

# PROPERTIES OF SYSTEMS

## STABILITY

- In most systems, we would like the output to remain finite for all finite inputs

That is, we want

$$|y(t)| \leq M_y < \infty$$

for all  $x(t)$  such that

$$|x(t)| \leq M_x < \infty$$

- Such systems are said to be bounded-input bounded-output stable
- There is a simple characterization of BIBO stable linear time-invariant systems which we will see later

## Memory

- A system is said to have memory if its current output depends on previous inputs.
- otherwise it is memoryless.

- Consider a resistor as a system, with input  $i(t)$  and output  $v(t)$

Since  $v(t) = Ri(t)$  the system is memoryless

In contrast, for a capacitor,

$$v(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

and hence the system has memory

- The "moving-average" system

$$y[n] = \frac{1}{3} (x[n] + x[n-1] + x[n-2])$$

has a memory of 2 samples

# CAUSALITY

- A system is said to be causal if the current output depends only on current and previous inputs.
- For example, the moving average

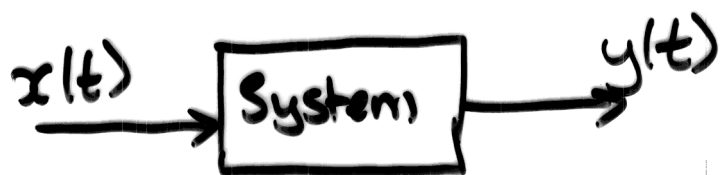
$$y[n] = \frac{1}{3} (x[n] + x[n-1] + x[n-2])$$

is causal, whereas

$$y[n] = \frac{1}{3} (x[n+1] + x[n] + x[n-1])$$

is not causal.

## TIME INVARIANCE



If we know that an input  $x(t)$  produces an output  $y(t)$ , then the system is said to be time-invariant if a delayed version of  $x(t)$  at the input produces a delayed version of  $y(t)$  at the output.

That is

$$x(t-t_0) \rightarrow y(t-t_0)$$

for all  $x(t)$  and  $t_0$

Almost all the circuits we dealt with in EE 2C34 were time invariant.

- Is any practical system truly time-invariant?

## LINEARITY

- A system is said to be linear if it obeys principle of superposition, for all inputs

- That is, if  $x_1(t)$  generates output  $y_1(t)$  and input  $x_2(t)$  generates output  $y_2(t)$

~~and~~ Then the system is linear if  
input  $a_1 x_1(t) + a_2 x_2(t)$

generates output  $a_1 y_1(t) + a_2 y_2(t)$

for all,  $a_1, a_2, x_1(t), x_2(t)$ .

- For example, the system

$$y(t) = c x(t) \text{ is linear}$$

- The system.

$$y(t) = c x(t)x(t-1) \text{ is non-linear}$$

- What about the system

$$y(t) = c x(t) + d$$

?

- Is any practical system truly linear?

# SUGGESTED PROBLEMS FROM CHAPTER 1

1.2

1.3

1.4 a, b, c, d

1.7

1.11

1.12 a, b, c

1.16 a)

1.21 a, c, c

1.28 a, c, d

1.31

1.34

1.41