ORTHOGONAL BLOCK CODE DESIGN FOR FREQUENCY-SELECTIVE MULTIPLE ANTENNA CHANNELS

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ABSTRACT

Orthogonal Space-Time Block Codes (OSTBCs) have several desirable properties for delay limited communication over a coherent narrowband (flat) fading multiple antenna channel. In particular, they provide full diversity, and maximum likelihood detection can be achieved via linear processing and symbol-by-symbol detection. As a result, the maximal symbol rate and minimal latencies of OSTBCs have been well studied. In this paper, we construct orthogonal block codes for frequency-selective multiple antenna channels, and study their symbol rate and latency properties. Our code construction is based on direct application of the orthogonality constraints. This enables us to show that the maximum symbol rate of an orthogonal block code for a channel with memory length L is $1/L$ times the maximum rate of an OSTBC for the same antenna configuration. Orthogonal codes that achieve this maximal rate can be simply constructed via the Kronecker product of a rate maximal OSTBC with an identity matrix of size L. This Kronecker design scheme can also be applied to construct orthogonal block codes for frequency-selective channels with other desirable properties, such as minimal latency.

1. INTRODUCTION

Orthogonal space-time block coding is a popular transmission scheme for narrowband coherent multiple antenna wireless links. Two key properties of the family of Orthogonal Space-Time Block Codes (OSTBCs) are that they enable full diversity gain, and that maximum likelihood detection can be achieved by linear processing and symbol-by-symbol detection. Several design schemes for OSTBCs have been established [1, 6, 10, 12–14], and the limits that the orthogonality constraint places on the maximum achievable (symbol) rate and the minimum achievable latency are well understood. There have been several approaches proposed for extending OSTBCs to frequency selective channels [3–5, 7–9, 11, 15]. These designs typically combine existing OSTBCs with certain transmission techniques for single-input single-output frequency-selective channels, such as Orthogonal Frequency Division Multiplexing (OFDM) or the time-reversal transmission scheme.

The goal of the present paper is obtain a direct construction of orthogonal space time block codes for frequency-selective multiple antenna channels, so that all the desirable properties of OSTBCs can be extended to wideband transmission systems. We seek a definitive statement of the limits that the orthogonality constraint places on the symbol rate and the latency of the code. We derive our construction using a "first principles" approach in which we directly enforce the orthogonality conditions. This enables us to show that the orthogonality constraint can be decoupled into independent orthogonality constraints in the space and delay domains. That allows us to show that the maximum symbol rate of any orthogonal block code for a frequency-selective channel with memory length L is $1/L$ times the maximal OSTBC rate for the flat-fading case with the same antenna configuration. Orthogonal codes which achieve that symbol rate can be simply constructed via the Kronecker product of a rate-maximal OSTBC for the flat-fading scenario [1, 6, 12] with an identity matrix of size L. The Kronecker coding scheme is also valid for non-maximal-rate designs, such as minimal latency codes [14]. If the OSTBC used in the Kronecker product operation has the minimum latency, then the resulting orthogonal code for the frequency-selective channel also has minimum latency.

To place our construction in context, we point out that several schemes have been proposed for applying OSTBCs to frequency-selective multiple antenna channels. A standard approach is to incorporate the principles of orthogonal frequency division multiplexing (OFDM), such as the space-time/space-frequency OFDM (ST/SF-OFDM) schemes [5], and the circulant generalized delay diversity (CGDD) scheme [3]. These schemes obtain a symbol rate which is a factor of $T/(T+(L-1))$ lower than that of the corresponding OSTBC, where T is the length of the data block. For long block lengths this factor can be close to 1, but if low latency is required the block length will be short and the effect of the cyclic prefix on the symbol rate will be significant. Furthermore, many of these coding schemes do not provide full diversity, because the channel matrix may drop rank at some subcarrier frequencies. An alternative technique is the time-reversal space-time block coding (TR-STBC) scheme [4, 9]. Its symbol rate reduction from the OSTBC case is a factor of $T/\big(T+2(L-1)\big)$, and it also requires a more complex receiver. The TR-STBC scheme lies within a significantly larger class of coding schemes for frequency-selective multiple antenna channels [15]. The symbol rate reductions of the schemes in that class are of a similar order to those above, and some of the schemes provide full diversity, but the (optimal) detection problem remains substantially more expensive than that of OSTBCs in flat fading.

It is well known that the price paid for the advantages of orthogonality in the flat-fading environment is a reduction in the maximum (symbol) rate of the system. That is, OSTBCs provide a particular trade-off between rate, diversity (BER performance), and system complexity[1]. In this paper, we show that in the case of frequency-selective channels, the rate penalty for orthogonality is L times larger than that for the flat-fading case. In spite

[1]Different coding schemes produce different trade-offs. For example, full-rate full diversity designs are available [2, 8], but they require a more complex receiver than an orthogonal design

of this, simulation results show that the orthogonal block code for frequency-selective channels can provide better performance than some existing designs, such as the ST/SF-OFDM [4, 5] and CGDD [3] schemes.

The notations used in this paper are reasonably standard: Bold upper case letters denote matrices, $\boldsymbol{A}_j$ is a submatrix of $\boldsymbol{A}$, and $\boldsymbol{A}_{ij}$ is a submatrix of $\boldsymbol{A}_j$. Bold lower case letters denote column vectors, $\boldsymbol{a}_j$ is the jth column in $\boldsymbol{A}$, $\boldsymbol{a}_{i,j}$ is the ith column in $\boldsymbol{A}_j$. The vector $\text{vec}(\boldsymbol{A})$ is formed by stacking the columns of $\boldsymbol{A}$, $(\cdot)^T$ denotes transpose, $(\cdot)^H$ denotes the conjugate transpose, and $\boldsymbol{I}_K$ denotes the identity matrix of size K. The symbol $\otimes$ denotes the Kronecker product.

2. SYSTEM MODEL

We consider a multiple antenna communication system with M transmit and N receive antennas. The channel between the jth transmit and ith receive antenna is frequency-selective with transfer function $H_{ji}(z) = \sum_{\ell=0}^{L-1} h_{ji}^{\ell} z^{-\ell}$. We assume that the coefficients $\{h_{ji}^{\ell}\}$ are independent zero-mean complex circular Gaussian random variables with equal variance. The realization of $\{h_{ji}^{\ell}\}$ is assumed to be constant over a block and known at receiver but not known at transmitter. The noise is zero-mean Gaussian and statistically independent among the N receive antennas. We consider a block transmission system and we assume that at least $(L-1)$ zeros are padded between consecutive transmitted blocks in order to avoid inter-block interference at the receiver. The transmitted signal from the jth transmit antenna over $(T+L-1)$ time slots is denoted by $\boldsymbol{s}_j = (s_{1,j} \quad s_{2,j} \quad \cdots \quad s_{T+L-1,j})^T$, where $s_{T+1,j} = \cdots = s_{T+L-1,j} = 0$. The received signal vector at the ith receive antenna is

$$\boldsymbol{y}_i = \sum_j \boldsymbol{\mathcal{H}}_{ji} \boldsymbol{s}_j + \boldsymbol{w}_i = \boldsymbol{\mathcal{H}}_i \boldsymbol{s} + \boldsymbol{w}_i, \tag{1}$$

where $\boldsymbol{y}_i = (y_{1,i} \quad \cdots \quad y_{(T+L-1),i})^T$, and $\boldsymbol{\mathcal{H}}_i = (\boldsymbol{\mathcal{H}}_{1i} \quad \cdots \quad \boldsymbol{\mathcal{H}}_{Mi})$, where $\boldsymbol{\mathcal{H}}_{ji}$ is the $(T+L-1) \times T$ lower triangular Toeplitz channel matrix between the jth transmit antenna and the ith receiver antenna. Its first column is equal to $(h_{ji}^0 \quad \cdots \quad h_{ji}^{L-1} \quad 0 \cdots 0)^T$. The vector $\boldsymbol{s} = (\boldsymbol{s}_1^T \quad \cdots \quad \boldsymbol{s}_M^T)^T$, stacks all the signals transmitted from the M transmit antennas, and $\boldsymbol{w}_i$ is the noise vector at the ith receive antenna. Our goal is to design orthogonal block codes for the system in (1). To do so we need to reformulate (1) such that the signal transmission matrix for the system is explicit:

$$\boldsymbol{y}_i = \sum_j \boldsymbol{S}_j \boldsymbol{h}_{ji} + \boldsymbol{w}_i = \boldsymbol{S} \boldsymbol{h}_i + \boldsymbol{w}_i, \tag{2}$$

where $\boldsymbol{S} = (\boldsymbol{S}_1 \quad \boldsymbol{S}_2 \quad \cdots \quad \boldsymbol{S}_M)$, and $\boldsymbol{S}_j$ is the $(T+L-1)\times L$ Toeplitz signal matrix transmitted from the jth transmit antenna,

$$\boldsymbol{S}_j = \begin{pmatrix} s_{1\,j} & 0 & \cdots & 0 \\ s_{2\,j} & s_{1\,j} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ s_{T\,j} & s_{(T-1)\,j} & \cdots & s_{(T-L+1)\,j} \\ 0 & s_{T\,j} & \cdots & s_{(T-L)\,j} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & s_{T\,j} \end{pmatrix}. \tag{3}$$

The vector $\boldsymbol{h}_{ji} = (h_{ji}^0 \quad \cdots \quad h_{ji}^{L-1})^T$ contains the channel coefficients from the jth transmit antenna to the ith receive antenna, and $\boldsymbol{h}_i = (\boldsymbol{h}_{1i}^T \quad \cdots \quad \boldsymbol{h}_{Mi}^T)^T$ is the length LM vector of stacked channel coefficients from the M transmit antennas to the ith receive antenna. Based on (2), the vector of all the received signals on all N receive antennas can be expressed as $\text{vec}(\boldsymbol{Y}) = (\boldsymbol{I}_N \otimes \boldsymbol{S})\,\text{vec}(\boldsymbol{H}) + \text{vec}(\boldsymbol{W})$, where the matrix $\boldsymbol{H} = (\boldsymbol{h}_1 \quad \cdots \quad \boldsymbol{h}_N)$.

3. ORTHOGONAL BLOCK CODE DESIGN

The orthogonality criterion for block codes for a frequency-selective channel is that the signal transmission matrix (code matrix) $\boldsymbol{S}$ in (2) is orthogonal. Following the standard conventions for OSTBCs for flat-fading channels in [6, 12, 14], we assume that the entries of the code matrix $\boldsymbol{S}$ are $0, \pm s_k, \pm s_k^*$ or their multiples by $i = \sqrt{-1}$. The symbols s_k, $k = 1, 2, \cdots, q$ are taken from a complex signal constellation. The orthogonality constraint can be written as

$$\boldsymbol{S}^H \boldsymbol{S} = \textstyle\sum_{k=1}^{q} |s_k|^2 \boldsymbol{I}. \tag{4}$$

The symbol rate of the code is $R = \frac{q}{T+L-1}$. Since $\boldsymbol{S}$ has a block Toeplitz structure, the Hurwitz-Randon theory used in the design of OSTBCs for the flat-fading case can not be applied directly. However, we will show that the existing classes of orthogonal codes for flat-fading channels can be used to design orthogonal block codes for frequency-selective channels. Our analysis will start with the orthogonality among the sub-matrices in $\boldsymbol{S}$. We then propose a two-step design scheme for orthogonal block codes for frequency-selective channels and establish the maximum symbol rate.

As indicated after (2), the code matrix $\boldsymbol{S}$ collects all the signals from all the transmit antennas; i.e., $\boldsymbol{S}$ is composed of submatrices $\boldsymbol{S}_j$, $j = 1, \cdots, M$. Therefore, a necessary condition for $\boldsymbol{S}$ to be orthogonal is that each $\boldsymbol{S}_j$ is orthogonal. Let $\boldsymbol{s}_{i,j}$ the ith column of $\boldsymbol{S}_j$. Because $\boldsymbol{S}_j$ has a Toeplitz structure, the largest number of non-zero entries in the first column of a matrix $\boldsymbol{S}_j$ that is orthogonal can be bounded, as we show in the following lemma. Furthermore, the zero entries have a certain pattern. This pattern plays an important part in our orthogonal design.

3.1. Channel length $L = 2$

For convenience we will begin by detailing the design procedure for the case of $L = 2$. The general case is summarized later.

Lemma 1 *Given $T \geq 2$, the largest number of independent variables $x_j \in \mathbb{C}$ in the equation $\sum_{j=1}^{T-1} x_j x_{j+1}^* = 0$ is $\lceil \frac{T}{2} \rceil$. One such arrangement is that in which the x_j with even indices are zero and those with odd indices are free.*

***Proof*:** The proof will proceed by induction on T. The statement is immediate for $T = 2$. For $T = 3$, the largest number of independent non-zero variables for $x_1 x_2^* + x_2 x_3^* = 0$ is 2, which is achieved when $x_2 = 0$. For $T = 4$, the largest number of independent variables for $x_1 x_2^* + x_2 x_3^* + x_3 x_4^* = 0$ is still 2. Possible arrangements include the case where $x_2 = x_4 = 0$. Thus the statement is true for $T = 2m + i$, $m = 1, i = 1, 2$.

Suppose that for a given $K \geq 2$, the statement is true for $T = 2K + i$, $i = 1, 2$, i.e., the largest number of independent variables is $K + 1$, and the non-zero variables have odd indices. For $T = 2(K+1) + i$, $i = 1, 2$, we have

$$\textstyle\sum_{j=1}^{2(K+1)+i} x_j x_{j+1}^* = \sum_{j=1}^{2(K+1)} x_j x_{j+1}^* + \Delta, \tag{5}$$

where $\Delta = \sum_{j=1}^{i} x_{2(K+1)+j-1} x^*_{2(K+1)+j}$. By the inductive hypothesis, $\sum_{j=1}^{2(K+1)} x_j x^*_{j+1} = 0$ and $x_{2m} = 0$, $m = 1, 2, \cdots, K+1$. Therefore, to prove the statement for $T = 2(K+1)+i$, $i = 1, 2$, we need only consider Δ. For the case where $i = 1$, $\Delta = x_{2(K+1)} x_{2(K+1)+1}$, but since $x_{2(K+1)} = 0$, $\Delta = 0$ independent of the value of $x_{2(K+1)+1}$. Therefore there are $K+2$ independent variables and the statement holds for $i = 1$. For the case where $i = 2$, $\Delta = x_{2(K+1)+1} x_{2(K+1)+2}$, where we have used the fact that $x_{2(K+1)} = 0$. To ensure that $\Delta = 0$ one of variables $x_{2(K+1)+1}$ and $x_{2(K+1)+2}$ must be zero. Hence, there are still $K+2$ independent variables and if we set $x_{2(K+1)+2} = 0$ the free variables have odd indices and the even indexed variables are zero. Therefore, the statement holds for $i = 2$ and the proof is complete. □

Orthogonality of $\boldsymbol{S}_j$ requires that $s^H_{1,j} s_{2,j} = 0$, $j = 1, \cdots, M$. By applying Lemma 1 to this case, the non-zero entries for $s_{1,j}$ can be chosen as $\{s_{n,j} | n = 1, 3, \cdots, 2(\lceil \frac{T}{2} \rceil - 1) + 1\}$. Therefore, $\boldsymbol{S}$ has the form,

$$\begin{pmatrix} s_{1,1} & 0 & s_{1,2} & 0 & \cdots & \cdots & s_{1,M} & 0 \\ 0 & s_{1,1} & 0 & s_{1,2} & \cdots & \cdots & 0 & s_{1,M} \\ s_{3,1} & 0 & s_{3,2} & 0 & \cdots & \cdots & s_{3,M} & 0 \\ 0 & s_{3,1} & 0 & s_{3,2} & \cdots & \cdots & 0 & s_{3,M} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ 0 & s_{b,1} & 0 & s_{b,2} & \cdots & \cdots & 0 & s_{b,M} \end{pmatrix}, \qquad (6)$$

where $b = 2\lceil \frac{T}{2} \rceil - 1$. Now it can be seen from (6) that $\boldsymbol{S}$ is orthogonal if and only if the £rst columns among all $\boldsymbol{S}_j$'s orthogonal; i.e., if the matrix $\boldsymbol{G} = (s_{1,1} \quad s_{1,2} \quad \cdots \quad s_{1,M})$ is orthogonal. Thus, the constraint for the orthogonality of $\boldsymbol{S}$ can be decoupled into constraints for orthogonality in the delay and spatial domains. Therefore, an orthogonal design strategy can be implemented by the following two steps.

- Step 1: Delay domain — Make the signal matrix for each transmit antenna, $\boldsymbol{S}_j$, orthogonal.
- Step 2: Spatial domain — Make the £rst signal vector among different transmit antennas orthogonal, i.e., make the matrix $\boldsymbol{G} = (s_{1,1} \quad s_{1,2} \quad \cdots \quad s_{1,M})$ orthogonal.

The maximum diversity gain for a frequency-selective channel is MNL [3, 15]. That gain can be decomposed into the multipath diversity gain L and spatial diversity gain MN. In our approach, the orthogonality constraint in Step 1 extracts the multipath diversity and that in Step 2 extracts the spatial diversity. Therefore, our approach provides full diversity, as expected. The design scheme leads to the following proposition.

Proposition 1 *The maximum symbol rate of an orthogonal block code for a frequency-selective channel of length $L = 2$ and M transmit antennas is $R = R_{¤at\text{-}max}/2$, where $R_{¤at\text{-}max}$ is the rate achieved by a rate-maximal orthogonal space time block code $\boldsymbol{Q}$ for a ¤at fading channel with M transmit antennas. The code matrix $\boldsymbol{S} = \boldsymbol{Q} \otimes \boldsymbol{I}_2$ achieves this rate, and the delay of that code is $d = 2d_{¤at}$, where $d_{¤at}$ is the delay of $\boldsymbol{Q}$.*

Proof: By applying Lemma 1, we need only consider code matrices of the form in (6). From the discussion above, the effective size of $\boldsymbol{G}$ is $\lceil \frac{T}{2} \rceil \times M$ if the zero entries are excluded. Suppose a code matrix $\boldsymbol{Q}$ for a ¤at fading channel has a maximum rate $R_{¤at\text{-}max} = q_{¤at}/d_{¤at}$, where $q_{¤at}$ is the number of symbols transmitted, and $d_{¤at}$ is the delay. The corresponding T for a frequency-selective channel is determined by $\lceil \frac{T}{2} \rceil = d_{¤at}$. The minimum T must be odd and is given by $T = 2(d_{¤at} - 1) + 1$. Given that $L - 1$ zeros are padded between the blocks in the frequency-selective case, the delay for a frequency-selective channel is $d = T + L - 1 = 2d_{¤at}$. Therefore, the maximal rate is $R = q_{¤at}/(2d_{¤at}) = R_{¤at\text{-}max}/2$. The non-zero entries in the £rst column in all $\boldsymbol{S}_j$'s can take the corresponding entries of $\boldsymbol{Q}$. Due to the block Toeplitz structure of the code matrix $\boldsymbol{S}$, it can be written as the Kronecker product of $\boldsymbol{Q}$ and an identity of size of 2, $\boldsymbol{S} = \boldsymbol{Q} \otimes \boldsymbol{I}_2$. □

Note that the proof of Proposition 1 indicates that if the OSTBC used in the Kronecker product operation has minimum latency, the resulting orthogonal code for the frequency-selective channel also has minimum latency. Therefore, the minimum latency of an orthogonal code for $L = 2$ is $2d_{¤at\text{-}min}$, where $d_{¤at\text{-}min}$ is the minimum delay of the OSTBC for the ¤at-fading case.

Let us examine some examples of Proposition 1 for systems with $L = 2$ and different numbers of transmit antennas M. For $M = 2$, the minimum block length is $T = 3$. The code matrix $\boldsymbol{S} = (\boldsymbol{S}_1 \quad \boldsymbol{S}_2) = (s_{1,1} \quad s_{2,1} \quad s_{1,2} \quad s_{2,2})$. The orthogonality constraint in delay domain (Step 1) makes $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ orthogonal. Hence, after Step 1, we have

$$\boldsymbol{S}^{(2,2)}_{\text{stp}_1} = \left(\begin{array}{cc|cc} s_{1,1} & 0 & s_{1,2} & 0 \\ 0 & s_{1,1} & 0 & s_{1,2} \\ s_{3,1} & 0 & s_{3,2} & 0 \\ 0 & s_{3,1} & 0 & s_{3,2} \end{array}\right), \qquad (7)$$

where the superscript $^{(2,2)}$ represents $M = 2$ and $L = 2$. The role of Step 2 is to make the £rst columns in $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ orthogonal, i.e., to make the matrix $\boldsymbol{G} = (s_{1,1} \quad s_{1,2})$ orthogonal. If the zero entries are excluded, $\boldsymbol{G}$ has an effective size of 2×2. The maximum number of symbols that can be transmitted is 2, and this rate is achieved by Alamouti's scheme [1]. The delay in this case is $d = T + L - 1 = 4$, the rate is $2/4 = 1/2$, and the code matrix can be constructed as

$$\boldsymbol{S}^{(2,2)} = \begin{pmatrix} s_1 & 0 & s_2 & 0 \\ 0 & s_1 & 0 & s_2 \\ -s_2^* & 0 & s_1^* & 0 \\ 0 & -s_2^* & 0 & s_1^* \end{pmatrix} = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \otimes \boldsymbol{I}_2. \qquad (8)$$

For the case of $M = 4$ antennas at the transmitter, the minimum block length is $T = 7$. The maximum symbol rate of an orthogonal code is 3/8, and a rate-maximal code is

$$\boldsymbol{S}^{(4,2)} = \left(\begin{array}{cc|cc|cc|cc} s_1 & 0 & s_2 & 0 & s_3 & 0 & 0 & 0 \\ 0 & s_1 & 0 & s_2 & 0 & s_3 & 0 & 0 \\ -s_2^* & 0 & s_1^* & 0 & 0 & 0 & -s_3 & 0 \\ 0 & -s_2^* & 0 & s_1^* & 0 & 0 & 0 & -s_3 \\ -s_3^* & 0 & 0 & 0 & s_1^* & 0 & s_2 & 0 \\ 0 & -s_3^* & 0 & 0 & 0 & s_1^* & 0 & s_2 \\ 0 & 0 & s_3^* & 0 & -s_2^* & 0 & s_1 & 0 \\ 0 & 0 & 0 & s_3^* & 0 & -s_2^* & 0 & s_1 \end{array}\right) = \boldsymbol{Q}_{\frac{3}{4}} \otimes \boldsymbol{I}_2. \qquad (9)$$

where $\boldsymbol{Q}_{\frac{3}{4}} = \begin{pmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & -s_3 \\ -s_3^* & 0 & s_1^* & s_2 \\ 0 & s_3^* & -s_2^* & s_1 \end{pmatrix}$, is the 3/4 OSTBC for a ¤at fading channel [12–14]. Observe that the codes $\boldsymbol{S}^{(2,2)}$ in (8) and $\boldsymbol{S}^{(4,2)}$ in (9) not only have the maximum rate but also have minimum latency, since Alamouti's scheme and $\boldsymbol{Q}_{\frac{3}{4}}$ have the minimum latency. For the case of $M = 3$, an orthogonal code can be obtained from the $M = 4$ case in (9) by removing 2 columns, say the last two columns. The maximal symbol rate remains the same.

For $M = 5$ and 6 transmit antennas, the maximum rate of an OSTBC for a ¤at-fading channel is 2/3, and this can be achieved with a with code length of 15 [6]. Applying the 2/3 code to our design approach, the maximum orthogonal code rate for $L = 2$ is $R = 1/3$ with delay of 30. The code has the form

$$S^{(5,2)} = Q_{\frac{2}{3}} \otimes I_2, \qquad (10)$$

where $Q_{\frac{2}{3}}$ is given in [6]. The code $S^{(5,2)}$ in (10) has the maximum rate but not minimum latency because $Q_{\frac{2}{3}}$ has the maximum rate but not minimum latency. The minimum latency of OSTBC for 5 or 6 transmit antennas in the flat-fading case is $2^{\lceil \log_2 M \rceil} = 8$, [14]. Therefore, the minimum latency of the orthogonal code for $M = 5, 6$ and $L = 2$ is 16, rather than the delay of 30 required by the code in (10). However, our design scheme can be applied to construct a minimal-latency orthogonal code for frequency-selective channels of $L = 2$ by simply using the minimum latency OSTBCs described in [14] in the Kronecker product. (The resulting codes are not necessarily rate maximal if $M \geq 5$.)

It is also possible to construct a non-maximal-rate and non-minimal-latency orthogonal block code using our design procedure. For example, for $L = 2$ and $M = 4$ a rate $1/4$ orthogonal block code can be constructed as $Q_{\frac{1}{2}} \otimes I_2$, where $Q_{\frac{1}{2}}$ is the rate $1/2$ OSTBC given in [13].

3.2. General case

For a frequency-selective channel with length $L \geq 3$, our results can be generalized to the following lemma and proposition. The proof of the lemma is given in the Appendix, and that of the proposition is analogous to the proof of Proposition 1.

Lemma 2 *Given $T \geq L$, the largest number of independent variables $x_j \in \mathbb{C}$ in the $L - 1$ equations*

$$\sum_{j=1}^{T-\ell} x_j x_{j+\ell}^* = 0 \qquad \ell = 1, 2, \cdots, L-1, \qquad (11)$$

is $\lceil \frac{T}{L} \rceil$. The free variables can be $x_1, x_{L+1}, \cdots, x_{L(\lceil \frac{T}{L} \rceil - 1)+1}$.

Proposition 2 *The maximum symbol rate of an orthogonal block code for a frequency-selective channel of length L and M transmit antennas is $R = R_{\text{flat-max}}/L$, where $R_{\text{flat-max}}$ is the rate achieved by a maximal rate orthogonal space time block code Q for a flat fading channel with M transmit antennas. The code matrix $S = Q \otimes I_L$ achieves this rate, and the delay of that code is $d = Ld_{\text{flat}}$, where d_{flat} is the delay of Q.*

Analogous to the case where $L = 2$, the minimum latency of the orthogonal code is $Ld_{\text{flat-min}}$.

4. SIMULATION

We consider a system with 2 transmit antennas and 1 receive antenna. The channel is a frequency-selective Rayleigh fading channel with memory length 2. We compare the proposed orthogonal code with the ST-OFDM and CGDD schemes. Each scheme is allocated the same transmission power and we consider schemes with a symbol block length of 4. Using Alamouti's code [1], the ST-OFDM and CGDD schemes [3, 5] will require 5 channel uses, and these schemes have a symbol rate of $4/5$. In contrast, the proposed scheme has a symbol rate of $1/2$.

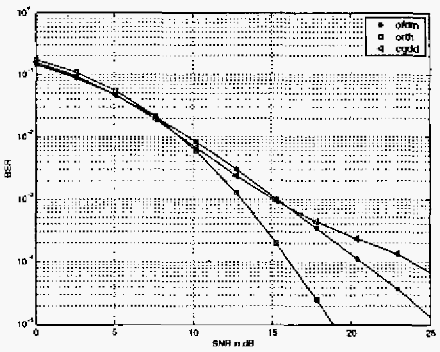

Fig. 1. BER performance against block SNR for our orthogonal design (at 1 bits per channel use) and the ST-OFDM and CGDD schemes (at $4/5$ bits per channel use).

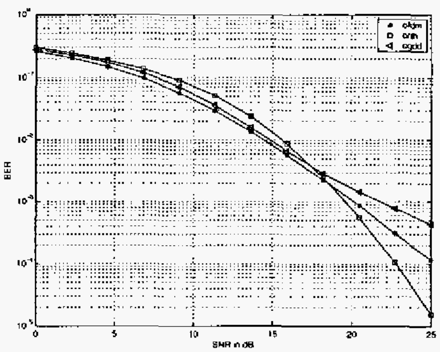

Fig. 2. BER performance against block SNR for our orthogonal design (at 2 bits per channel use) and the ST-OFDM and CGDD schemes (at $8/5$ bits per channel use).

In Fig 1 we have provided the BER performance of the proposed scheme when 4-QAM signalling is used. In this case the bit rate is 1 bit per channel use. In that figure we have also plotted the BER performance of the ST-OFDM and CGDD schemes with BPSK signalling, which results in a slightly lower bit rate of $4/5$ bits per channel use. In Fig 2 we have provided the BER performance of the proposed scheme with 16-QAM signalling (2 bits per channel use) and that of the ST-OFDM and CGDD schemes with 4-QAM signalling ($8/5$ bits per channel use).

Despite the fact that these configurations result in the proposed scheme having a higher data rate than its competitors, the simulation results demonstrate improved performance at moderate-to-high SNRs. This is due to the fact that the proposed scheme provides full diversity. Actually, the performance advantage of the proposed scheme is slightly larger than shown, because we have neglected the power used to transmit the cyclic prefix in the calculation of the SNR for the ST-OFDM and CGDD schemes.

5. CONCLUSION

A direct construction of orthogonal block codes for frequency-selective multiple antenna channels has been derived. The maximum achievable (symbol) rate was found and a rate-maximizing code structure was specifed. The orthogonal design can be realized by a two-step orthogonality strategy. The first step is to make the signal matrix transmitted from each antenna self-orthogonal. This step obtains delay diversity. The second step is to make the first signal vector among all transmit antennas orthogonal. This step obtains spatial diversity. For a flat-fading channel only the second step needs to be considered. This two-step design strategy reveals the relationship between the orthogonal block codes for frequency-selective channels and those for flat-fading channels. An orthogonal block code for a frequency-selective channel of memory length of L can be simply constructed by the Kronecker product of an existing orthogonal space time block code matrix for the flat-fading channels with an identity matrix of size of L. If the orthogonal space-time block code for the flat-fading channels used in the Kronecker product operation is rate maximal or latency minimal, the resulting orthogonal code for the frequency-selective channels is rate maximal or latency minimal, respectively.

Acknowledgement: The authors wish to thank Dr. Zhi-Quan (Tom) Luo of The University of Minnesota and Dr. Jian-Kang Zhang of McMaster University for valuable discussions.

6. APPENDIX: PROOF OF LEMMA 2

For the case of $L = 2$, the statement collapses to that in Lemma 1, the proof of which involved induction on T. To prove the more general statement in Lemma 2, we will employ an additional (outer) induction on L.

The (outer) inductive hypothesis that the statement holds for $L = P$ states that for $T \geq P$ there are $\lceil \frac{T}{P} \rceil$ free variables in

$$\left\{x_j | \mathcal{S}_{T,\ell} \triangleq \sum_{j=1}^{T-\ell} x_j x_{j+\ell}^* = 0,\ \ell = 1, 2, \ldots, P-1\right\}_{j=1}^{T}$$

and that they may be chosen to be $x_1, x_{P+1}, \cdots, x_{P(\lceil \frac{T}{P} \rceil - 1)+1}$. To show that this hypothesis implies that when $L = P + 1$ the statement holds for all $T \geq P + 1$ we will use a structured induction argument on T. We will treat the case of $T = P+1$ separately, and then will use induction on K and i to show that the lemma holds for $T = K(P + 1) + i$ for all $K \geq 1$ and $1 \leq i \leq P + 1$.

For $T = P + 1$ we have that $\mathcal{S}_{P+1,\ell} = \mathcal{S}_{P,\ell} + \Delta_{P+1,\ell}$, where $\Delta_{P+1,\ell} = x_{P+1-\ell} x_{P+1}^*$. By the (outer) inductive hypothesis (on L) we have that $\mathcal{S}_{P,\ell} = 0$ and $\Delta_{P+1,\ell} = 0$ for all $1 \leq \ell \leq P - 1$. Therefore, we need only enforce $\mathcal{S}_{P+1,P} = x_1 x_{P+1}^* = 0$. To do so, we can simply set $x_{P+1} = 0$, which results in one free variable, x_1, as stated in the lemma.

For $T = P + 2$, we have $\mathcal{S}_{P+2,\ell} = \mathcal{S}_{P+1,\ell} + \Delta_\ell$, where $\Delta_{P+2,\ell} = x_{P+2-\ell} x_{P+2}^*$. Under the (outer) inductive hypothesis (on L), $\mathcal{S}_{P+1,\ell} = 0$ for $1 \leq \ell \leq P - 1$ and $\Delta_{P+2,\ell} = 0$ for $2 \leq \ell \leq P$. Therefore, the equations which remain to be satisfed are $\mathcal{S}_{P+1,P} = x_1 x_{P+1}^* = 0$ and $\Delta_{P+2,1} = x_{P+1} x_{P+2} = 0$. By setting $x_{P+1} = 0$ we satisfy both equations and retain the largest number of free variables. Hence the lemma holds for $T = P + 2$. We now make the (inner) hypothesis that the lemma holds for $T = P + 1 + i$ for some $1 \leq i \leq P$, and examine the equations when $T = P + 2 + i$. Under this hypothesis, $\mathcal{S}_{P+2+i,\ell} = 0$, except when $\ell = i$. When $\ell = i$ we have $\mathcal{S}_{P+2+i,i} = x_{P+2} x_{P+2+i}^*$, and hence we must choose $x_{P+2+i} = 0$ in order to satisfy (11). We have now shown that if the lemma holds for $L = P$ then it holds for $L = P + 1$ and $P + 1 \leq T \leq 2(P + 1)$. What remains to be shown is that the lemma also holds for larger values of T.

Suppose that for some $K \geq 2$ the statement holds for $L = P + 1$ and $T = K(P + 1) + m$, for all $1 \leq m \leq P + 1$, and consider the case where $T = (K + 1)(P + 1) + i$, for $1 \leq i \leq P + 1$. For $i = 1$ we have $\mathcal{S}_{(K+1)(P+1)+1,\ell} = \mathcal{S}_{(K+1)(P+1),\ell} + x_{(P+1)(K+1)+1-\ell} x_{(K+1)(P+1)+1}^*$. The inductive hypothesis on K ensures that $\mathcal{S}_{(K+1)(P+1),\ell} = 0$ and $x_{(K+1)(P+1)+1-\ell} = 0$ for all $1 \leq \ell \leq P$. Hence, $x_{(K+1)(P+1)+1}$ may be chosen freely, and the lemma holds. If we assume that the lemma holds for $T = (K + 1)(P + 1) + i$ for some $1 \leq i \leq P$, then for $T = (K + 1)(P + 1) + i + 1$ we have that $\mathcal{S}_{(K+1)(P+1)+i+1,\ell} = 0$, except when $\ell = i$. Therefore, in order to satisfy (11) we must choose $x_{(K+1)(P+1)+i+1} = 0$. Hence, the lemma holds for $T = (K + 1)(P + 1) + i$, $1 \leq i \leq P + 1$. These induction arguments on K and i verify that the outer induction argument on L holds for all values of $T \geq 2(P + 1) + 1$, and hence the proof is complete.

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