Gain of Pyramidal Horn Antennas

EXERCISE OBJECTIVE

When you have completed this exercise, you will be familiar with the characteristics of pyramidal horn antennas and with the techniques used to calculate and measure their gain.

DISCUSSION

Free-space propagation loss

The power received by an antenna decreases as it is moved away from the transmitting antenna. In free space, the received signal power is inversely proportional to the square of the distance that separates the transmitting and receiving antennas. The power loss due to the separation between the antennas is called the **free-space propagation loss L_F**. The mathematical expression for determining the free-space propagation loss is

$$L_{F(dB)} = 10 \log\left(\frac{4\pi r}{\lambda}\right)^2 = 20 \log\frac{4\pi r}{\lambda} \qquad (1)$$

where r is the distance between the antennas
λ is the wavelength in free-space (in same units as r)

For a given wavelength, Equation (1) shows that L_F depends only on the distance between the antennas. This relationship can be determined experimentally by transmitting a signal from one antenna and measuring the power received at another antenna for different separations. However, since the antennas used generally have directive properties, the same orientation must be kept between them as the experiment is performed. If the different antenna separations are known, the attenuation of the received signal power obtained at one distance relative to that obtained at another distance can easily be calculated with the use of Equation (2).

$$A_{dB} = 20 \log\frac{r_2}{r_1} \qquad (2)$$

where A is the attenuation in dB
r_1 and r_2 are the first and second distances, respectively.

To characterize numerically the directional properties of antennas, the concept of directivity or directive gain is usually used. As seen in Exercise 1-2, directivity is the maximum radiation intensity in a given direction relative to the average radiation intensity (i.e. relative to the radiation intensity of an isotropic antenna transmitting the same total power). For a lossless antenna, the antenna gain (directive gain) is the same as the directivity.

Antenna gain measurements

There are different methods for measuring the gain of an antenna. The simplest, which is called the **reference antenna method**, the **comparison method**, or the **substitution method**, consists in comparing the power received by a reference antenna P_{Ref} to the power received by the antenna under test P_{Test}. The gain of the unknown antenna is given by Equation (3).

$$G_{Test} = \frac{P_{Test}}{P_{Ref}} G_{Ref} \tag{3}$$

Equation (4) expresses the same relation in dB.

$$G_{Test(dB)} = P_{Test(dB)} - P_{Ref(dB)} + G_{Ref(dB)} \tag{4}$$

Before using the substitution method, the reference antenna must be calibrated. One way to do this is to use two identical antennas. Once the transmitted and received powers have been measured, the gain can be calculated using Equation (5).

$$G = \frac{4\pi r}{\lambda} \sqrt{\frac{P_{Rec}}{P_o}} \tag{5}$$

where G is the gain
r is the distance between the antennas
P_{Rec} and P_o are the received and transmitted powers, respectively
λ is the wavelength in free space (in same units as r).

Types of horn antennas

Horn antennas provide a smooth transition for electromagnetic waves between the waveguide and free space. Horn antennas are made in a variety of shapes, depending on the gain, radiation pattern, and impedance desired. Figure 1-26 shows some common types.

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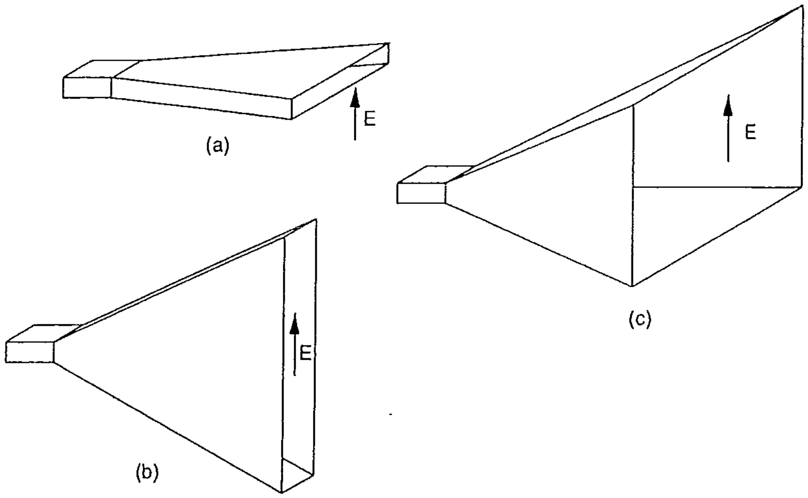


(a) H-plane sectoral horn, (b) E-plane sectoral horn, (c) pyramidal horn

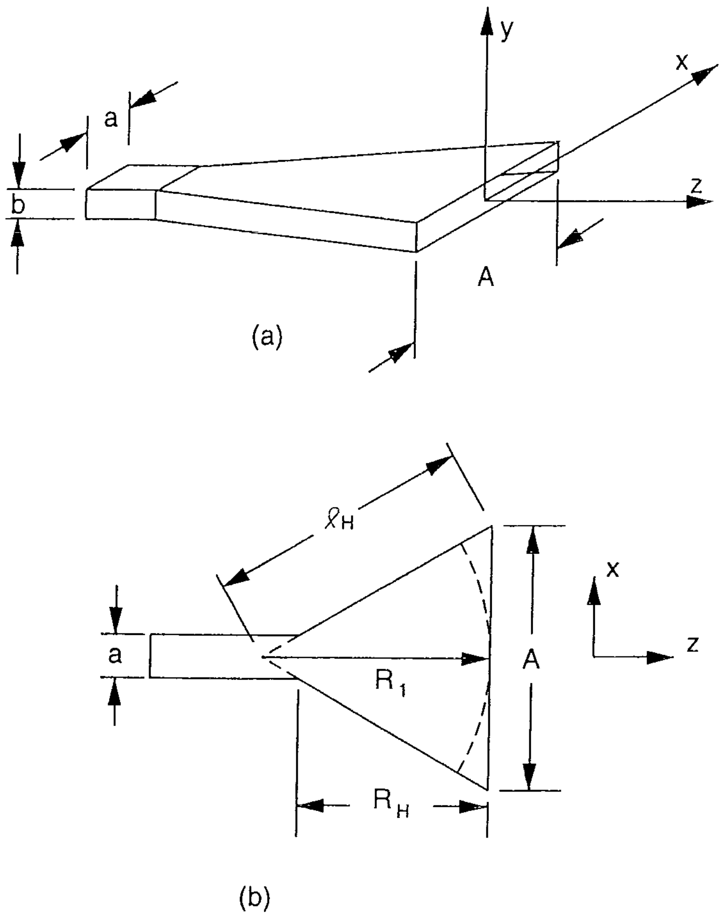
Figure 1-26. Rectangular-waveguide horn antennas

The pyramidal horn is often used as a reference in antenna gain measurements since its gain can be calculated accurately from its physical dimensions. Sectoral horns are flared in only one plane, and are therefore special cases of pyramidal horns. Other types of horn antennas exist in addition to the antennas shown in the figure. Some, such as conical horn antennas, are used with round waveguides.

The characteristics of sectoral horn antennas are closely related to those of pyramidal horn antennas. By studying sectoral horn antennas, it is possible to develop a method of approximating the gain of a pyramidal horn antenna from its physical dimensions.

H-plane sectoral horn antennas

The geometry of an H-plane sectoral antenna is shown in Figure 1-27.



(a) geometry, (b) cross section through the H plane

Figure 1-27. H-plane sectoral antenna

The following relationships apply to the physical dimensions:

$$\ell_H^2 = R_1^2 + \left(\frac{A}{2}\right)^2 \tag{6}$$

$$R_1 = A\sqrt{\left(\frac{\ell_H}{A}\right)^2 - \frac{1}{4}} \tag{7}$$

$$\frac{A}{R_1} = \frac{A - a}{R_H} \tag{8}$$

$$R_H = (A - a)\,\frac{R_1}{A} \tag{9}$$

By replacing R_1 by its value in Equation (7), we obtain

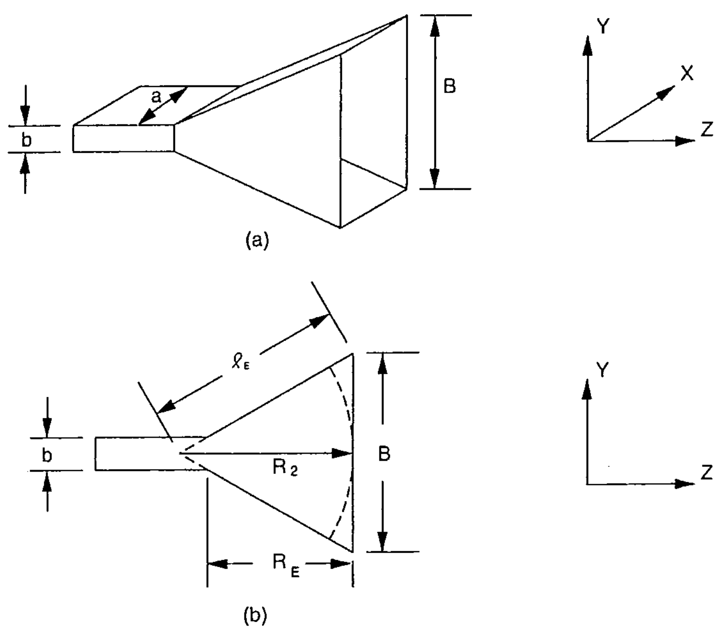
$$R_H = (A - a)\sqrt{\left(\frac{\ell_H}{A}\right)^2 - \frac{1}{4}} \tag{10}$$

It can be shown that, for an optimal horn, that is, a horn having the maximum gain for its dimensions,

$$A = \sqrt{3\lambda R_1} \tag{11}$$

E-plane sectoral horn antennas

The geometry of an E-plane sectoral antenna is shown in Figure 1-28.



(a) geometry, (b) cross section through the E plane

Figure 1-28. E-plane sectoral antenna

The following relationships apply to the physical dimensions:

$$\ell_E^2 = R_2^2 + \left(\frac{B}{2}\right)^2 \tag{12}$$

$$R_E = (B - b)\sqrt{\left(\frac{\ell_E}{B}\right)^2 - \frac{1}{4}} \tag{13}$$

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For an optimal horn, that is, a horn having the maximum gain for its dimensions,

$$B = \sqrt{2\lambda R_2} \qquad (14)$$

Pyramidal horn antennas

The pyramidal horn antenna is one of the most popular antenna forms. Its geometry is illustrated in Figure 1-29.

(a) geometry, (b) cross section through H plane, (c) cross section through E plane

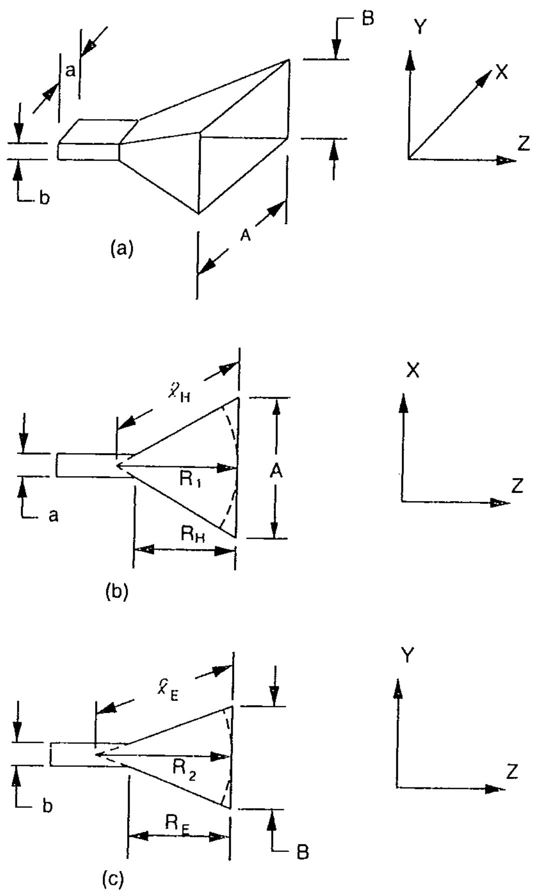

Figure 1-29. Pyramidal horn antenna

A waveguide can propagate an infinite number of different types or modes of electromagnetic waves. Each mode has its own electrical and magnetic field configurations. If the opening angle of the flare of a pyramidal horn antenna is small enough, only the dominant mode of the waveguide will be significant.

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The field lines of the dominant mode will expand into a cylindrical form for a sectoral horn antenna, or into a spherical form for a pyramidal horn antenna. This is shown in Figure 1-30.

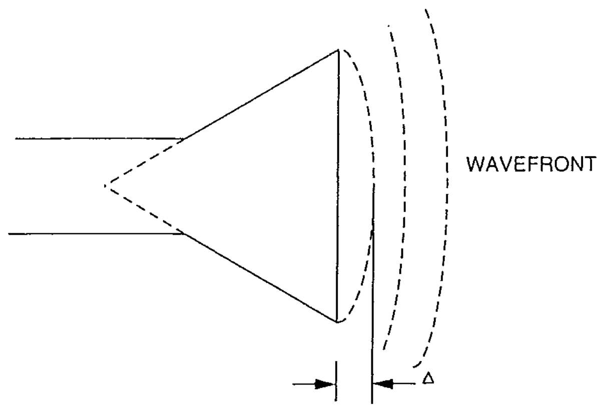

Figure 1-30. Phase error (Δ) due to curvature of the wavefront in a horn antenna

As shown in the figure, the wavefront is not flat, but is curved. This introduces phase errors which must be taken into account in analysing the antenna characteristics. The phase errors are described by the normalized path errors s and t.

$$s = \frac{\Delta_E}{\lambda} = \frac{B^2}{8\lambda \ell_E} \tag{15}$$

$$t = \frac{\Delta_H}{\lambda} = \frac{A^2}{8\lambda \ell_H} \tag{16}$$

where s and t are the normalized path errors
λ is the wavelength
A, B, ℓ_E and ℓ_H are dimensions shown in Figure 1-29.

The approximate gain of a pyramidal horn antenna can be calculated using Equation (17).

$$G = \frac{32}{\pi}\left(\frac{A}{\lambda}\right)\left(\frac{B}{\lambda}\right) L_E \, L_H \tag{17}$$

where L_E and L_H represent the losses due to the phase error caused by the flare.

Expressed in dB, this equation is

$$G_{(dB)} = 10.08 - 10 \log_{10}\left[\left(\frac{A}{\lambda}\right)\left(\frac{B}{\lambda}\right)\right] - L_{E(dB)} - L_{H(dB)} \tag{18}$$

The values of $L_{E(dB)}$ and $L_{H(dB)}$ can be found by first calculating the values of s and t using Equations (15) and (16), then reading the values of $L_{E(dB)}$ and $L_{H(dB)}$ from Figure 1-31.

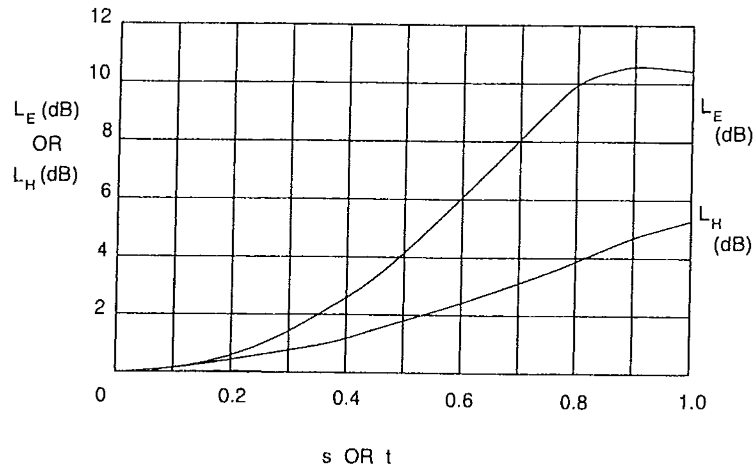

Figure 1-31. Loss factors for E- and H-plane flares.

Procedure Summary

In this exercise you will observe the power loss, called the free-space propagation loss, due to the separation between two antennas. You will study the characteristics of a pyramidal horn antenna, especially its half-power beamwidth, its front-to-back ratio, its gain and its effective area. To calculate the gain of an antenna you will first calibrate a large horn antenna and then use this reference with the substitution method to evaluate the gain of a small horn antenna.

PROCEDURE

Setting up the equipment

☐ 1. The main elements of the Antenna Training and Measuring System, that is the Data Acquisition Interface/Power Supply, the RF Generator, the Antenna Positioner and the computer, must be properly set up before beginning this exercise. Refer to Section 4 of the User Manual for setting up the Antenna Training and Measuring System, if this has not already been done.

☐ 2. Place an antenna mast with locking ring on the transmission support. Couple a large horn onto the waveguide-to-coax adapter as seen in Exercise 1-2. Using the plastic holder, install the antenna on the mast with the aperture oriented in the H plane (refer to Figure 1-32). Connect the adapter to the 10 GHz OSCILLATOR OUTPUT of the RF Generator using the long SMA cable.

☐ 3. Place the other antenna mast with locking ring on the sliding support of the Antenna Positioner. Connect the second large horn to the waveguide-to-coax adapter. Install the antenna on the mast.

Using the sliding support, ensure that the aperture of your antenna is in line with the rotation centre of the Antenna Positioner and oriented to rotate in the H plane, as shown in Figure 1-32.

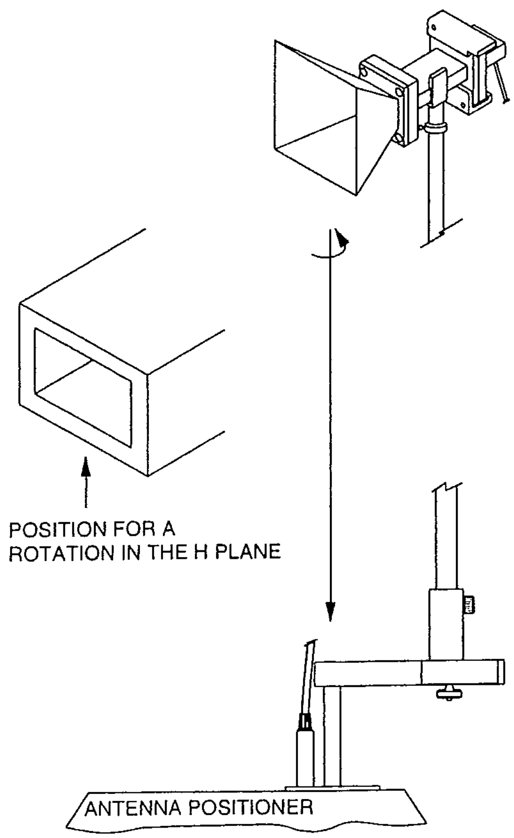

Figure 1-32. Set-up of the receiving antenna

Connect the receiving antenna to the RF input on top of the Antenna Positioner using the intermediate SMA cable.

☐ 4. Position the antennas a distance of r = 80 cm apart. Adjust them so that they are at the same height and directly facing each other.

☐ 5. Make the following adjustments:

On the RF Generator

10 GHz OSCILLATOR MODE .... 1 kHz
10 GHz OSCILLATOR RF POWER .... OFF
1 GHz OSCILLATOR RF POWER .... OFF

Power up the RF Generator and the Power Supply.

Turn on the computer and start the LVDAM-ANT software.

Propagation loss

☐ 6. Knowing that you have a distance r_1 between two antennas at the beginning of your experiment, use Equation (2) to calculate the attenuation of the received signal power if you move them apart to a distance of $r_2 = 2\,r_1$.

$$A = 20 \log \frac{r_2}{r_1} \text{ (dB)}$$

$$= \_\_\_\_\_\_\_\_ \text{ dB}$$

☐ 7. Set the 10 GHz OSCILLATOR RF POWER switch on the RF Generator to the ON position.

CAUTION!

For your own safety, never look directly into the horn antenna while the RF POWER switch is ON.

Use the Attenuation control to optimize the acquisition of the radiation pattern.

☐ 8. Start the acquisition and store the radiation pattern in the antenna1 data box, be sure to select the correct plane (E or H).

☐ 9. Position your antennas 1.6 m apart. Do not change the attenuation level you set in the first acquisition. Plot a new radiation pattern and store it in the antenna2 data box.

Compare your acquisitions. You should observe a difference in the signal levels of the two patterns. Does this confirm your previous calculation?

Print both patterns to obtain a more convenient graph for your comparison (do not forget to save your patterns before printing).

HPBW, side lobes and gain

☐ 10. Your antennas are still 1.6 m apart, oriented in the H plane and facing each other. Using the Attenuation control, optimize reception of the signal, then do another acquisition. Store this pattern in the antenna3 data box and set its MSP to 0°.

☐ 11. Using the second metal post on the plastic holder, turn both antennas on their sides; they are now oriented in the E plane. Perform a new acquisition. Store this radiation pattern in the antenna3 data box and set its MSP to 0°.

You now have the radiation patterns of the E and H planes of the horn antenna. Save the antenna3 data, then print them in this 2-D configuration.

Using the E-H and 3-D options, observe the spatial representation of these patterns.

Plot these two patterns.

☐ 12. Evaluate the half-power beamwidth of the E and H planes of the horn antenna.

$HPBW_E$ = ______° $HPBW_H$ = ______°

Plot the E-plane and the H-plane 2-D polar patterns together with the tables showing the values of $HPBW_E$ and $HPBW_H$.

☐ 13. Evaluate the front-to-back (F/B) ratio of the antenna's E plane.

$F/B_{E(dB)}$ = Main lobe (dB) - Back lobe (dB)

$F/B_{E(dB)}$ = ______ dB = ______ (ratio)

☐ 14. Calculate the gain of the large pyramidal horn at 10.5 GHz, knowing that it has the following dimensions:

ℓ_H = 11 cm ℓ_E = 9.4 cm

Measure the inside dimensions of the horn aperture.

A = ______ cm B = ______ cm

Calculate the wavelength at 10.52 GHz

λ = ______ m = ______ cm

You can now calculate

$$s = \frac{B^2}{8\lambda \ell_E} = \_\_\_\_\_\_$$

and from Figure 1-31 you obtain: $L_{E\,dB}$ = ______

$$t = \frac{A^2}{8\lambda \ell_H} = \_\_\_\_\_\_$$

and from Figure 1-31 you obtain: $L_{H\,dB}$ = ______

Finally, using Equation (18), calculate the gain of the antenna.

$$G_{dB} = 10.08 + 10 \log_{10}\left[\left(\frac{A}{\lambda}\right) \times \left(\frac{B}{\lambda}\right)\right] - L_{E(dB)} - L_{H(dB)} \text{ (dB)}$$

G_{dB} = ______ dB

☐ 15. Knowing the half-power beamwidth of the large horn antenna in the E and H planes, you can calculate an approximate value for its actual gain from the following formula, seen in Exercise 1-2:

$$D \approx G = \frac{26000}{HPBW_E \cdot HPBW_H}$$

G = ______

$G_{dB} = 10 \log G$ = ______ dB

☐ 16. You have seen that the actual gain of an antenna can be evaluated using Equation (5).

$$G = \frac{4\pi r}{\lambda}\sqrt{\frac{P_{Rec}}{P_o}}$$

where r (the antenna separation) and λ should be in the same units.

To calculate this gain you will use relative values. The following procedures will allow you to determine the received (P_{Rec}) and the transmitted (P_o) powers.

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In order to accurately evaluate the antenna gains in Step 16 and 17, screw the 10 dB attenuator onto the RF input on top of the Antenna Positioner and connect the SMA cable to the attenuator.

a. Remove both horn antennas from their masts and disconnect them from the waveguide-to-coax adapters.
b. Connect the adapters together, as shown in Figure 1-33.
c. On the RF Generator turn the RF POWER ON.
d. Optimize the signal using the Attenuation control.
e. Note the power of the received signal.

P_o: ______ dB

f. Turn the RF POWER OFF, disconnect the two adapters and once again set up the pyramidal horn antennas on their masts so they are 1 m apart, **directly** facing each other.

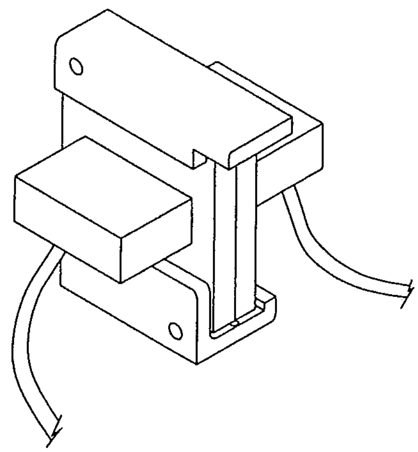

Figure 1-33. Connecting the waveguide-to-coax adapters together.

Note: *Refer to Step 13 of Exercise 1-2 to accurately evaluate the maximum signal level.*

g. Turn the RF POWER ON. Do not modify the attenuation level.
h. Record the following values:

P_{Rec}: ______ dB

Since your values are in dB, Equation (5) must be changed to allow for the use of this unit. Using this new formula, calculate the actual gain of your antenna.

$$G = 10\ \log_{10} 4\pi r - 10\ \log_{10}\lambda + 0.5\ (P_{Rec} - P_o)\ (\text{dB})$$

G = ______ dB

Compare the actual gain of the pyramidal horn antenna with your first two results (Steps 14 and 15).

☐ 17. Using your last result you can calculate the gain of a small horn antenna using the substitution method (the large horn becomes the reference antenna):

a. The large horn antennas are still 1 m apart and facing each other. Use the Attenuation control to optimize reception of the signal and note the power received.

P_{Ref}: ______ dB

b. Remove the receiving antenna from the mast and replace the large horn with the small horn antenna. Install this new set-up on the mast. DO NOT change the attenuation level. Note the power received.

P_{Test}: ______

Note: *Make sure you record the* ***maximum*** *signal level in steps a and b.*

Using Equation (4) and the real gain of the large horn as the reference gain (G_{Ref}), calculate the gain (G_{Test}) of the small horn antenna.

$$G_{Test} = P_{Test} + G_{Ref} - P_{Ref} = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ \text{ (dB)}$$

☐ 18. To complete your familiarization with the horn antenna, estimate the effective area A_e of the large horn using the formula

$$G = \frac{4\pi A_e}{\lambda^2}$$

To obtain the linear gain you need to solve the above equation, making the following substitution:

$$G = 10^{\frac{G_{dB}}{10}} = \_\_\_\_\_\_\_\_\_\_$$

Then

$$A_e = \_\_\_\_\_\_ = \_\_\_\_\_\_ \text{ m}^2$$

The aperture efficiency η_{ap}, (refer to Exercise 1-2), should be close to ½. Verify this assumption.

$$\eta_{ap} = \frac{A_e}{A_p} = \frac{A_e}{A \times B}$$

η_{ap} = ______

☐ 19. Make sure you have saved your radiation patterns if you expect to use them in the future, then exit the LVDAM-ANT software. Place all power switches in the O (off) position, turn off the computer, disassemble the set-up, and return all components to their storage compartments.

CONCLUSION

In this exercise, you learned that a signal is reduced by 6 dB every time the distance from the source is doubled. You learned and demonstrated how the real gain of an antenna can be evaluated and you compared this with the theoretical gain. You used the substitution method to calculate the gain of a small horn antenna. Finally, you calculated the effective area of a large horn and verified that its aperture efficiency is close to ½.

REVIEW QUESTIONS

1. For a given wavelength, which parameter influences the free-space propagation loss L_F?

   ____

   ____

2. You want to design an optimal H-plane sectoral horn antenna operating at 10.52 GHz. The interior dimensions of the waveguide are a = 2.3 cm and b = 1 cm, and the horn will have an inside depth of R_1 = 5 cm. What will be the width of the aperture (A) and the outside depth (R_H) of this horn?

   ____

   ____

   ____

3. Does the following formula $G = \frac{26000}{HPBW_E \cdot HPBW_H}$ give a good approximation of the gain of a horn antenna? Explain why.

   ____

   ____

   ____

4. What is the influence of the opening angle flare of a pyramidal horn antenna on the propagation mode?

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___

5. Which parameters should be considered in the evaluation of the pyramidal horn antenna gain?

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___

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