LUMPED PARAMETER CIRCUIT MODEL

Assume

- Dimensions are small with respect to wavelength

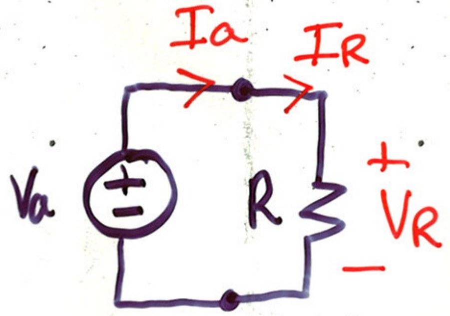
e.g., for 60 Hz; $\lambda \approx \frac{3 \times 10^8}{60} \approx$ 500 km

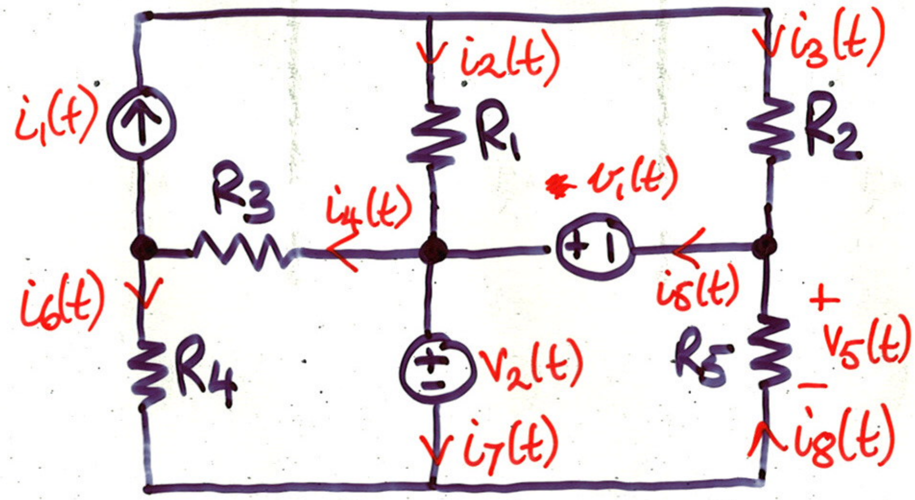
for 3 GHz, $\lambda \approx$ 10 cm

- wire resistance <<< other resistance

hence ignored (or lumped as separate resistance)

CONNECTING CIRCUIT ELEMENTS





Node
Loop
Branch

KIRCHOFF'S VOLTAGE LAW

- lumped circuits are closed systems
- they conserve work/energy

⇒ ➡ work done to increase potential is released elsewhere (energy transfer by movement of charge)

⇒ Algebraic sum of voltage $\left\{ \begin{matrix} \text{drops,} \\ \text{rises} \end{matrix} \right\}$ around a loop is zero

- Typically choose voltage drop to be positive

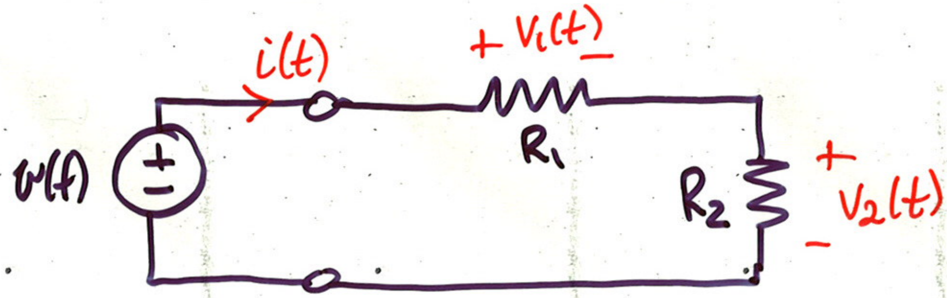
e.g., $-V_a + V_R = 0$

$$-v_2(t) + v_1(t) + v_5(t) = 0$$

- Alternatively (not recommended)

sum of voltage rises = sum of voltage drops

APPLICATIONS



Find $v_1(t)$ and $v_2(t)$ as functions of $v(t)$

KVL: $-v(t) + v_1(t) + v_2(t) = 0$ ①

Ohm's Law: $v_1(t) = R_1\, i(t)$ ②

$v_2(t) = R_2\, i(t)$ ③

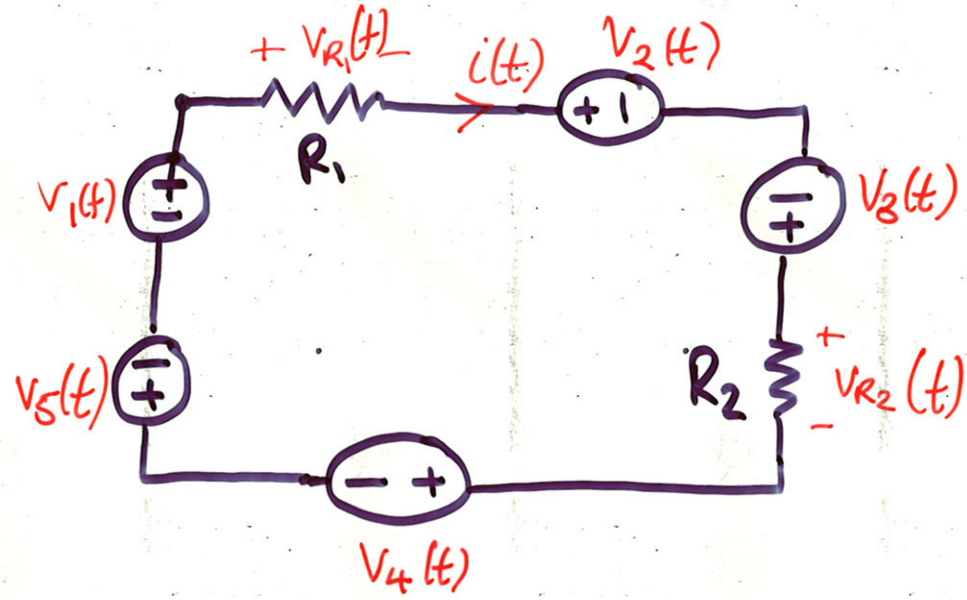
Substitute ②, ③ into ①

$$\Rightarrow v(t) = (R_1 + R_2)\, i(t) \quad ④$$

$$\Rightarrow \quad v_1(t) = \frac{R_1}{R_1 + R_2} v(t)$$

$$v_2(t) = \frac{R_2}{R_1 + R_2} v(t)$$

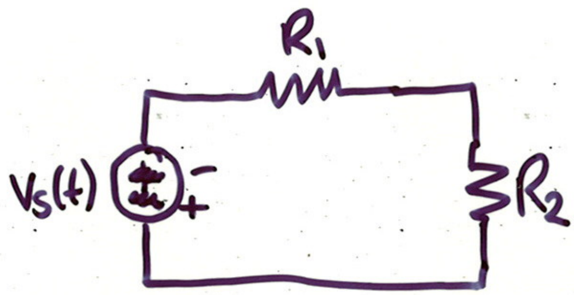
- voltage division
- more voltage required to overcome larger resistance
- Using ④, from source circuit looks like



KVL: $v_5(t) - v_1(t) + v_{R_1}(t) + v_2(t) - v_3(t) + v_{R_2}(t) + v_4(t) = 0$

$$\Rightarrow \underbrace{v_5(t) - v_1(t) + v_2(t) - v_3(t) + v_4(t)}_{v_s(t)} + v_{R_1}(t) + v_{R_2}(t) = 0$$

$$\Rightarrow v_s(t) + v_{R_1}(t) + v_{R_2}(t) = 0$$



KIRCHOFF'S CURRENT LAW

- Ideal conductor cannot store charge
- Current is the flow of charge

⇒ Sum of currents entering a node

= sum of currents departing

eg.,

$$I_a = I_R$$

$$i_2(t) + i_5(t) = i_4(t) + i_7(t)$$

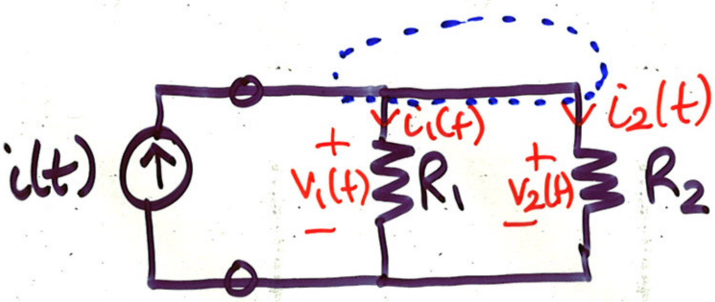
Alternatively,

Algebraic sum of all currents entering a node = 0

e.g.,

$$I_a - I_R = 0$$

$$i_2(t) + i_5(t) - i_4(t) - i_7(t) = 0$$



Find $i_1(t), i_2(t)$

KCL: $i(t) = i_1(t) + i_2(t)$

Ohm's Law: $v_1(t) = R_1 i_1(t)$

$V_2(t) = R_2 i_2(t)$

KVL, right loop: $-v_1(t) \pm v_2(t) = 0$

$$\Rightarrow \quad i_1(t) = \frac{R_2}{R_1+R_2} i(t)$$

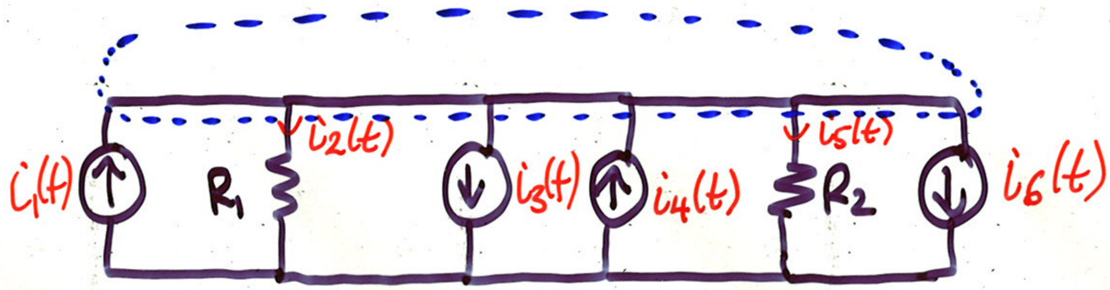
$$i_2(t) = \frac{R_1}{R_1+R_2} i(t)$$

- Current division
- More current on the path with lower resistance

· Note. $v(t) = \frac{R_1 R_2}{R_1+R_2} i(t)$; $\frac{R_1 R_2}{R_1+R_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

· From source, cirwit looks like

$R_p = \frac{R_1 R_2}{R_1+R_2}$



KCL: $i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_5(t) - i_6(t) = 0.$

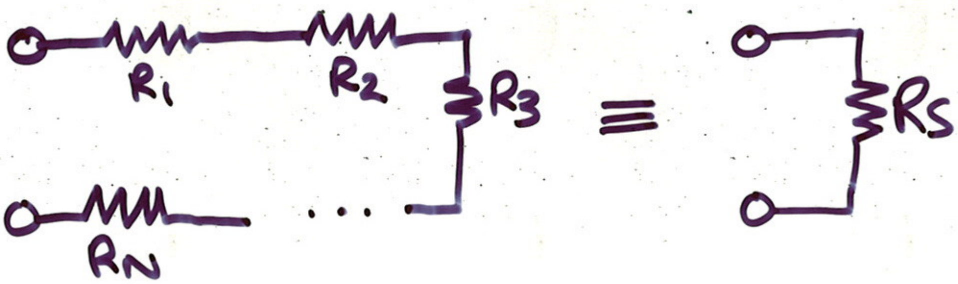
$$\Rightarrow \quad \underbrace{i_1(t) - i_3(t) + i_4(t) - i_6(t)} - i_2(t) - i_5(t) = 0$$

$$= \quad i_s(t) \quad - i_2(t) - i_5(t) = 0$$

RESISTANCES IN SERIES AND PARALLEL

- We have derived, <u>from first principles</u>, some special cases of more general results
- Try to prove these yourself.

SERIES.



$$R_S = \sum_i R_i$$

PARALLEL

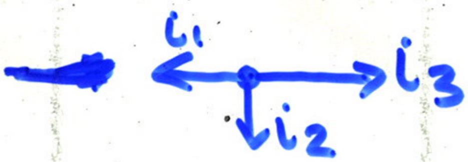
$$\frac{1}{R_P} = \sum_i \frac{1}{R_i}$$

$$G_P = \sum_i G_i$$

KCL

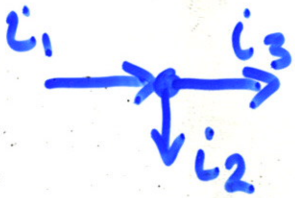
$i_1 + i_2 + i_3 = 0.$

sum all currents in $= 0$.



$i_1 + i_2 + i_3 = 0$

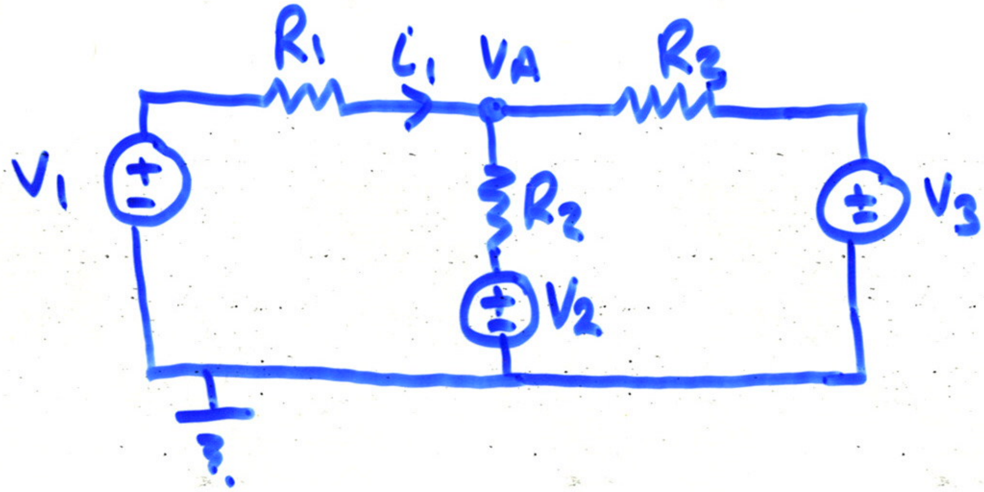
sum all currents out $= 0$.



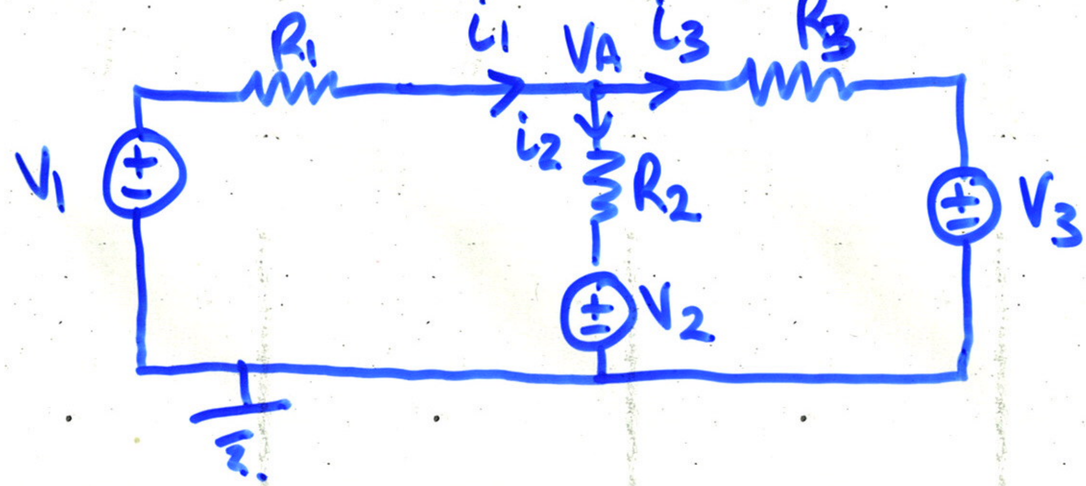
$i_1 - i_2 - i_3 = 0$

$\Rightarrow i_1 = i_2 + i_3.$

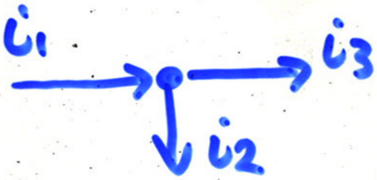
Sum currents in = sum currents out.



What is V_A?

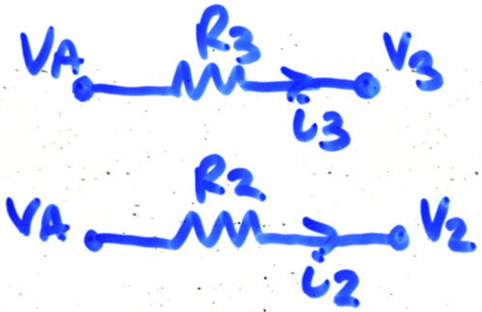


Break up the problem into small pieces.



KCL: $i_1 = i_2 + i_3$.

Ohms Law $\dfrac{V_1 - V_a}{R_1} = i_1$



$$\frac{V_A - V_3}{R_3} = i_3$$

$$\frac{V_A - V_2}{R_2} = i_2.$$

4 unknowns V_A, i_1, i_2, i_3
4 equations $\Rightarrow$ can be solved.

$$\Rightarrow \frac{V_1}{R_1} - \frac{V_A}{R_1} = \frac{V_A}{R_2} - \frac{V_2}{R_2} + \frac{V_A}{R_3} - \frac{V_3}{R_3}$$

$$\Rightarrow \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\Rightarrow V_A = \frac{1}{G_1 + G_2 + G_3}\left(G_1 V_1 + G_2 V_2 + G_3 V_3\right), \quad G = \frac{1}{R}.$$