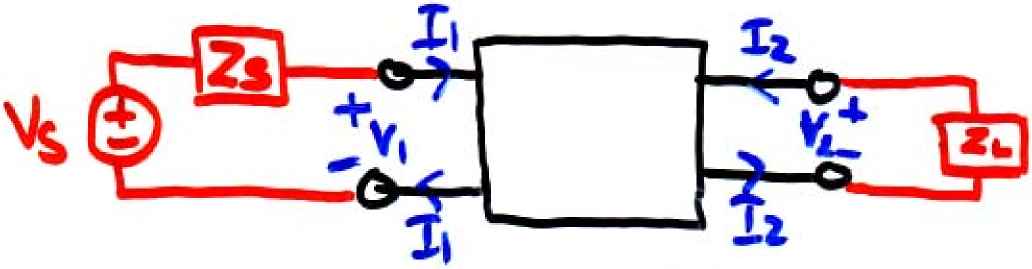
A TERMINATED TWO-PORT NETWORK.



What might we be interested in?

Voltage gain $\frac{V_2}{V_S}$

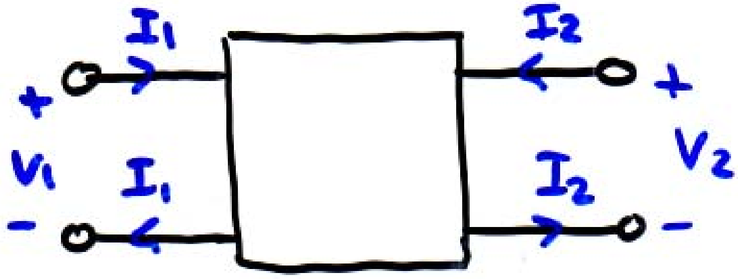
Current gain $-I_2/I_1$

Input impedance at source $Z_{in} = \frac{V_1}{I_1}$

Thevenin equiv. from perspective of load

Let's find them

Impedance Models for Two-Port Networks



Impedance Model

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Interpretation

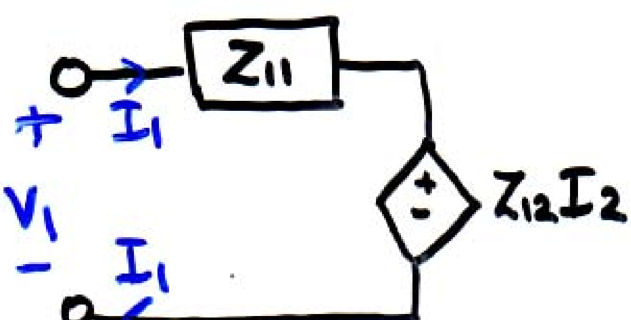
$Z_{11} = \frac{V_1}{I_1}$ when port 2 open

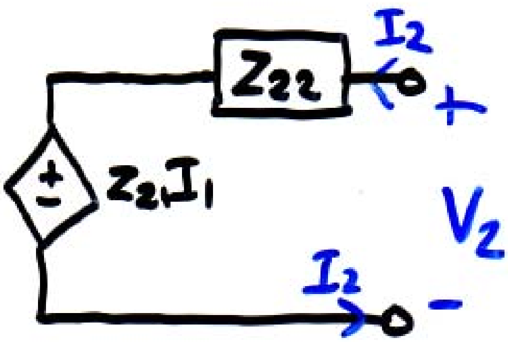
$Z_{22} = \frac{V_2}{I_2}$ when port 1 open.

$Z_{12} = \frac{V_1}{I_2}$ when port 1 open

$Z_{21} = \frac{V_2}{I_1}$ when port 2 open.

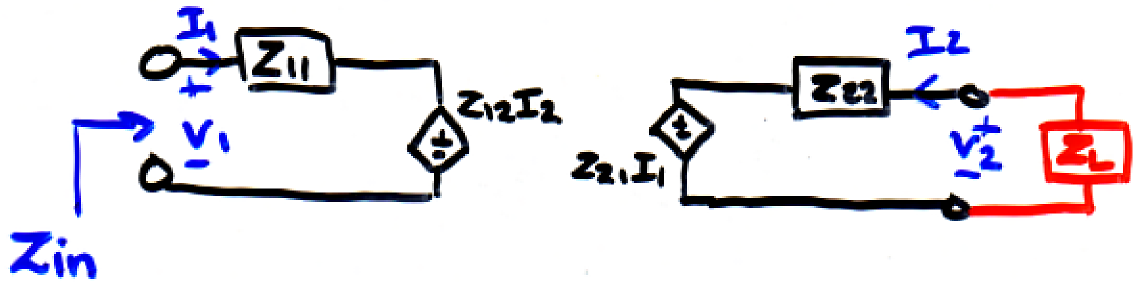
[redacted]





Input impedance

Use partioned model



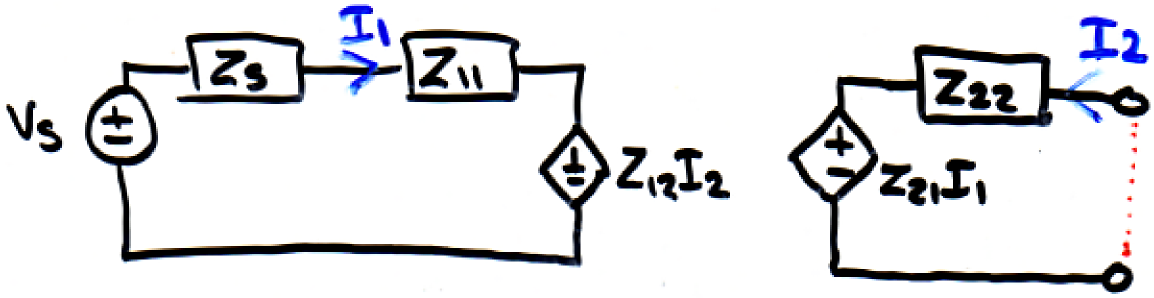
$V_1 =$ (left mesh)

I_2 (right mesh)

$$\Rightarrow \quad V_1 = \left(Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}+Z_L} \right) I_1$$

$$\Rightarrow \quad Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}+Z_L}$$

Thevenin equivalent from load's perspective



Find V_{oc}

$V_{oc} =$ ☐ (right mesh)

$I_1 =$ ☐ (left mesh)

$I_2 = 0$ (open circuit)

$$\Rightarrow V_{oc} = \frac{Z_{21}}{Z_{11}+Z_s} V_s$$

Find I_{sc}

$I_{sc} =$ ☐ (right mesh)

$I_1 =$ ☐ (left mesh)

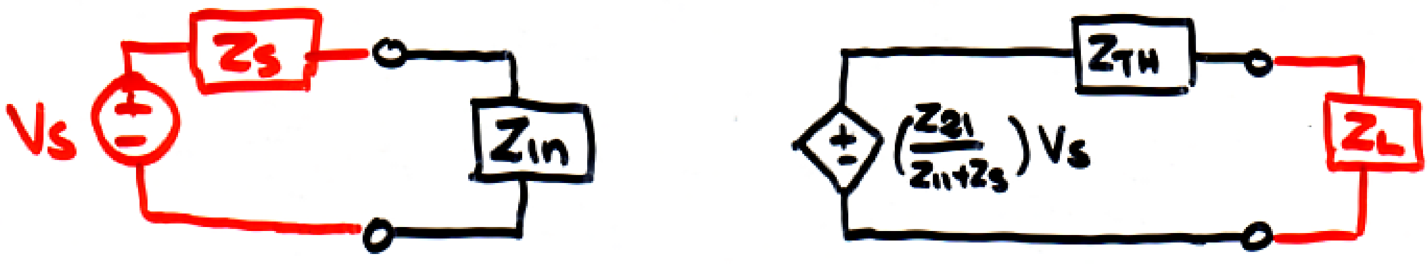
$I_2 =$ ☐

Some algebra

$$I_{sc} = \frac{Z_{21}}{Z_{22}(Z_{11}+Z_s) - Z_{12}Z_{21}}$$

$$Z_{TH} = \frac{V_{oc}}{I_{sc}} = \text{some algebra} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11}+Z_s}$$

With Z_{in} + Thevenin equivalent at output, we have a new model



Note that Z_{in} depends on Z_L

Z_{TH} depends on Z_S

Use this new model to find gains

Voltage gain

$$V_2 = \left(\frac{z_{21}}{z_{11}+z_S}\right) V_S \cdot \frac{Z_L}{Z_L + Z_{TH}}$$

$$\Rightarrow \frac{V_2}{V_S} = \frac{z_{21}}{z_{11}+z_S} \cdot \frac{Z_L}{Z_L + z_{22} - \frac{z_{21} z_{12}}{z_{11}+z_S}}$$

$$= \frac{z_{21} Z_L}{(z_{11}+z_S)(Z_L+z_{22}) - z_{12} z_{21}}$$

Current gain

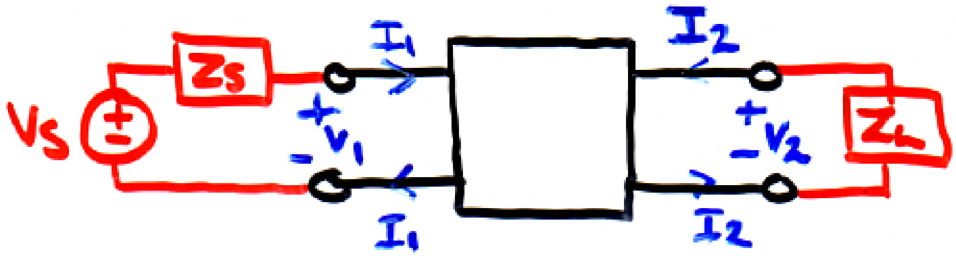
$$I_1 = \frac{V_S}{Z_S + Z_{in}} \qquad -I_2 = \frac{\left(\frac{z_{21}}{z_{11}+z_S}\right) V_S}{Z_{TH} + Z_L}$$

$$\Rightarrow \frac{I_2}{I_1} = -\frac{z_{21}}{(z_{11}+z_S)} \cdot \frac{1}{(Z_{TH}+Z_L)} \cdot (Z_S + Z_{in})$$

$$= \text{some algebra}$$

$$= \frac{-z_{21}}{z_{22}+Z_L}$$

Summary



$$Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L}$$

$$V_{TH} = \frac{Z_{21}}{(Z_{11} + Z_S)} V_S.$$

$$Z_{TH} = Z_{22} - \frac{Z_{12} Z_{21}}{(Z_{11} + Z_S)}$$

$$\frac{V_2}{V_1} = \frac{Z_{21} Z_L}{(Z_{11} + Z_S)(Z_L + Z_{22}) - Z_{12} Z_{21}}$$

$$\frac{I_2}{I_1} = \frac{-Z_{21}}{Z_{22} + Z_L}$$

Interpretation

if we want to increase Z_{in} increase Z_L

increase V_{TH} decrease Z_S

decrease Z_{TH} decrease Z_S

increase $\frac{I_2}{I_1}$ decrease Z_L

increase $\frac{V_2}{V_1}$ increase Z_L, decrease Z_S