ELEMENTARY SIGNALS

- There are several elementary signals which feature prominently in the study of signals + systems
- They are often the key to developing intuition
- Familiarize yourself with them

EXPONENTIALS

$x(t) = Be^{at}$

Decays for $a<0$
Grows for $a>0$

$x[n] = Br^n$
$= Be^{n \times \ln(r)}$

Decays for $0<r<1$
Grows for $r>1$

Decays with alternating sign for $-1<r<0$

Grows with alternating sign for $r<-1$

Sinusoidal Signals

$x(t) = A\cos(\omega t + \phi)$ $\qquad$ $x[n] = A\cos(\Omega n + \phi)$

Fundamental period $= \frac{2\pi}{\omega}$ secs

Note: $x[n]$ is periodic if there is a positive integer N such that

$$A\cos(\Omega n + \Omega N + \phi) = A\cos(\Omega n + \phi)$$

This requires that $\Omega N = 2\pi m$

for some integer m

Therefore, Ω must be of the form

$\frac{2\pi m}{N}$ rads per sample

If Ω is not of this form, then $x[n]$ is not periodic

Complex Exponentials

Recall that $e^{j\theta} = \cos\theta + j\sin\theta$

If $B = Ae^{j\phi}$, then.

$$Be^{j\omega t} = A\cos(\omega t+\phi) + jA\sin(\omega t+\phi)$$

$$Be^{j\Omega n} = A\cos(\Omega n+\phi) + jA\sin(\Omega n+\phi)$$

Exponentially Damped Sinusoids

$$x(t) = Ae^{-\alpha t}\sin(\omega t+\phi), \quad \alpha>0$$

$$x[n] = Br^n\sin(\Omega n+\phi)$$

$$0<|r|<1$$

Note that these are not periodic signals

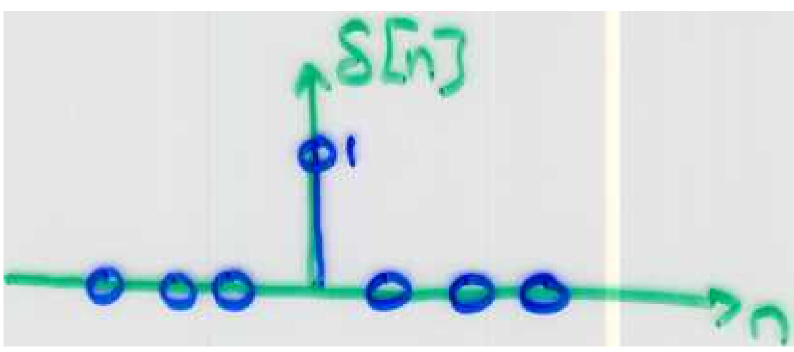
STEP FUNCTION

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad ; \quad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

DISCRETE-TIME IMPULSE FUNCTION

- RARELY OF DIRECT PHYSICAL USE, BUT DRAMATICALLY SIMPLIFIES MATHEMATICAL MODELS

$$\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$$



NOTE THAT THIS IS A BURST OF ENERGY IN A SHORT PERIOD OF TIME

Properties of $\delta(t)$

- From the definition, $\delta(t)$ is even symmetric

$$\delta(-t) = \delta(t)$$

- For the impulse $\delta(t)$ to have a physical meaning, it must appear inside an integral

For example,

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = \text{[redacted]}$$

- [redacted]

How can we compute this integral?

- Use the definition:

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot \lim_{\varepsilon \to 0} p_\varepsilon(t-t_0)\,dt.$$

$$= \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \int_{-\varepsilon/2+t_0}^{\varepsilon/2+t_0} x(t)\,dt.$$

Now for small ε and $x(t)$ which are continuous ~~[redacted]~~ in an interval around t_0,

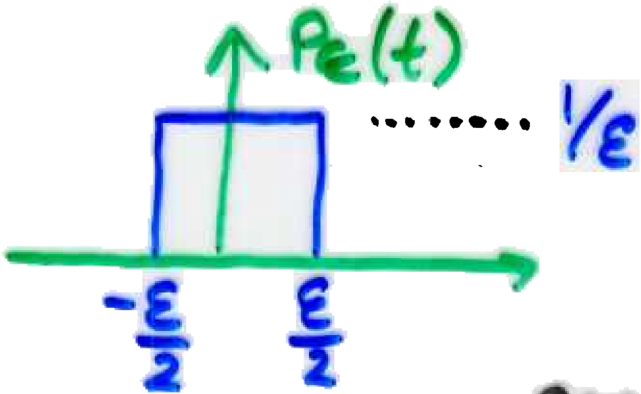
Continuous Time Impulse

- It is also mathematically convenient to be able to represent a short burst of energy in continuous time
- This is described by the Dirac delta function, which is any function which satisfies

$$\delta(t) = 0 \quad \text{for } t \neq 0 \qquad \text{(zero almost everywhere)}$$

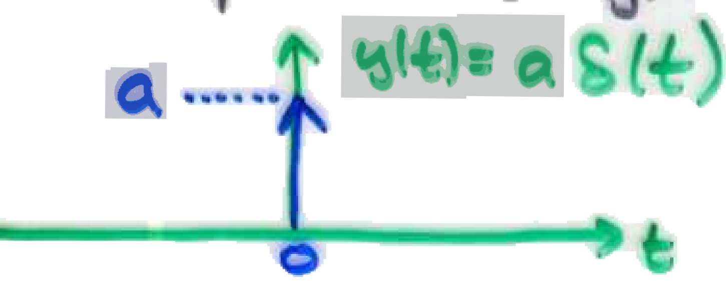
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \qquad \text{(unit area)}$$

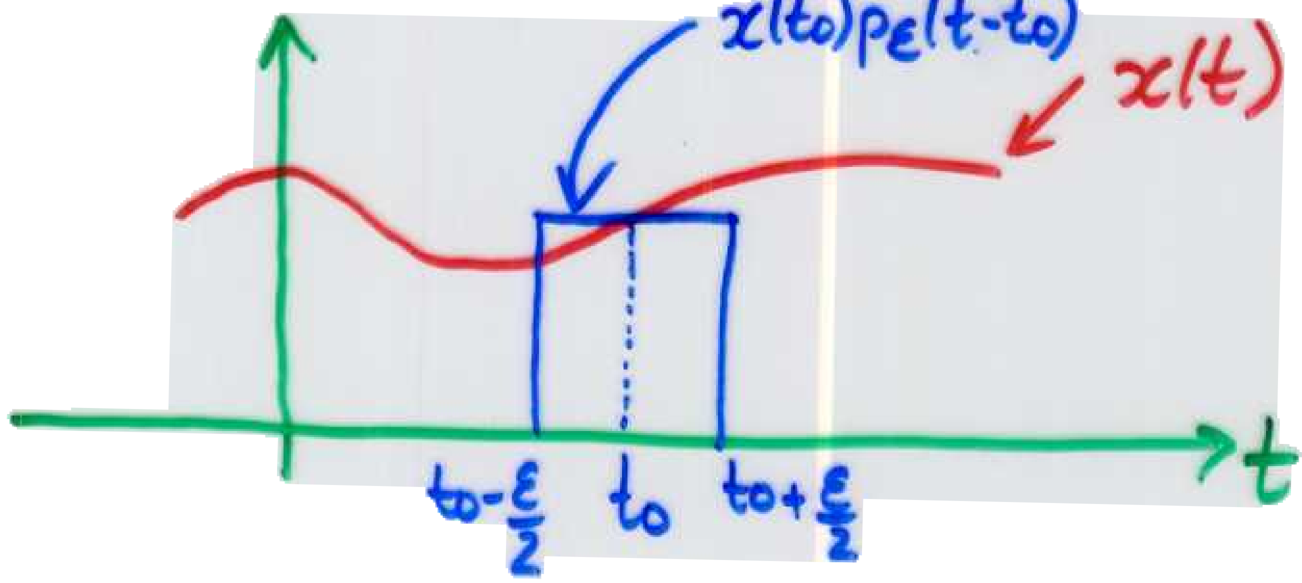
- There are many ways to generate this function. one is as follows



$$\delta(t) = \lim_{\varepsilon \to 0} p_\varepsilon(t)$$

- We usually represent an impulse of "strength" (ie, area) a as.





$x(t_0)p_\varepsilon(t-t_0)$
$x(t)$
t
$t_0-\frac{\varepsilon}{2}$
t_0
$t_0+\frac{\varepsilon}{2}$

$$\int_{-\varepsilon/2+t_0}^{\varepsilon/2+t_0} x(t)\,dt \approx x(t_0)\,\varepsilon$$

This approximation becomes exact as $\varepsilon \to 0$

Hence,

$$\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \int_{-\frac{\varepsilon}{2}+t_0}^{\frac{\varepsilon}{2}+t_0} x(t)\,dt$$

$$= \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \; x(t_0)\cdot\varepsilon$$

$$= x(t_0)$$

Therefore, for continuous functions $x(t)$,

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

This is often called the "sifting property" of the impulse

Ramp Functions

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$= t\,u(t)$$

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$= n\,u[n]$$