APPLICATIONS OF FOURIER REPRESENTATIONS (CHAPTER 4)

FREQUENCY RESPONSE OF LTI SYSTEMS

Recall that an LTI system is BIBO stable if

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \quad \text{or} \quad \sum_n |h[n]| < \infty$$

2. Recall that for a $\underbrace{\ }_{\text{non-periodic}}$ signal x, its FOURIER Transform exists if

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty \quad \text{OR} \quad \sum_n |x[n]| < \infty$$

plus some conditions of maxima, minima and discontinuities

Recall that the frequency response of an LTI system is.

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\, dt \quad \text{or} \quad H(e^{j\Omega}) = \sum_n h[n] e^{-j\Omega n}$$

THEREFORE

$$\text{Impulse response} \xleftrightarrow{\text{FT/DTFT}} \text{Frequency Response.}$$

Furthermore, in the LTI discrete-time case

- Stable systems are guaranteed to have a frequency response, because BIBO stability implies existence of DTFT

- Not quite true in continuous time, but is true for all reasonable systems

Therefore, for stable systems.

$$y(t) = x(t) * h(t) \overset{FT}{\longleftrightarrow} Y(j\omega) = H(j\omega)X(j\omega)$$

$$y[n] = x[n] * h[n] \overset{DTFT}{\longleftrightarrow} Y(e^{j\Omega}) = H(e^{j\Omega})X(e^{j\Omega})$$

If the system is unstable, the FT or DTFT may not exist and you cannot use the above formulae

The relationship

convolution in time $\xleftrightarrow{FT/DTFT}$ multiplication in frequency

is the key to filter design

- The nature of low pass / band pass / high pass filters you are familiar with in continuous time

- However we must be a bit careful in discrete time because the DTFT is periodic in frequency with period 2π. Hence we restrict attention to the region $-\pi < \Omega \leq \pi$

- Fig 4. contains pictures of the frequency response of ideal filters

- Remember that in practice filters
  - will not be completely flat in the pass band.
  - will have a "transition band" between pass and stop bands
  - the stop band level will be small but not zero

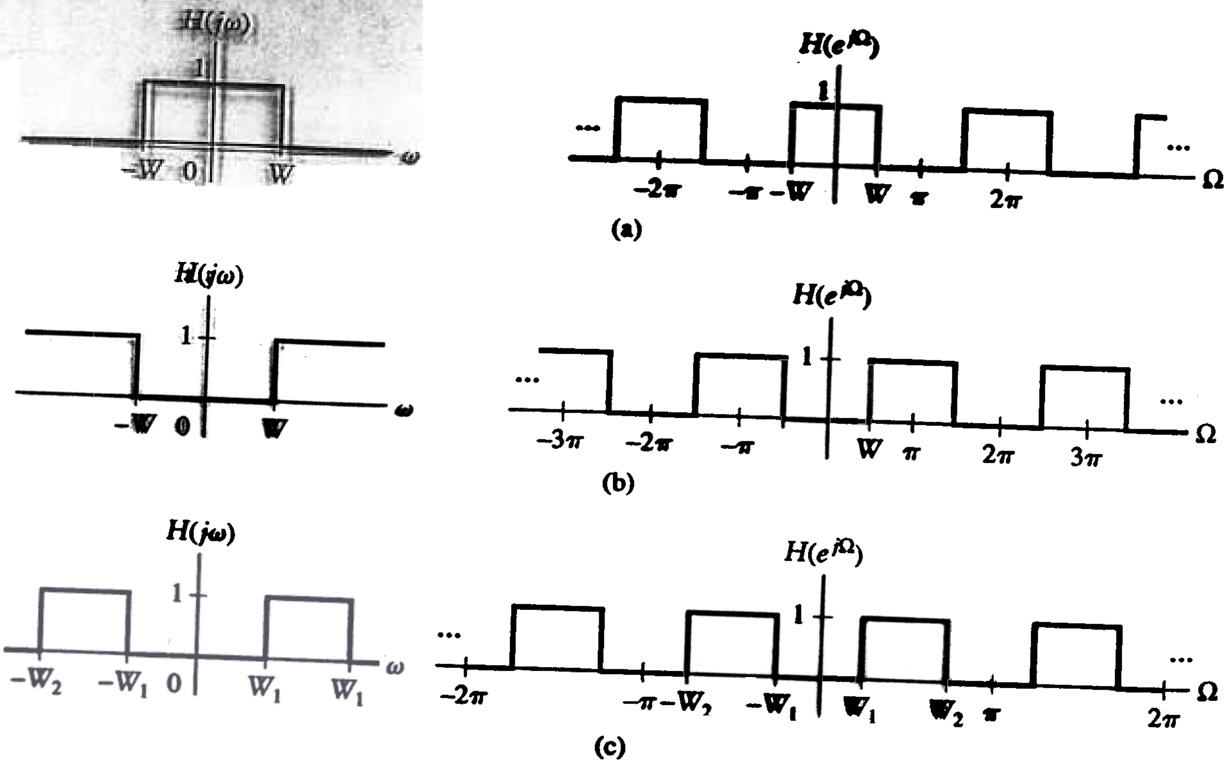

FIGURE 4.1 Frequency response of ideal continuous- and discrete-time filters. (a) Lowpass characteristic. (b) Highpass characteristic. (c) Bandpass characteristic.