TRANSFORM ANALYSIS OF SYSTEMS

$$y(t) = h(t) * x(t)$$

$$\Rightarrow \quad Y(s) = H(s)\,X(s)$$

find output given $x(t)$ and $h(t)$

$$H(s) = \frac{Y(s)}{X(s)}$$

system identification

Relationships with differential equations

$$a_0\, y(t) + a_1 \frac{dy}{dt} + \quad + a_N \frac{d^N y}{dt^N}$$

$$= b_0\, x(t) + b_1 \frac{dx}{dt} + \quad + b_M \frac{d^M x}{dt^M}$$

Take Laplace Transforms of both sides

Recall that $\quad \dfrac{d^k w(t)}{dt^k} \overset{L}{\longleftrightarrow} s^k W(s)$

$$\Rightarrow \quad a_0 Y(s) + a_1 s\, Y(s) + \quad + a_N s^N\, Y(s)$$

$$= b_0 X(s) + b_1 s\, X(s) + \dots + b_M s^M X(s)$$

$$H(s)=\frac{Y(s)}{X(s)} \qquad \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$$

Now factorize

$$H(s) - \frac{(b_M/a_N)\prod_{k=0}^{M}(s\ c_k)}{\prod_{k=0}^{N}(s-d_k)}$$

are the zeros

d_k are the poles

In the rest of this section we will seek intuition from the positions of poles and zeros.

CAUSALITY AND STABILITY

We know that

$$H(s) \xleftrightarrow{\mathcal{L}^{-1}} h(t)$$

but to do the inverse Laplace Transform, we need to know the ROC of $H(s)$

That is, we need to know more about the system

CAUSALITY

If the system is known to be causal, then $h(t)=0$, $t<0$. That is, $h(t)$ is right sided

Therefore the ROC is to the right of <u>all</u> pole. It must be to the right of <u>all</u> the poles because the ROC cannot contain any poles.

Example: A causal system has a transfer function

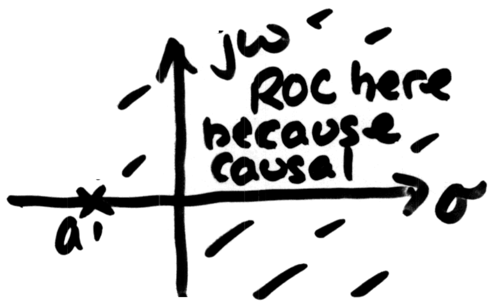
$H(s) = \frac{1}{s-a}$ Find $h(t)$.

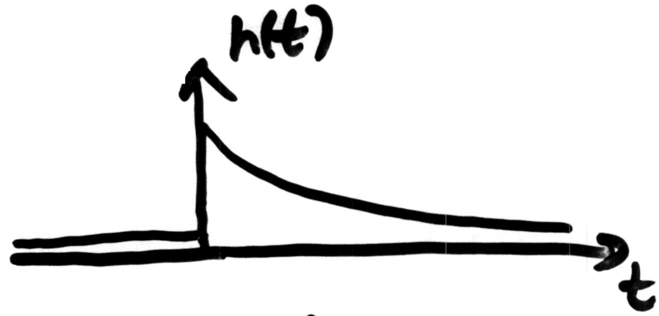
Since the system is causal, $h(t)$ is right sided

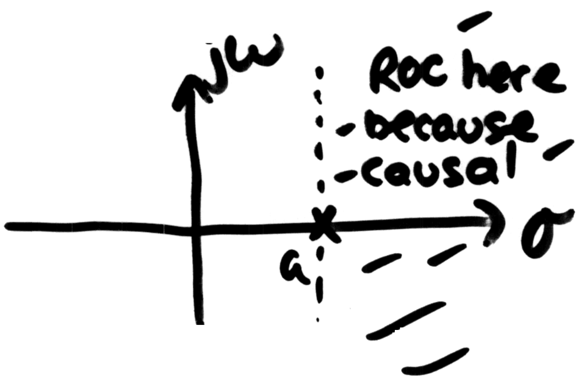
$\Rightarrow h(t) \quad e^{at}u(t)$

$a<0$, exponential decays

$a>0$, exponential grows







STABILITY

Recall that BIBO stable systems have

$$\int |h(t)|\,dt < \infty$$

Recall that $I(\sigma) = \int |h(t)| e^{-\sigma t} dt$

$\Rightarrow$ For a stable system $I(0) < \infty$

$\Rightarrow$ $j\omega$ axis is in the ROC.

Also note that $H(j\omega) = H(s)\big|_{s=j\omega}$

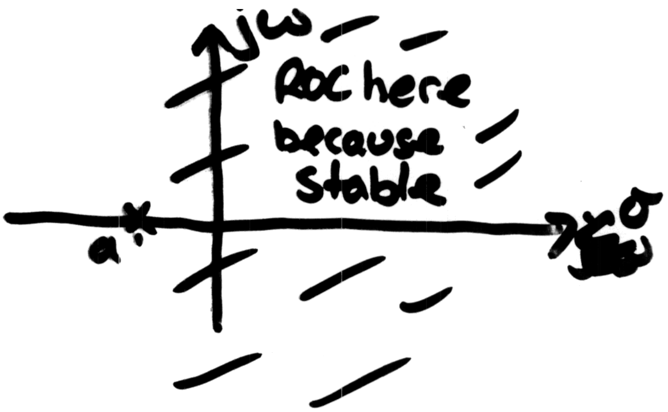
$\Rightarrow$ stable systems have a frequency response

$\boxed{\text{If } j\omega\text{-axis is not in the ROC, frequency response is not defined.}}$

Example A stable system has a transfer function

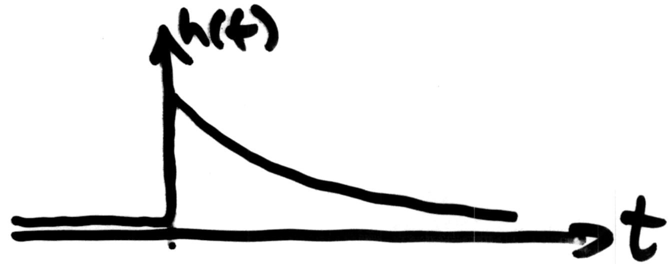
$$H(s) = \frac{1}{s-a}$$ Find $h(t)$.

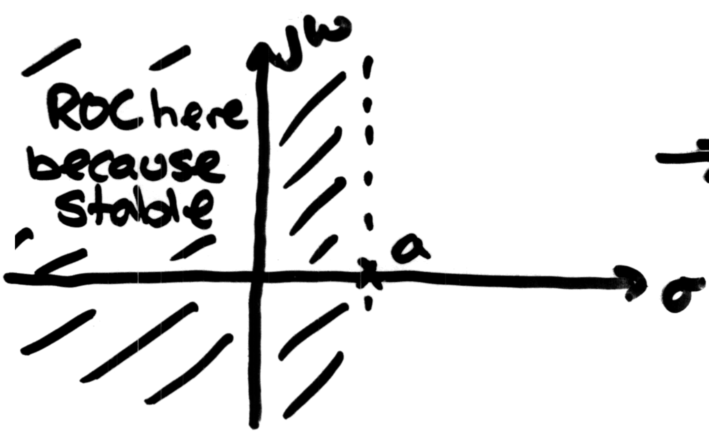
Since $h(t)$ is stable, ROC must contain $j\omega$ axis



$\rightarrow$

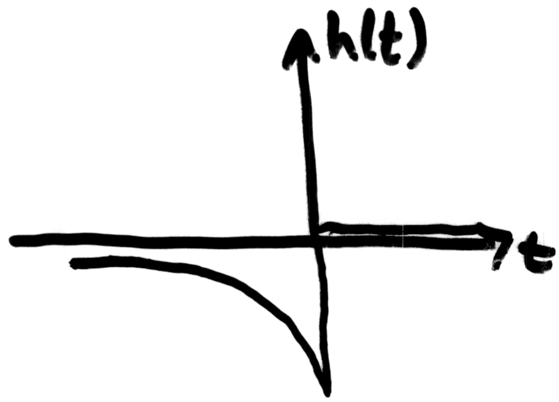
$h(t) = e^{at}u(t)$





$\rightarrow$

$h(t) = e^{at}u(-t)$



Note that if the pole is in the right half plane, the system must be anti-causal to be stable.

CAUSAL AND STABLE

ROC must be to the right of right most pole

ROC must include the $j\omega$-axis

- Therefore, for a system to be causal and stable all poles must be in the left half plane

Since most control system design is done in the Laplace transform domain this is an important constraint

EXAMPLE 6.19

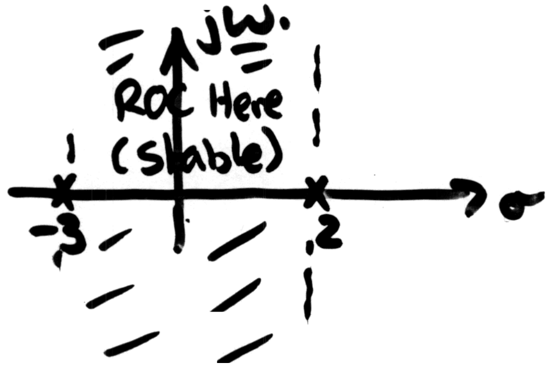
$$H(s) = \frac{2}{s+3} + \frac{1}{s-2}$$

Find $h(t)$ if

a) System is stable

b) System is causal.

a)



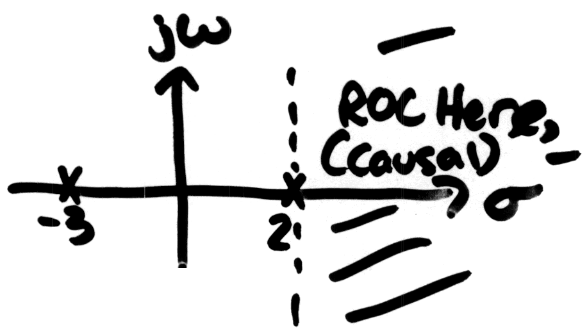
$\frac{2}{s+3} \overset{L^{-1}}{\longleftrightarrow} 2e^{-3t}u(t)$, ROC is on the Right of $s=-3$

$\frac{1}{s-2} \overset{L^{-1}}{\longleftrightarrow} e^{2t}u(-t)$, ROC is on the left of $s=2$.

$\Rightarrow h(t) = 2e^{-3t}u(t) \quad e^{2t}u(-t)$

[Non-Causal]

b)



$\frac{2}{s+3} \overset{L^{-1}}{\longleftrightarrow} 2e^{-3t}u(t)$ $\Rightarrow h(t) = 2e^{-3t}u(t) + e^{2t}u(t)$

$\frac{1}{s-2} \overset{L^{-1}}{\longleftrightarrow} e^{2t}u(t)$

[Unstable]