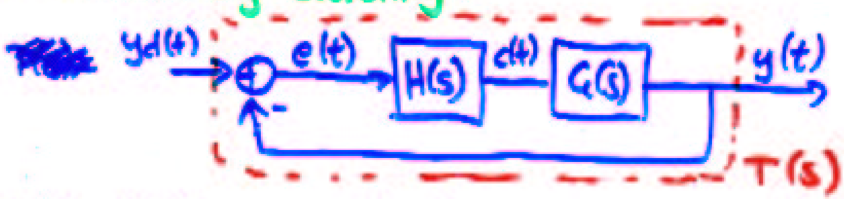
STABILITY

Large loop gain large $G(s)H(s)$

Good: $\Rightarrow$ loop is
i) less sensitive to parameter variations
ii) less sensitive to disturbances
iii) reduces distortion due to non-linear effects.

Bad, can cause instability in systems in which $G(s)H(s)$ has 3 or more poles. These are very common!

what do we mean by stability?



$$T(s) = \frac{Y(s)}{Yd(s)} = \frac{G(s)H(s)}{1 + G(s)H(s)} \quad ; \quad G(s) = \frac{N_G(s)}{D_G(s)} \quad H(s) = \frac{N_H(s)}{D_H(s)}$$

- This is a causal system, so for it to be stable, the poles of $T(s)$ must be in the left half plane.
- But what are the poles, because $G(s)$ and $H(s)$ are both ratios of polynomials?
- Using previous factorization $G(s)H(s) = \frac{P(s)}{s^k Q_1(s)}$

$$T(s) = \frac{P(s)}{s^k Q_1(s) + P(s)} \quad ; \quad P(s), Q_1(s) \text{ are polynomials}$$

- Hence poles of $T(s)$ are roots of

$$s^k Q_1(s) + P(s) = 0$$

- We will now explore the effects of a simple gain on the position of the poles.
- This is actually a precursor to the root locus which we will look at more closely next week

First-order plant, proportional controller

Consider the case where

$$G(s) = \frac{1}{\tau_0 s + 1}, \quad H(s) = K.$$

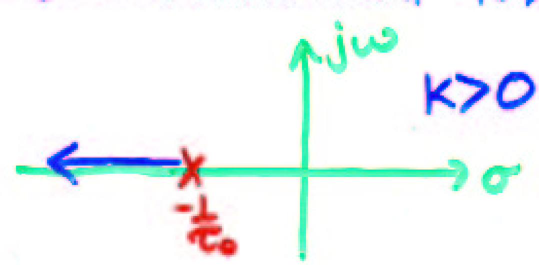
$$\Rightarrow \quad T(s) = \frac{K}{\tau_0 s + K + 1} = \frac{K/\tau_0}{s + \frac{K+1}{\tau_0}}$$

This has a single pole at $s = -\frac{K+1}{\tau_0}$

When $K=0$ pole is at $-1/\tau_0$

As K increases pole moves along negative real axis

$\Rightarrow$ system is stable for all $K \geq 0$



For $K<0$, pole moves to the right along the real axis

$\Rightarrow$ system is stable for $K>-1$

pole at origin for $K=-1$

system unstable for $K<-1$.

- How does our choice of gain affect the transient response?

- $$y_{step}(t) = \mathcal{L}_t^{-1}\left\{\frac{T(s)}{s}\right\}$$
$$= \frac{K}{K+1}u(t) - \frac{K}{K+1}e^{-\frac{(K+1)}{\tau_0}t}\,u(t).$$

- Steady state error $= \underbrace{}_{}\, = y_d(t) - y_{step}(t)$ $\; 1 - \frac{K}{K+1} = \frac{1}{1+K}$, as predicted by previous theory

- Transient response

  time constant is $\frac{\tau_0}{K+1}$

  $\Rightarrow$ as gain increases time constant decreases

  $\Rightarrow$ faster response.

Second order loop transfer function

- Now consider the case where

$$G(s)H(s) = \frac{K}{s(s\tau+1)}$$

- This arises when $G(s) = \frac{1}{s(s\tau+1)}$ and $H(s) = K$ (as in the lab)

  or when $G(s) = \frac{1}{s\tau+1}$ and $H(s) = \frac{K}{s}$ (integral control).

- In this case

$$T(s) = \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{K}{\tau s^2 + s + K}$$

- closed loop poles are at.

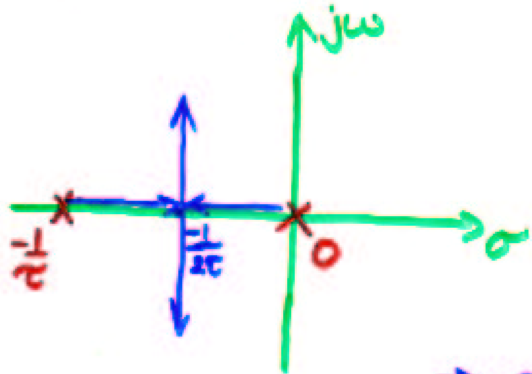
$$s = -\frac{1}{2\tau} \pm \sqrt{\frac{1}{4\tau^2} - \frac{K}{\tau}}$$

$\Rightarrow$ when $K=0$, poles are at $s = -\frac{1}{\tau}$, $s=0$

as K increases, poles approach each other on real axis

when $K = \frac{1}{4\tau}$, poles coincide at $s = -\frac{1}{2\tau}$

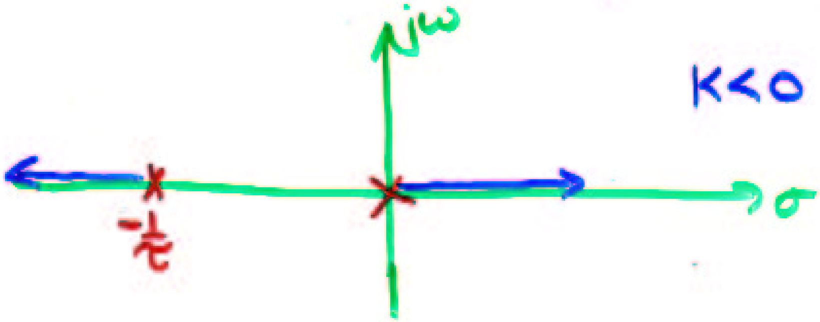
when $K > \frac{1}{4\tau}$, poles become complex conjugate pairs



K≥0

⇒ system is stable for all K>0

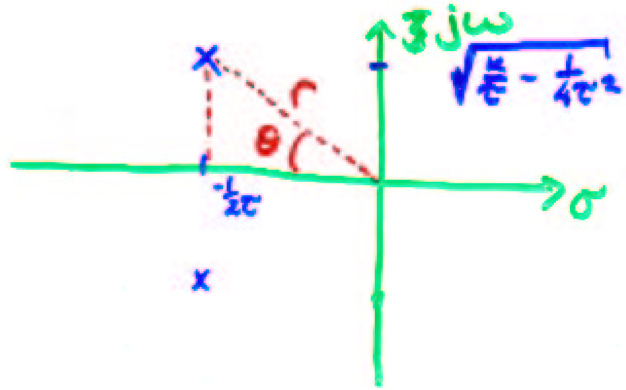
For K<0 poles move away from eachother
⇒ closed loop immediately becomes unstable



K<0

- Recall from yesterday's lecture that we can find ζ and ω_n of standard form of second order system from pole geometry.

- Assume $K > \frac{1}{4\tau}$



$\zeta = \cos\theta$

$\omega_n = r$

- Homework: Show that.

$$\zeta = \frac{1}{2\sqrt{\tau K}} \; ; \quad \omega_n = \sqrt{\frac{K}{\tau}}$$

Third and higher order loops

- If $G(s)H(s)$ is of order 1 or 2 (ie denominator polynomial is of order 1 or 2).

  then closed loop is stable for all $K>0$

- This is not true when $G(s)H(s)$ is of order ≥ 3

- For example.

$$G(s)H(s) = \frac{6K}{(s+1)(s+2)(s+3)}$$

$$\Rightarrow T(s) = \frac{6K}{s^3 + 6s^2 + 11s + 6(K+1)}$$

  The variation of poles with $K>0$ is given in Fig 9.26.

- Note that it is now hard to find analytic expressions for the poles.

- However we can develop rules which make it easy to sketch "root locus" diagrams

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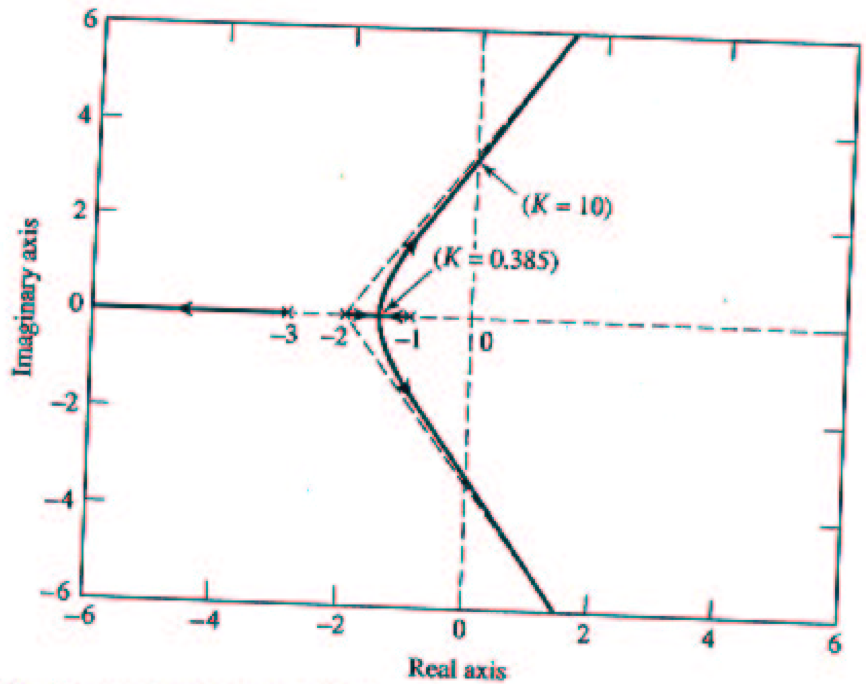

FIGURE 9.26 Root locus of third-order feedback system with loop transfer function

$$L(s) = \frac{6K}{(s+1)(s+2)(s+3)}$$