Poles + Zeros (finally).

$$F(s) = \frac{N(s)}{D(s)}$$

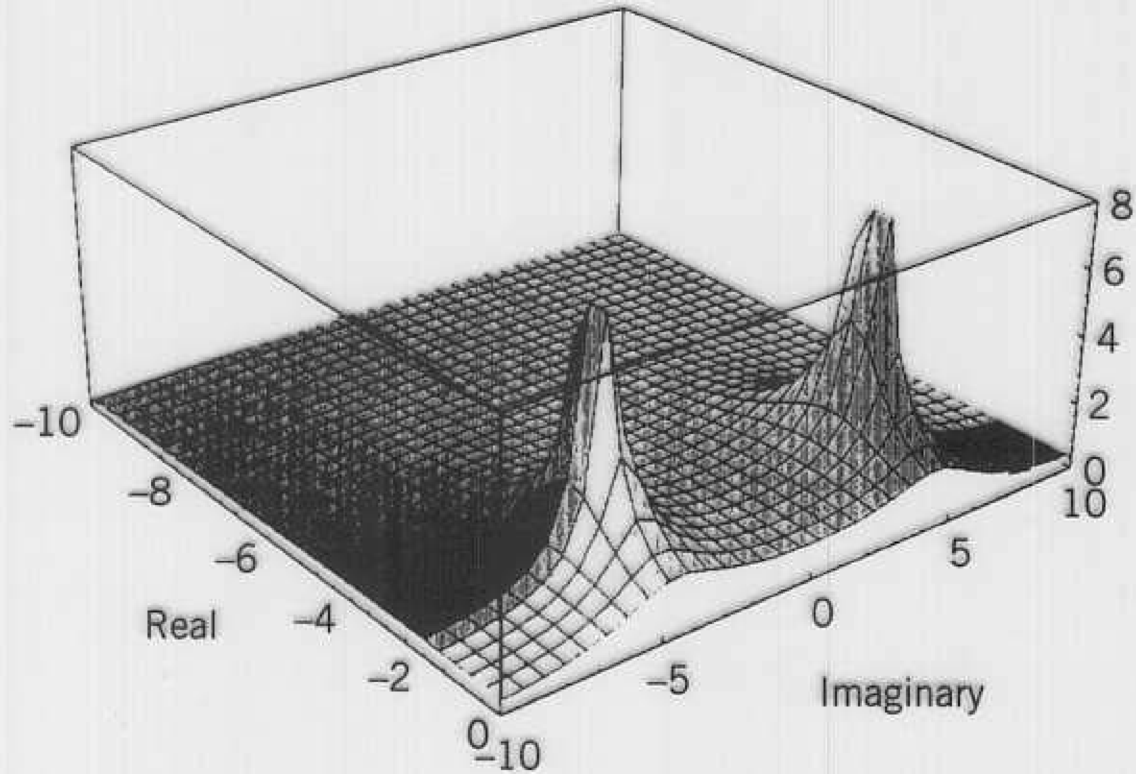
where N and D are polynomials

Zeros are the values of s for which $N(s)=0$
Poles are the values of s for which $D(s)=0$

Why are they called poles?

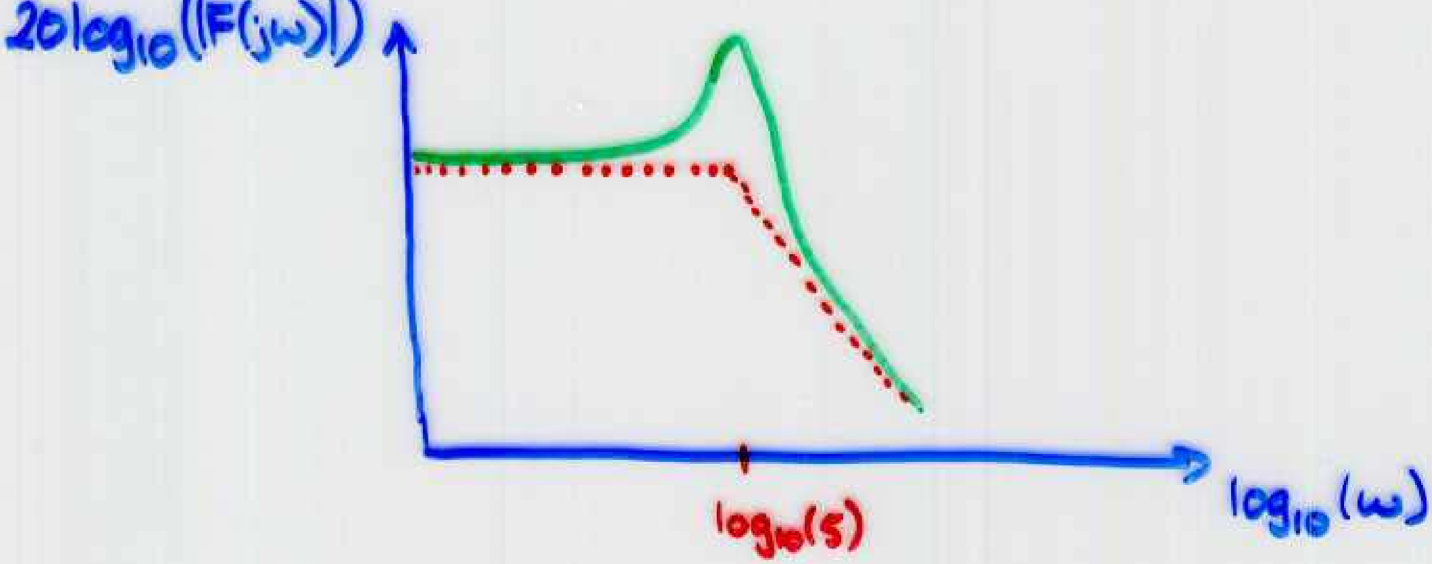
* If you plot $|F(s)|$ as a function of $s=\sigma+j\omega$

  around the pole, $|F(s)| \to \infty$
* looks like a pole or tower in the middle of a field, see Fig 14.5-1.



8
6
4
2
0
−10
−8
−6
Real
−4
−2
0
−10
−5
0
5
10
Imaginary

What happens for $s = j\omega$?



Does this suggest a relationship at all ?

INITIAL & FINAL VALUE THEOREMS

A function's initial value can be found from.

$$f(0^+) = \lim_{s \to \infty} sF(s)$$

and its final value from

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

if the poles of $F(s)$ are in the left half plane.
(real part is negative)

Proof:

Initial value:

$$\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \int_0^\infty \left[f(t) s e^{-st} \right] dt$$

e^{-st} goes to zero much faster than $s \to \infty$
Hence integrand is small except near $t=0$

$$\Rightarrow \lim_{s \to \infty} sF(s) \approx \lim_{s \to \infty} f(0) \int_0^\infty s e^{-st} dt$$

$$= f(0) \lim_{s \to \infty} \left. -\frac{se^{-st}}{s} \right]_0^\infty$$

$$= f(0)$$

This proof can be made rigorous

Final value.

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

but $\mathcal{L}\left\{\frac{df}{dt}\right\} = \int_0^\infty \frac{df}{dt} e^{-st} dt.$

take limit as $s \to 0$.

$$\Rightarrow \lim_{s \to 0} \mathcal{L}\left\{\frac{df}{dt}\right\} = f(\infty) - f(0)$$

Hence $\lim_{s \to 0} sF(s) = f(\infty)$.

Examples

$$f(t) = Ae^{-\alpha t}\cos\beta t\, u(t)$$

$$\Rightarrow F(s) =$$

Initial value $= A$, $\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \frac{As^2 + \alpha s}{s^2 + 2\alpha s + \alpha^2 + \beta^2}$

$$= A$$

Final value $= 0$, $\lim_{s \to 0} sF(s) = \lim_{s \to 0} \frac{As^2 + \alpha s}{s^2 + 2\alpha s + \alpha^2 + \beta^2}$

$$= 0$$

Now look at $f(t) = \cos(\omega_0 t)\,u(t)$

What is its final value?

Perhaps we should try the final value theorem.

$$\lim_{s \to 0} sF(s) = \lim_{s \to 0} \frac{s^2}{s^2 + \omega_0^2}$$

$$= 0.$$

But this is wrong!

What happened?

$F(s)$ has poles at $s = \pm j\omega_0$

These have 0 real part

Hence Theorem does not apply!