several cascaded components: antenna, transmission line (or waveguide) assembly and receiver (see figure below). All these system components, the antenna included, have their contributions to the system noise. The system noise level is a critical factor in determining its sensitivity and SNR.



3.1. Noise Analysis of Cascaded Matched Two-port Networks[3]

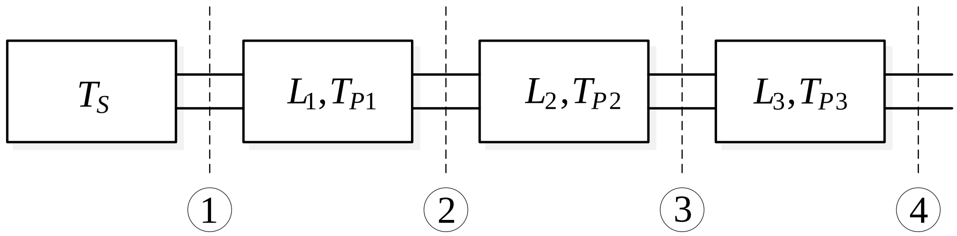
To understand the noise analysis of the radio receiver system, we must first review the basics of the noise analysis of cascaded two-port networks. For simplicity, we will assume that all networks are impedance matched, which is close to what is in fact happening in a realistic receiver system.

In the figure below (case (a)), a generic cascaded network is shown where the first component on the left is the noise source (e.g., the antenna picking up noise from the sky) with noise temperature T_S. The remaining two-port components are characterized by their physical temperatures T_{Pi} and by their *loss factors* (or *loss ratios*) L_i, $i = 1, 2\ldots$. In the case of a passive lossy two-port network (such as a waveguide or a transmission line), L is the inverse of the efficiency. In some analyses, the antenna can be viewed as a two-port network as well, such that its "input port" is its aperture receiving the noise signals from the environment and its output port is its connection to the transmission line.

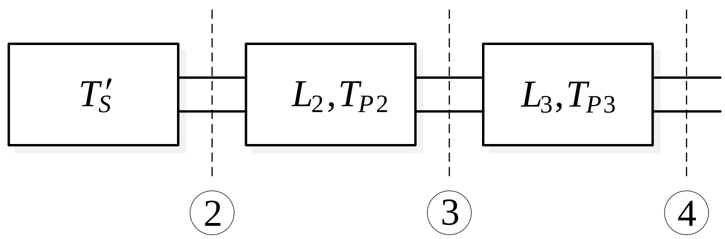
The efficiency is defined as the output-to-input power ratio $e = P_{\text{ou}} / P_{\text{in}}$ and $e \le 1$ for a passive two-port component. In contrast, $L = P_{\text{in}} / P_{\text{ou}}$ and $L \ge 1$. Thus, the loss ratio of an antenna as a two-port component is $L_A = e_A^{-1}$. In noise theory, any two-port component for which $L \ge 1$, i.e., it exhibits power loss, is referred to as "attenuator" although this component does not necessarily need to

[3] From T.Y. Otoshi, "Calculation of antenna system noise temperatures at different ports—revisited," *IPN Progress Report*, Aug. 15, 2002.

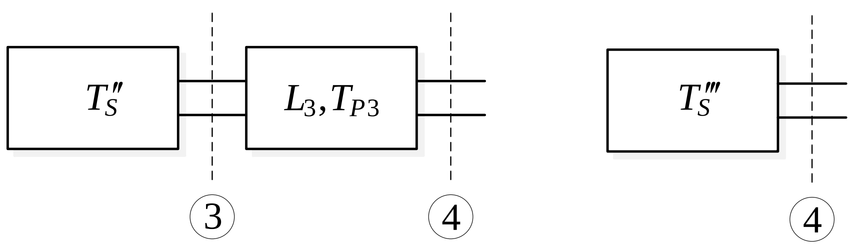
be an attenuator; it could be, for example, the entire antenna-plus-feed assembly. On the other hand, if $L<1$, we have a component which exhibits gain and it is referred to as an "amplifier". In this case, the efficiency is replaced by the gain G, which, just like the efficiency, is the output-to-input power ratio $P_{\text{ou}}/P_{\text{in}}$ but it is greater than 1. As with the efficiency, the relationship $L=G^{-1}$ holds.



(a) original network



(b) equivalent source noise temperature at location 2



(c) equivalent source noise temperatures at locations 3 and 4

Figure (a) above shows a cascaded network of 2-port components, each characterized by its loss factor L_i, $i=1,2,3$, and by its physical temperature T_{Pi}, $i=1,2,3$. The first component of the cascaded network is a noise source with temperature T_S. Figure (b) shows a network where an equivalent source of temperature T_S' replaces the original source plus its neighboring 2-port network (L_1, T_{P1}). The equivalent source T_S' at location 2 is

$$T_S' = L_1^{-1} T_S + (1 - L_1^{-1}) T_{P1}. \tag{7.28}$$

From (7.28), it is evident that in addition to the usual "attenuated" source noise-power term $L_1^{-1}T_S$, there is a contribution due to the physical temperature of the 1st two-port network. This contribution is referred to as the ***device equivalent noise temperature at its output***,

$$T_{D1}^{\text{ou}} = (1 - L_1^{-1})T_{P1}. \quad (7.29)$$

This contribution is entirely determined by the device physical temperature and its loss factor, i.e., it does not depend on the source.

To understand where (7.28) comes from, we can re-write it as

$$L_1 = \frac{T_S - T_{P1}}{T_S' - T_{P1}}. \quad (7.30)$$

This is indeed the ratio of input-to-output noise power for the 1st network. T_S represents the power traveling toward the device input while its own noise power, represented by T_{P1} travels away from it. Thus, the total power at the input is represented by $T_S - T_{P1}$.[4] At the same time, at the output, as per figure (b), the total noise power incident toward network #2 is given by T_S'. However, the portion that relates to the attenuation L_1 (i.e., the power at the network #1 input) does not include the intrinsic device noise power T_{P1}, which is always present at the device output regardless of whether there is a noise source at the input or not. Thus, T_{P1} has to be subtracted from T_S'.

Using the same methodology, we can find the equivalent source noise temperature T_S'' at location 3 as

$$T_S'' = L_2^{-1}T_S' + (1 - L_2^{-1})T_{P2} \quad (7.31)$$

where

$$T_{D2}^{\text{ou}} = (1 - L_2^{-1})T_{P2} \quad (7.32)$$

is the 2nd device equivalent noise temperature at its output.

We can repeat this step for the network location 4 where we obtain the equivalent source noise temperature T_S'''. In each case, in addition to the "attenuated" source power we have to add the respective network ***equivalent output device noise temperature***,

$$T_{Di}^{\text{ou}} = (1 - L_i^{-1})T_{Pi}, \; i = 1, 2, \ldots. \quad (7.33)$$

As an illustration of the general procedure, we show the the equivalent source noise temperature T_S''' at location 4:

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[4] Remember the expression $|a|^2 - |b|^2$ for the total power at the input of a microwave network where a and b are the incident and the scattered (outgoing) root-power waves, respectively. $|a|^2$ represents the incoming power whereas $|b|^2$ represents the outgoing power.

$$T_S''' = T_S(L_1L_2L_3)^{-1} + (1-L_1^{-1})T_{P1}(L_2L_3)^{-1} + (1-L_2^{-1})T_{P2}L_3^{-1} + (1-L_3^{-1})T_{P3}. \quad (7.34)$$

3.2. Transferring System Noise Temperature along Lossy Networks

The rule of transferring noise temperature from the output port of a lossy network to its input port (or *vice versa*) is simple:

$$T_{\text{in}} = LT_{\text{ou}} = T_{\text{ou}} / e \quad (7.35)$$

where e is the device efficiency (or gain). This rule, while simple, is not immediately obvious. A formal proof can be found in the Appendix of

B.L. Seidel and C.T. Stelzried, "A radiometric method for measuring the insertion loss of radome materials," *IEEE Trans. Microw. Theory Thech.*, vol. MTT-16, No. 9, Sep. 1968, pp. 625−628.

We can now define the ***equivalent noise temperature of a lossy component at its input*** (also known as ***equivalent input device noise temperature***) by substituting T_{Di}^{ou} from (7.33) as T_{ou} in (7.35):

$$T_{Di}^{\text{in}} = L_iT_{Di}^{\text{ou}} = (L_i - 1)T_{Pi}. \quad (7.36)$$

It is worth noting that (7.36) suggests that T_{Di}^{in} could be much larger than the physical temperature T_{Pi} if the device is very lossy, i.e., if $L_i \gg 1$ ($e_i \ll 1$).

Finally, we discuss the physical meaning of the ***equivalent input device noise temperature*** through an alternative way of deriving the relationship in (7.36). We omit the subscript i hereafter. Consider a noise source of temperature T_S at the device input. Its noise power is then $kT_S\Delta f$. To find the output noise power of the device, we add the two input contributions − that of the noise source and that due to the equivalent input device noise temperature, and then multiply the result by the device efficiency:

$$P_{\text{N,ou}} = e(kT_S\Delta f + kT_D^{\text{in}}\Delta f). \quad (7.37)$$

To find the relation between the equivalent input device noise temperature T_D^{in} and its physical temperature T_P, we consider the particular case when the temperature of the source T_S is equal to the physical temperature T_P of the device. In this case, the output noise power must be $P_{\text{N,ou}} = kT_P\Delta f$ because the whole system of the lossy device plus the source is at the physical temperature T_P. Substituting $T_S = T_P$ in (7.37) results in

$$P_{\text{N,ou}} = e(kT_P\Delta f + kT_D^{\text{in}}\Delta f) = kT_P\Delta f \quad (7.38)$$

which, when solved for T_D^{in}, produces (7.36). Note that we have not imposed any restrictions on the actual values of T_S and T_P but have only required that

T_D^{in} depends solely on T_P (i.e., it is independent of the noise source at the input) and that (7.37) holds in the special case of $T_S = T_P$.

3.3. <u>The atmosphere as an "attenuator"</u>

An illustration of the above concepts in noise analysis is the impact of the atmosphere on the sky noise, e.g., the cosmic microwave background ($T_{\text{CMB}} \approx$ 2.725° K). The atmosphere, depending on the time of the day and the weather conditions, exhibits loss, which we describe by the loss factor L_{atm}. L_{atm} can be calculated if we know the averaged attenuation constant in the atmosphere α_{atm} and its thickness H, e.g., $L_{\text{atm}} \approx \exp(2\alpha_{\text{atm}} H)$. This atmospheric "attenuator" lies between the cosmic microwave background noise source and the antenna. Therefore, the actual external noise temperature perceived by the antenna is

$$T_{\text{sky}} = L_{\text{atm}}^{-1} T_{\text{CMB}} + (1 - L_{\text{atm}}^{-1}) T_{\text{atm},P} \tag{7.39}$$

where $T_{\text{atm},P}$ is the physical temperature of the atmosphere, as per (7.28). The 1st term in (7.39) is the *space noise* whereas the 2nd one is the *atmospheric noise*. The impact of the atmosphere is often considered negligible. For a pencil-beam antenna pointed at the sky, $T_A = T_{\text{sky}}$.

3.4. <u>Antenna noise due to the antenna physical temperature</u>

If the antenna has losses, the noise temperature at its terminals includes not only the antenna temperature T_A due to the environment surrounding the antenna (the *external* antenna temperature) but also the antenna equivalent noise temperature T_{AP} due to its physical temperature T_P. Here, we note that the antenna acts as an "attenuator" in the cascaded network consisting of the external noise, the antenna, the waveguide and the receiver; see Figure on p. 10.

We first describe the antenna noise contribution at reference location #1, the antenna aperture, or, equivalently, its "input". Here, we view the antenna as a lossy two-port component. Its equivalent ***input*** noise temperature T_{AP} is

$$T_{AP} = \left(\frac{1}{e_A} - 1 \right) T_P = \frac{R_l}{R_r} T_P, \text{ K} \tag{7.40}$$

where e_A is the radiation efficiency $(0 \le e_A \le 1)$, R_l is the antenna loss resistance and R_r is its radiation resistance. Eq. (7.40) is essentially an application of Eq. (7.36) to the case of an antenna. It describes the thermal noise contribution of the antenna due to its physical temperature T_P referred to its "input" (the antenna aperture). T_{AP} must be added to T_A in order to obtain the system operating noise temperature at location #1. In fact, additional terms exist due to the noise contributions of the lossy TL (or waveguide) and the receiver electronics.

3.5. Noise due to the physical temperature of the transmission line

We now consider the transmission line (TL) as a source of noise when it has conduction losses. In a manner analogous to the one applied to the antenna, the TL is considered as a two-port "attenuator". Thus, its noise contribution at the antenna terminals (the input to the TL or reference location #2) is

$$T_{L2} = \left(\frac{1}{e_L} - 1 \right) T_{LP}, \text{ K.} \qquad (7.41)$$

Here, $e_L = e^{-2\alpha l}$ is the ***line thermal efficiency*** $(0 \le e_L \le 1)$, T_{LP} is the physical temperature of the TL, α (Np/m) is the attenuation constant of the TL, and l is its length.

To transfer the TL noise contribution to the reference location #1, we use (7.35) which leads to

$$T_{L1} = \frac{T_{L2}}{e_A} = \frac{1}{e_A} \left(\frac{1}{e_L} - 1 \right) T_{LP}. \qquad (7.42)$$

Together with T_{AP}, T_{L1} must be added to T_A in order to obtain the system operating noise temperature at location #1.

3.6. System noise referred to the antenna aperture (location #1)

The system temperature referred to the antenna aperture includes the contributions of the antenna (external noise temperature plus equivalent input antenna thermal noise temperature), the transmission line and the receiver as

$$T_{sys}^A = \underbrace{T_A}_{\text{antenna external}} + \underbrace{T_P \left(\frac{1}{e_A} - 1 \right)}_{T_{AP}\text{, antenna internal}} + \underbrace{\frac{1}{e_A} T_{LP} \left(\frac{1}{e_L} - 1 \right)}_{\text{TL internal}} + \underbrace{\frac{1}{e_A e_L} T_R}_{\text{receiver}}. \qquad (7.43)$$

Here, T_A is the external temperature that corresponds to the antenna temperature provided the antenna is loss-free, as discussed in Section 2. T_R is the receiver noise temperature (at its input, reference location #3). It is given by

$$T_R = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots, \text{ K.} \tag{7.44}$$

Here,

T_1 is the noise temperature of the first amplifying stage;
G_1 is the gain of the first amplifying stage ($G_1 = L_1^{-1}$, see (7.35));
T_2 is the noise temperature of the second amplifying stage;
G_2 is the gain of the second amplifying stage ($G_2 = L_2^{-1}$).

Notice that T_R is divided by the efficiencies e_L and e_A in order to refer it to the TL input (location #2) and on to the antenna aperture (location #1); see (7.35).

3.7. System noise referred to the antenna terminals (TL input, location #2)

The reference location is changed by considering the efficiency of the antenna. As per (7.35), we have

$$T_{sys}^{TL} = T_{sys}^{A} \cdot e_A \tag{7.45}$$

since T_{sys}^{TL} is the system noise temperature at the antenna "output" and T_{sys}^{A} is that at its "input". Substituting (7.43) into (7.45) produces

$$T_{sys}^{TL} = \underbrace{T_A e_A}_{\substack{\text{antenna}\\ \text{external}}} + \underbrace{T_P (1 - e_A)}_{\substack{\text{antenna}\\ \text{internal}}} + \underbrace{T_{LP} \left(\frac{1}{e_L} - 1 \right)}_{\text{TL internal}} + \underbrace{\frac{1}{e_L} T_R}_{\text{receiver}} . \tag{7.46}$$

3.8. System noise referred to the receiver input (location #3)

The reference location is changed once again by considering the efficiency of the TL:

$$T_{sys}^{R} = T_{sys}^{TL} \cdot e_L . \tag{7.47}$$

Therefore,

$$T_{sys}^{R} = \underbrace{T_A e_A e_L}_{\substack{\text{antenna}\\ \text{external}}} + \underbrace{T_P (1 - e_A) e_L}_{\substack{\text{antenna}\\ \text{internal}}} + \underbrace{T_{LP} (1 - e_L)}_{\text{TL}} + \underbrace{T_R}_{\text{receiver}}, \text{ K.} \tag{7.48}$$

Example (from Kraus, p. 410, modified): A receiver has an antenna with an external noise temperature 50° K, a physical temperature of 300° K, and an efficiency of 99%. Its transmission line has a physical temperature of 300° K and an efficiency of 90%. The first three stages of the receiver all have 80° K noise temperature and 13 dB gain (13 dB is about 20 times the power). Find the system temperature at: (a) the antenna aperture, (b) the antenna terminals, and (c) the receiver input.

The receiver noise temperature is

$$T_R = 80 + \frac{80}{20} + \frac{80}{20^2} = 84.2 \text{ °K.} \tag{7.49}$$

(a) Then, the system temperature at the antenna aperture is

$$\begin{aligned} T_{sys}^A &= T_A + T_P\left(\frac{1}{e_A} - 1\right) + \frac{1}{e_A} T_{LP}\left(\frac{1}{e_L} - 1\right) + \frac{1}{e_A e_L} T_R, \\ T_{sys}^A &= 50 + 300\left(\frac{1}{0.99} - 1\right) + \frac{300}{0.99}\left(\frac{1}{0.9} - 1\right) + \frac{84.2}{0.99 \cdot 0.9} \approx 181.2009 \text{ K.} \end{aligned} \tag{7.50}$$

(b) The system temperature at the antenna terminals is

$$T_{sys}^{TL} = T_{sys}^A \cdot e_A \approx 181.2009 \cdot 0.99 \approx 180.3889 \text{ °K.}$$

(c) The system temperature at the receiver input is

$$T_{sys}^R = T_{sys}^{TL} \cdot e_L = 180.3889 \cdot 0.9 \approx 162.35 \text{ °K.}$$

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4. Minimum Detectable Temperature (Sensitivity) of the System

The minimum detectable temperature, or sensitivity, of a receiving system $\Delta T_{\min}$ is the *RMS* noise temperature of the system ΔT_{rms}, which, when referred to the antenna aperture (reference location #1), is

$$\Delta T_{\min} = \Delta T_{\text{rms}} = \frac{k' T_{sys}^A}{\sqrt{\Delta f \cdot \tau}}, \tag{7.51}$$

where

k' is a system constant (commensurate with unity), dimensionless;
Δf is the pre-detection bandwidth of the receiver, Hz;
τ is the post-detection time constant, s.

The post-detection time constant τ is mostly determined by the averaging (or the observation) time. As per the Nyquist–Shannon sampling theorem, the temporal length of the observation determines the minimum frequency bandwidth as $\Delta f_{\min} = 1/(2T_{\max})$. If we set $T_{\max} = \tau$ and $\Delta f_{\min} = \Delta f$, we see that the denominator in (7.51) is $\sqrt{\Delta f \cdot \tau} = 1/\sqrt{2}$. This explains why the system constant k' is commensurate with unity.

The RMS noise temperature ΔT_{rms} is determined experimentally by pointing the antenna at a uniform bright object and recording the signal for a sufficiently long time. Assume the output of the receiver is in the form of real-positive numbers proportional to the received noise power. Modern receivers are digital and their output is actually in the form of integers. Then, the *RMS* deviation D_{rms} of the numbers produced by the receiver represents (is proportional to) the *RMS* noise temperature at the receiver:

$$D_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(a_n - a_{av})^2} = \kappa \cdot \Delta T_{\text{rms}}^R \text{ where } a_{av} = \frac{1}{N}\sum_{n=1}^{N} a_n . \tag{7.52}$$

Here, κ is a known constant converting the integers to noise-temperature values. ΔT_{rms} (at reference location #1) can be obtained from ΔT_{rms}^R by

$$\Delta T_{\text{rms}} = \frac{\Delta T_{\text{rms}}^R}{e_A e_L} = \Delta T_{\min} . \tag{7.53}$$

This is the *sensitivity* of the system in terms of noise temperature.

In order a source to be detected, it has to create an incremental antenna temperature ΔT_A which exceeds $\Delta T_{\min}$, $\Delta T_A > \Delta T_{\min}$. The *minimum detectable power* $P_{\min}$ is thus

$$P_{\min} = 0.5 A_e p_{\min} = k \Delta T_{\min} \Delta f \tag{7.54}$$

where A_e is the effective antenna area, $p_{\min}$ is the power-flux density (magnitude of Poynting vector) due to the source at the location of the antenna, and the factor of 0.5 accounts for the randomness of the wave polarization. It follows that the minimum detectable power-flux density is

$$p_{\min} = \frac{2k\Delta T_{\min}\Delta f}{A_e} . \tag{7.55}$$

The signal-to-noise ratio (SNR) for a signal source of incremental antenna temperature ΔT_A is given by

$$SNR = \frac{\Delta T_A}{\Delta T_{\min}} . \tag{7.56}$$

This SNR is used in radio-astronomy and remote sensing.

In the previous example, we found that the system temperature at the antenna aperture is $T_{sys}^A \approx 181.2009$ K. Assume that the receiver bandwidth is $\Delta f = 100$ Hz, that the system constant is $k' = 1$ and that the post-detection constant is $\tau = 1$ s. Find the minimum detectable power at the antenna aperture $P_{\min}$.

$$P_{\min} = k\Delta T_{\min}\Delta f = k\Delta f \cdot \frac{k' T_{sys}^A}{\sqrt{\Delta f \cdot \tau}} = k\sqrt{\frac{\Delta f}{\tau}} k' T_{sys}^A$$

$$\approx 1.38 \cdot 10^{-23} \cdot \sqrt{100} \cdot 181.2009 \approx \underline{\underline{2.5 \cdot 10^{-20}\ \text{W}}}$$

5. System Signal-to-Noise Ratio (SNR) in Communication Links

The system noise power at the antenna terminals (location #2) is

$$P_N = k T_{sys}^{TL} \Delta f_r \text{, W.} \tag{7.57}$$

Here, Δf_r is the bandwidth of the receiver and $T_{sys}^{TL} = e_A T_{sys}^A$. From Friis' transmission equation, we can express the received power at the antenna terminals as

$$P_r = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)\text{PLF}\left(\frac{\lambda}{4\pi R}\right)^2 G_t(\theta_t, \varphi_t) G_r(\theta_r, \varphi_r) \cdot P_t \,. \tag{7.58}$$

Finally, the SNR becomes

$$SNR = \frac{P_r}{P_N} = \frac{(1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)\text{PLF}\left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r \cdot P_t}{k T_{sys}^{TL} \Delta f} \,. \tag{7.59}$$

The above equation is fundamental in the design of telecommunication systems. More specifically, if the SNR necessary for the adequate operation of the receiver is known, Eq. (7.59) allows for determining the maximum range over which the communication link is stable.