PROPERTIES OF SYSTEMS

STABILITY

- In most systems, we would like the output to remain finite for all finite inputs

  That is, we want

$$|y(t)| \leq M_y < \infty$$

  for all $x(t)$ such that

$$|x(t)| \leq M_y < \infty$$

- Such systems are said to be bounded-input bounded-output stable

- There is a simple characterization of BIBO stable linear time-invariant systems which we will see later

Memory

- A system is said to have memory if its ~~cu~~ current output depends on previous inputs.
- otherwise it is memoryless.

- ~~For~~ Consider a resistor as a system, with input $i(t)$ and output $v(t)$

  Since $v(t) = R\,i(t)$ the system is memoryless

  In contrast, for a capacitor,

  $$v(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau)\,d\tau$$

  and hence the system has memory

- The "moving-average" system

  $$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

  has a memory of 2 samples

CAUSALITY

- A system is said to be causal if the current output depends only on current and previous inputs.

- For example, the moving average

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

is causal, whereas

$$y[n] = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$$

is not causal.

Time Invariance

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

If we know that an input $x(t)$ produces an output $y(t)$, then the system is said to be time-invariant if a delayed version of $x(t)$ at the input produces a delayed version of $y(t)$ at the output

That is

$$x(t-t_0) \longrightarrow y(t-t_0)$$

for all $x(t)$ and t_0

Almost all the circuits we dealt with in EE 2CJ4 were time invariant

- Is any practical system truly time-invariant ?

LINEARITY

- A system is said to be linear if it obeys principle of superposition, for all inputs
- That is, if $x_1(t)$ generates output $y_1(t)$ and input $x_2(t)$ generates output $y_2(t)$

  Then the system is linear if input $a_1 x_1(t) + a_2 x_2(t)$

  generates output $a_1 y_1(t) + a_2\, y_2(t)$

  for all, $a_1, a_2, x_1(t), x_2(t)$.

- For example, the system

  $$y(t) = c\, x(t)$$ is linear

- The system.

  $$y(t) = c\;\; x(t)\, x(t-1)$$ is non-linear

- What about the system

  $$y(t) = c\;\, x(t) + d$$?

- Is any practical system truly linear?

SUGGESTED PROBLEMS FROM CHAPTER 1

1.2
1.3
1.4 a, b, c, d
1.7
1.11
1.12 a, b, c

1.16 a)
1.21 a, c, e
1.28 a, c, d
1.31
1.34
1.41