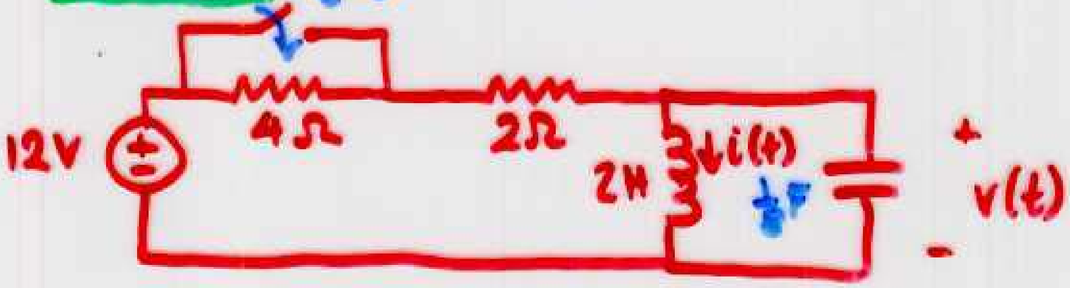
CIRCUIT ANALYSIS in the "s-domain"

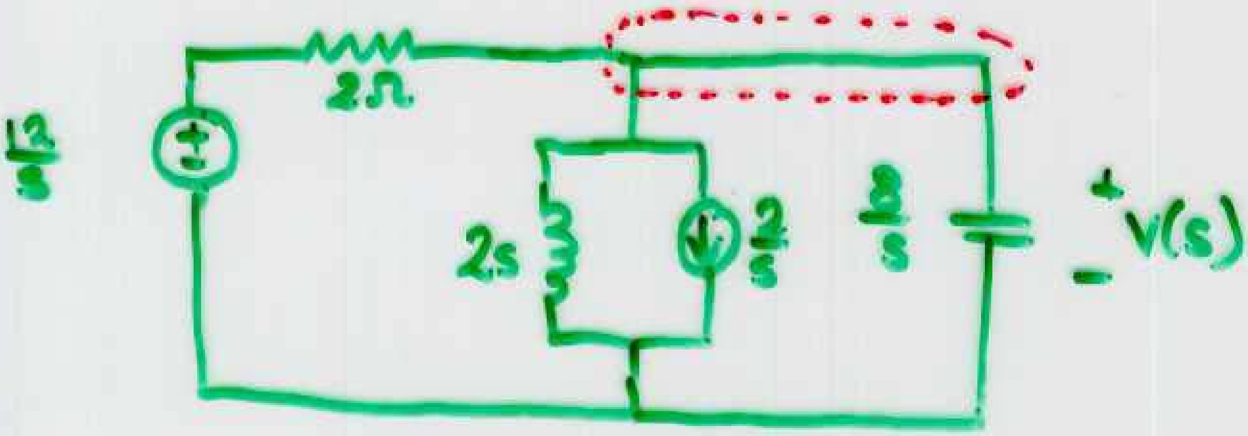
1. Transform all signals into "s-domain"
2. Replace circuit elements by their "s-domain" equivalents; eg impedance + initial condition
3. Do node/mesh analysis in "s-domain" to obtain Laplace transform of desired signal
4. Take inverse Laplace Transform to obtain time domain signal.

Example:



Find $v(t)$, $t \geq 0$

Steps ① and ②, for $t \geq 0$ we have



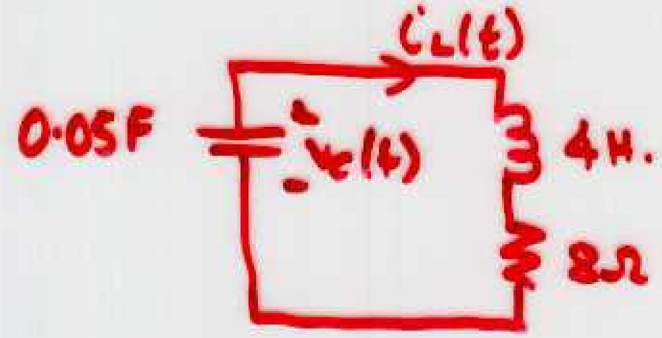
NOTE THAT $i(0) = 2A$, $v(0) = 0V$

③ KCL at marked node.

$\Rightarrow V(s) = \frac{32}{s^2+4s+4} = \frac{32}{(s+2)^2}$

using tables + linearity $\Rightarrow v(t) = 32te^{-2t}u(t)$

EXAMPLE:



Find $i_L(t)$, $t \geq 0$ if $V_c(0) = 8V$ and $i_L(0) = 1A$.

EQUIVALENT MODEL.

KVL

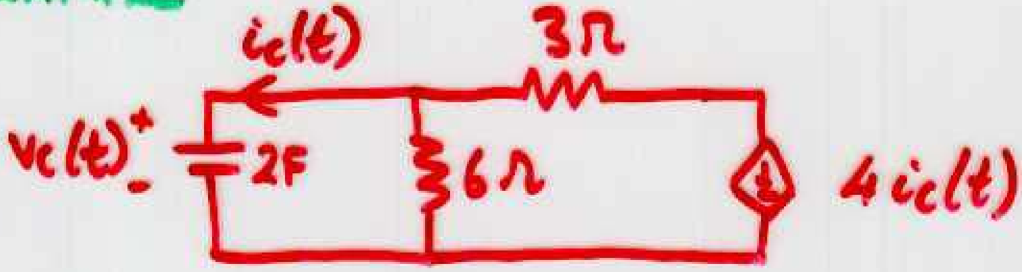
$$\frac{8}{s} - \frac{1}{Cs} I(s) = 4sI(s) - 4 + 8I(s).$$

$$\Rightarrow \quad I(s) = \frac{4 + 8/s}{1/Cs + 4s + 8} = \frac{s+2}{s^2+2s+5}$$

$$= \frac{s+1}{(s+1)^2+4} + \frac{1}{2}\left(\frac{2}{(s+1)^2+4}\right)$$

$$\Rightarrow \quad i_L(t) = \left[e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t\right]u(t)$$

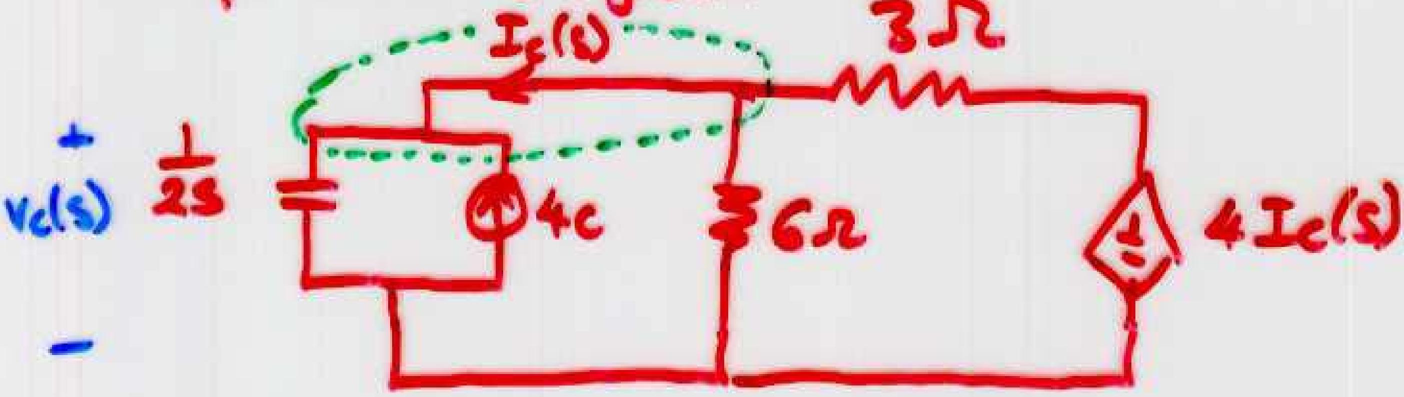
EXAMPLE



Find $v_c(t)$ for $t \geq 0$ if $v_c(0) = 4V$

SOLUTION

Equivalent diagram



KCL at green node

Capacitor current $I_c(s) =$

⇒ $V_c(s) = \dfrac{4}{s - 3/4}$

⇒ $V_c(t) =$

Does this ring alarm bells?

TRANSFER FUNCTION

The transfer function of a circuit is defined as the ratio

$$H(s) = \frac{\text{Laplace Transform of output signal}}{\text{Laplace Transform of input signal}}, \quad (*)$$

provided all initial conditions are zero.

* This is sometimes called the "zero-state response"
* It is closely related to the "network function" for the ~~ss~~ steady state response to sinusoidal signals.

* if the signals are voltages, then (*) implies that

$$V_{out}(s) = H(s)\, V_{in}(s).$$

So once we have $H(s)$ we can find the zerostate response to almost any input signal

A particularly important signal is the unit step. The response is called the "step response"

since ~~□~~ $\mathcal{L}\{u(t)\} = \frac{1}{s}$

if $v_{sr}(t)$ denotes the step response, then

$$V_{sr}(s) = \frac{H(s)}{s}$$

* When input is a current and output is a voltage, H(s) is often said to be an "impedance"

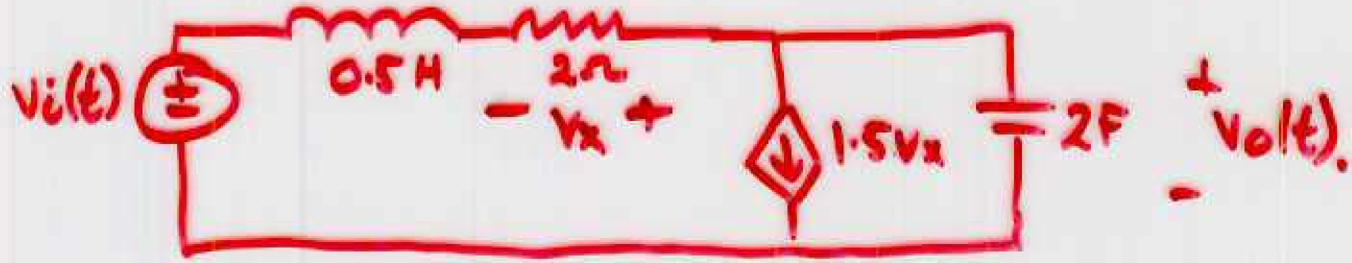
$$H(s) = Z(s) = \frac{V_{out}(s)}{I_{in}(s)}$$

* When input is a voltage and output is a current, H(s) is often said to be an "admittance"

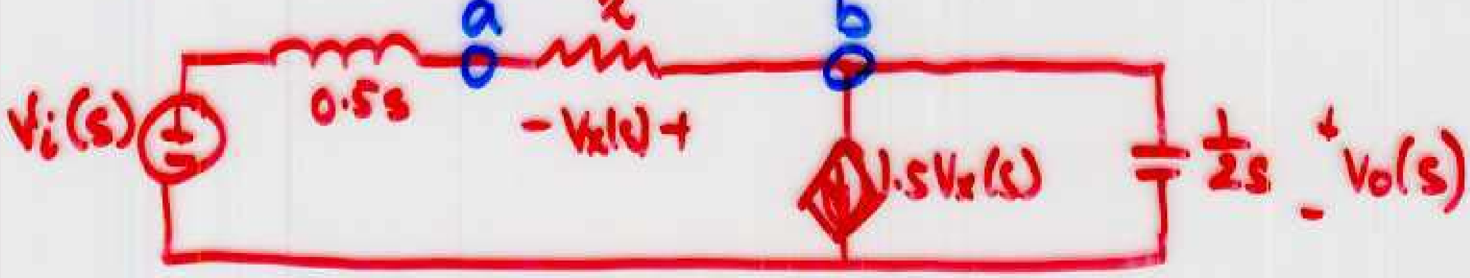
$$H(s) = Y(s) = \frac{I_{out}(s)}{V_{in}(s)}$$

STEP RESPONSE EXAMPLE

Determine the step response of the following circuit



Equivalent diagram with zero initial conditions



Node equations at a and b.

$$\Rightarrow V_b(s) = \frac{4}{(s+2)^2} V_i(s)$$

Since $V_b = V_o$, $H(s) = \frac{4}{(s+2)^2}$

Step response $= \mathcal{L}^{-1}\left\{ \frac{H(s)}{s} \right\}$

$= \mathcal{L}^{-1}\left\{ \quad \right\}$

$\Rightarrow$ stepresponse $= \left[1 - e^{-2t} - 2te^{-2t}\right] u(t)$