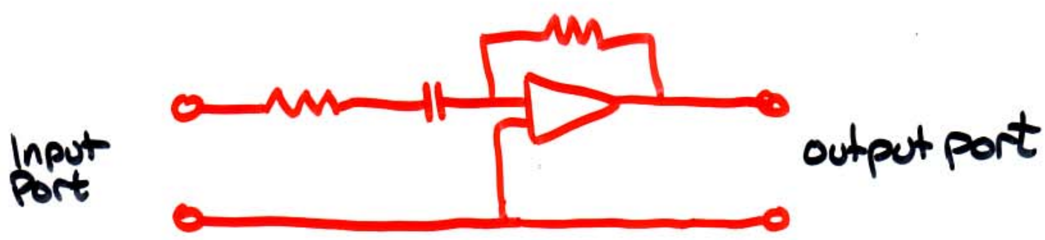
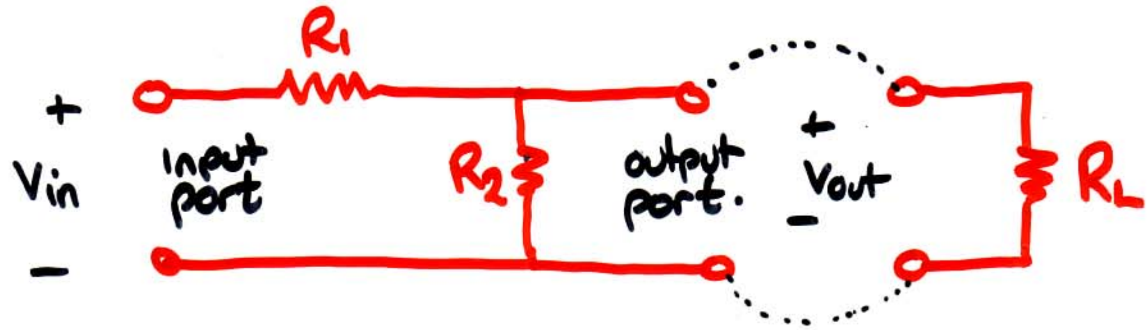
Two- Port Networks

Many circuits have input + output ports. e.g.



We often deal with such circuits in isolation

In practice, they are often connected to other circuits and these circuits influence the behaviour of the current circuit; e.g.,



$$V_{out} = \frac{(R_2 \| R_L) V_{in}}{R_1 + R_2 \| R_L} = \frac{R_2 V_{in}}{R_1 + R_2 + R_1 R_2 / R_L}$$

$$= \frac{R_2 V_{in}}{(R_1 + R_2)\left(1 + \frac{R_1 R_2}{(R_1 + R_2) R_L}\right)}$$

when $R_L \gg \frac{R_1 R_2}{R_1 + R_2}$

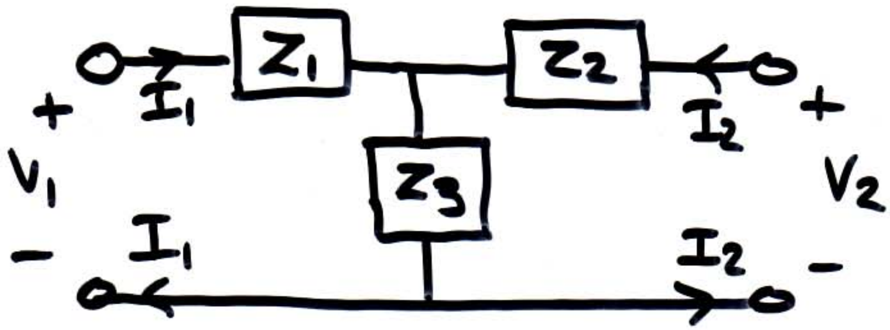
$$V_{out} \approx \frac{R_2}{R_1 + R_2} V_{in}$$

Purpose of the theory of two-port networks.

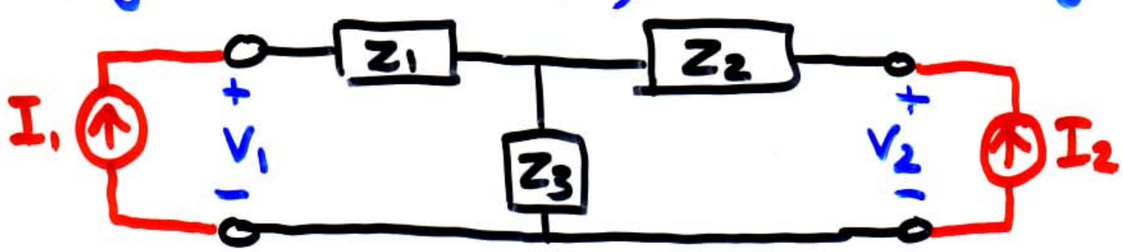
- provide a structured way to analyse interconnected circuits

Lets explore some examples.

The T-NETWORK



Apply current sources, measure voltages.



Mesh analysis:

$$V_1 - Z_1 I_1 - Z_3 (I_1 - I_2) = 0.$$
$$V_2 - Z_2 I_2 - Z_3 (I_2 - I_1) = 0$$

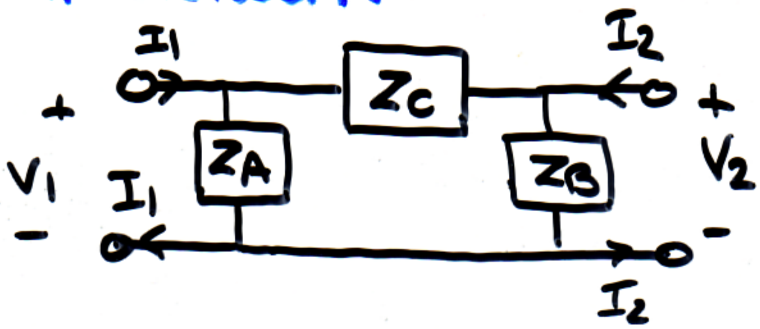
$$\Rightarrow \qquad \begin{aligned} V_1 &= (Z_1 + Z_3) I_1 \;\square\; Z_3 I_2. \\ V_2 &= Z_3 I_1 + (Z_2 + Z_3) I_2. \end{aligned}$$

Matrix form

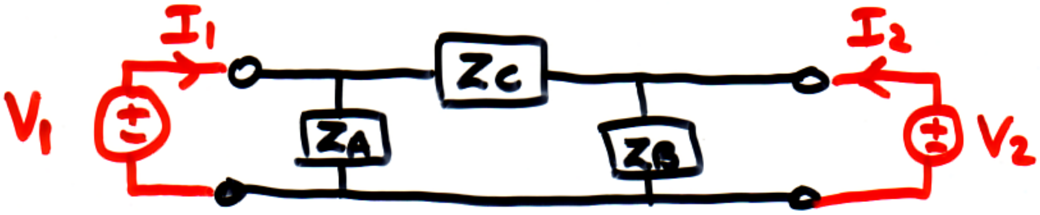
$$V = Z\, I$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The π Network



Apply voltage sources, measure currents.



Node analysis

$I_1 =$

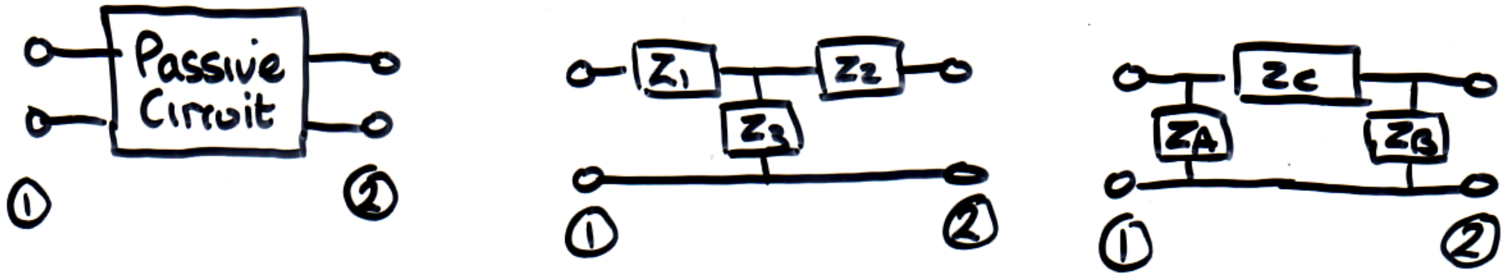
$I_2 =$

$$\Rightarrow \quad I_1 = (Y_A + Y_C)V_1 - Y_C V_2$$

$$I_2 = -Y_C V_1 + (Y_B + Y_C)V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For A Given Passive Circuit, How Are T and Π Models Related?



Input impedance at terminal 1, with terminal 2 open.

$$Z_{in} = Z_1 + Z_3 \quad ; \quad Z_{in} = Z_A \| (Z_C + Z_B)$$

$$= \frac{Z_A(Z_C + Z_B)}{Z_A + Z_B + Z_C}$$

Input impedance at terminal 1, with terminal 2 shorted.

$$Z_{in} = Z_1 + (Z_2 \| Z_3) \quad ; \quad Z_{IN} = Z_A \| Z_C$$

$$= Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \qquad = \frac{Z_A Z_C}{Z_A + Z_C}$$

Doing similar things with roles of terminals reversed gives more equations, which we can then solve (later)

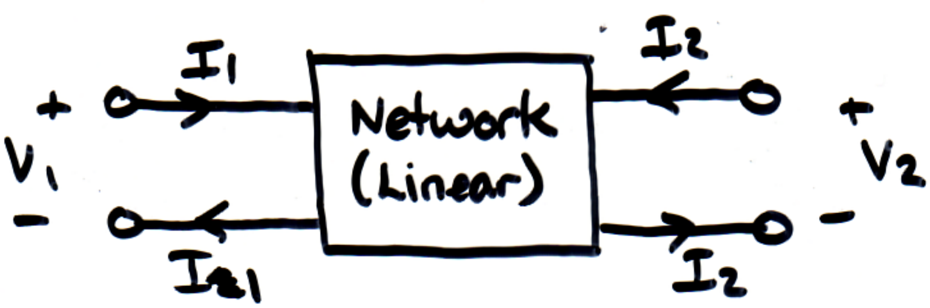
What is more important

is to observe that Z_{in} (at terminal 1) depends on the "load" at terminal 2.

Similarly,

Z_{in} at terminal 2 depends on load at terminal 1

- It is this interdependence that complicates the analysis
- The idea behind ~~two~~ the theory of two port networks is to simplify and structure the analysis



4 quantities, $V_1, V_2,$ ~~&~~ I_1, I_2.

2 will be inputs (independent variables, sources)

2 will be outputs

We have already seen:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Impedance model.

outputs

inputs

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Admittance model.

We can also have

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Hybrid model.

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

Inverse hybrid model

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Transmission model.

We will study these models and their relationships

why so many?

In different scenarios, one will be a easier to use than others.