TIME DOMAIN SPECIFICATIONS

- In many applications, some of the design objectives can be expressed in terms of the step response
- Typically these are.

    - Rise time, T_r : time taken to rise from 10% to 90% of ~~[illegible]~~ $y_{step}(\infty)$
    - Peak time, T_p: time when $y_{step}(t)$ reaches its maximum value, y_{max}
    - Percentage Overshoot, P.O.

$$P.O. = \frac{y_{max} - y(\infty)}{y(\infty)} \times 100$$

    - Settling time, T_s : time taken to settle within 5% of $y(\infty)$.

SECOND-ORDER UNDERDAMPED SYSTEMS

- These systems are important because they can be used to approximately model many practical systems

- Lets see if we can determine T_p and P.O. as functions of ω_n and ζ.
- Recall that

$$y_{step}(t) = \left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \phi\right)\right] u(t)$$

- We find the turning points of the function by differentiating it and setting to zero.

$$t_{\text{turning points}} = \frac{n\pi}{\omega_n\sqrt{1-\zeta^2}} \quad , \quad n = 0, 1, 2, \ldots$$

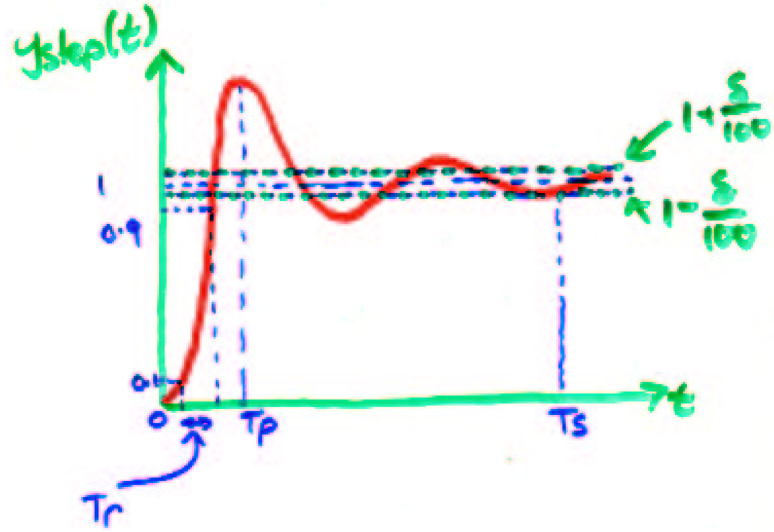
and $t = \infty$

If the system is underdamped, the first turning point has the largest overshoot.

$$\Rightarrow T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

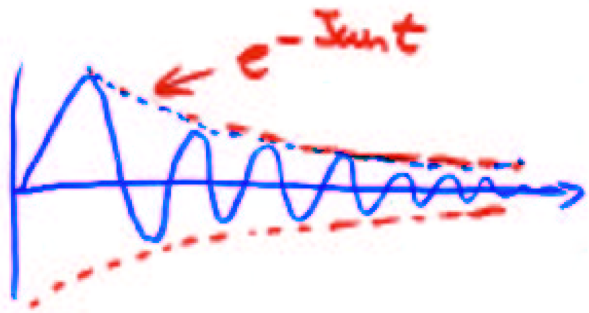
Furthermore, $y_{max} = y_{step}(T_p) = 1 + e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}}$

Hence $$P.O = 100 \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$



$$T(s) = \frac{T(0)\,\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$y_{step}(t) - 1 \propto e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \phi\right)$$



The values of T_r and T_s are more difficult to find exactly.

- However, the settling time can be approximated by determining when

$$e^{-\zeta \omega_n t} = \frac{5}{100}$$

This is an over estimate because it ignores the effects of the sine term

Hence $$T_s \approx \frac{-\log\left(\frac{5}{100}\right)}{\zeta \omega_n}$$

- The relationship betwen T_r and ω_n and ζ is even more complicated, but can be approximated numerically. For typical values of ζ,

$$T_r \approx \frac{0.6 + 2.16\zeta}{\omega_n}$$