Formulae for Arithmetic Series

Here are some formulae which will help you compute the sum of a finite arithmetic series. First of all,

$$\sum_{n=1}^{N}(a + nd) = Na + \frac{N(N+1)d}{2}.$$

(1)

This formula can be used to find related sums. For example,

$$\sum_{n=0}^{N}(a + nd) = a + \sum_{n=1}^{N}(a + nd)$$
$$= (N+1)a + \frac{N(N+1)d}{2}.$$

Equation (1) can also help us build a more general formula:

$$\sum_{n=N_1}^{N_2}(a + nd) = \sum_{n=0}^{N_2=N_1}\big((a + N_1 d) + nd\big)$$
$$= (N_2 - N_1 + 1)(a + N_1 d) + \frac{(N_2 - N_1)(N_2 - N_1 + 1)d}{2}$$
$$= (N_2 - N_1 + 1)a + \frac{(N_2 - N_1 + 1)(N_2 + N_1)d}{2}.$$