THREE PHASE CIRCUITS.

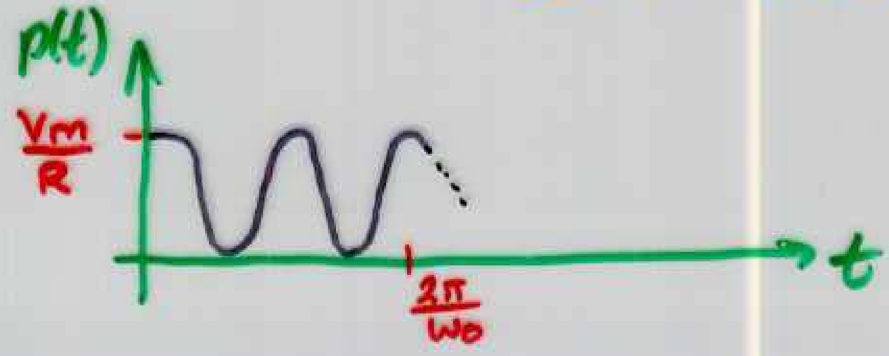
* Last section of work, but is simply a special case of phasor analysis.

* That being said, the application is very important — generation, transmission and consumption of electrical power.

* To get an indication of why three phase circuits are important consider the following single phase circuit, with a sinusoidal source

What is the instantaneous power delivered to the resistor?

$$
\begin{aligned}
P(t) &= v(t)\,i(t) \\
&= v^2(t)/R. \\
&= \frac{V_m^2}{R}\cos^2 \omega_0 t \\
&= \frac{V_m^2}{2R}\left(1+\cos 2\omega_0 t\right)
\end{aligned}
$$



What is the average power?

$$P_{av} = \frac{V_m}{2R}$$

* Notice the "pulsing" nature of the power.
* At low frequencies this will make lights flicker, and more importantly, make motors "jerky"
* The idea of transmitting "three phase power" is to reduce the fluctuations in the instantaneous power, for example, consider the following circuit

$v_a(t) = V_m \cos w_0 t$

$v_b(t) = V_m \cos(w_0 t - 120°)$

$v_c(t) = V_m \cos(w_0 t - 240°)$

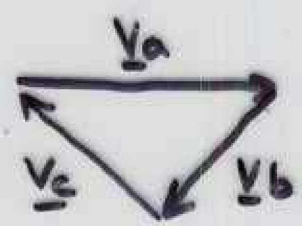
What is the instantaneous power delivered to the load?

Lets do phasor analysis

$\underline{V}_a = V_m \angle 0$; $\underline{V}_b = V_m \angle -120$, $\underline{V}_c = V_m \angle -240$

Since $V_n = 0$, KCL at "neutral node" gives

but look at "vector addition" of $\underline{V}_i$'s.



$\Rightarrow \underline{I}_n = 0$

Now look at instantaneous power:

$$p(t) = \frac{V_m^2}{R}\left[\cos^2 \omega_0 t + \cos^2(\omega_0 t - 120^\circ) + \cos^2(\omega_0 t - 240^\circ)\right]$$

$$= \frac{V_m^2}{2R}\left[3 + \cos 2\omega_0 t + \cos(2(\omega_0 t - 120^\circ)) + \cos(2(\omega_0 t - 240^\circ))\right]$$

$$= \frac{3V_m^2}{2R}.$$

$\Rightarrow$ instantaneous power is constant!

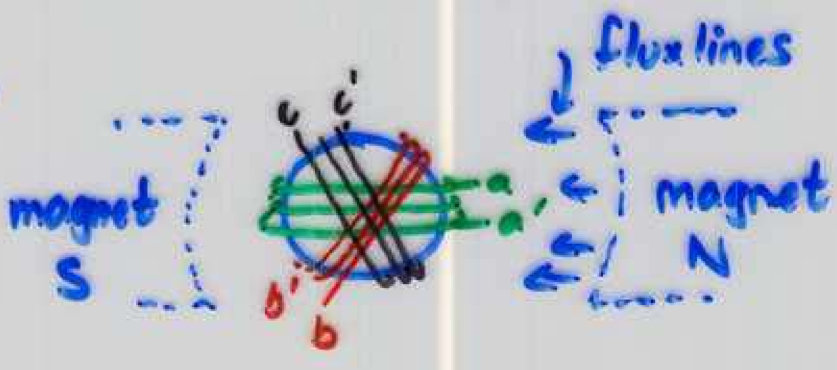
The instantaneous power in a 3-phase circuit is not always constant, but the fact that it can be made constant makes it very attractive.

Definition—

A "three phase circuit' generates, distributes and uses energy in the form of three voltages which are equal in magnitude and symmetric in phase.

For example, a three phase generator, might consist of three "windings" on a cylindrical rotator in a constant magnetic field.

End view.



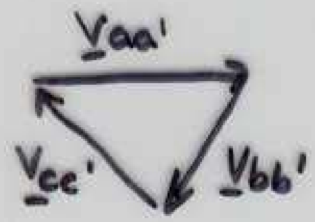
Note that the coils cut the flux lines 120° apart, + hence produce voltages which are separated by 120° in phase

if $\underline{V}_{aa'} = V\angle 0$, then $\underline{V}_{bb'} = V\angle -120$

and $\underline{V}_{cc'} = V\angle -240$

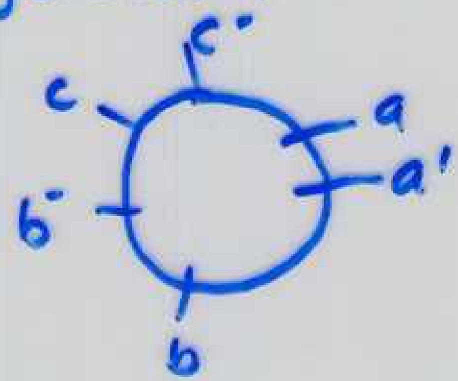
These voltages are "balanced" in that they have equal amplitudes and are symmetric in phase. Hence

$$\underline{V}_{aa'} + \underline{V}_{bb'} + \underline{V}_{cc'} = 0$$

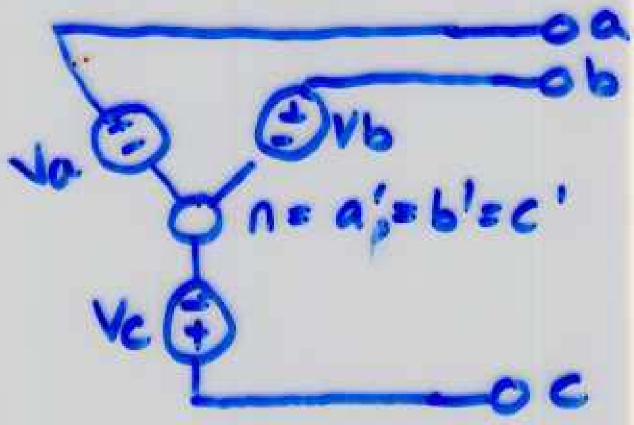


* For notational convenience, we set $\underline{V}_{aa'} = \underline{V}_a$, etc
* We will also often write $\underline{V}_a = V_p \angle 0$,

  to remind us that V_p is the "phase voltage"
* The "ordering" a, b, c is called the "positive phase sequence"
* How do we connect the 6 terminals of the generator?



One way is to connect all the "dashed" terminals together to form a "neutral" terminal.



This is called a Y-connection

* What is the "line to line" voltage $\underline{V}_{a,b}$ if the phase voltage is V_p?

* The source voltages (phasors) are

$$\underline{V}_a = V_p \angle 0 \ , \ \underline{V}_b = V_p \angle -120 \ , \ \underline{V}_c = V_p \angle -240$$

$$\underline{V}_{ab} = \underline{V}_a - \underline{V}_b = V_p - V_p(\cos(-120^\circ) + j\sin(-120^\circ))$$

$$\approx V_p(1.5 + j\,0.866)$$

$$= \sqrt{3}\, V_p \angle 30^\circ$$

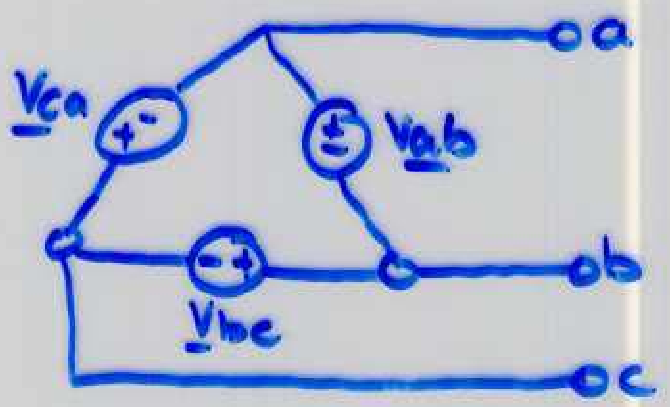
Similarly, $\underline{V}_{bc} = \sqrt{3}\, V_p \angle -90^\circ$

$\underline{V}_{ca} = \sqrt{3}\, V_p \angle -210^\circ$

* Therefore, the line-to line voltages are $\sqrt{3}$ times the phase voltage and are displaced by 30° in phase

* As is clear from the diagram, the current supplied at a terminal (the "line current") is equal to the current supplied by the source (the "phase current").

* Since the line voltages are related to the phase voltages, an equivalent model for the generator is



This "delta connection" is not usually used in practice because it is sensitive to imbalances in the sources

However, it is used in the construction of loads.

Just as a rotor turning in a magnetic field generates power, suppling power to coils on a rotor can cause the rotor to move ..... This is one way to construct an electric motor.

Since we would like constant power to be supplied to the motor, rather than pulsed power; a three phase winding on the motor is natural.

These windings can be connected as a Y or Δ connection, but there are equivalances between the two.

We will look at the Y case first.