LOW-COMPLEXITY ROBUST MISO DOWNLINK PRECODER OPTIMIZATION FOR THE LIMITED FEEDBACK CASE

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ABSTRACT

We consider the design of the linear precoder for a multiple-input single-output (MISO) downlink in a system that employs limited feedback using Grassmannian quantization. The goal is to minimize the outage probability of a target signal-to-interference-and-noise ratio (SINR) under a transmitted power constraint. By approximating the outage constraint by a zero-outage region, employing a semidefinite relaxation, and applying an extension of the S-Lemma, the problem is converted into a quasi-convex problem. Insights into the structure of the solution of that problem generate an alternate design formulation that provides greater robustness in the presence of significant uncertainties and has a quasi-closed form solution.

***Index Terms*—** Broadcast channel, beamforming, quality-of-service, Grassmannian limited feedback, chance constraints.

1. INTRODUCTION

In the communication of inelastic data traffic from a base station (BS) to multiple receivers, an effective strategy for the design of the transmitter is to seek to minimize the transmission power required to enable reliable communication to each receiver at the chosen data rates. Given the complexity of implementing the optimal encoding structure for the downlink [1], such quality-of-service (QoS) design problems are typically formulated for linear transmitters; e.g., [2], [3]. In scenarios in which the receivers have a single antenna and are coherent, and the channels are memoryless, the linear QoS design problem reduces to optimizing the BS's precoding matrix so as to minimize the power required to satisfy SINR constraints at the receivers [2–4]. In order to perform this design, the BS must be able to determine the SINRs at the receivers as a function of the precoder. However, doing so requires the BS to obtain accurate information on the state of the channels to the receivers; i.e., accurate CSI. Acquiring that CSI requires channel resources that are consequently not available for the downlink communication task. As a result, in practice the BS has only estimates of the CSI and can only estimate the SINR at each receiver. The precoder design must be performed in the presence of this uncertainty.

One approach to dealing with the uncertainty in the BS's CSI is to develop a model for the uncertainty and incorporate that model into the design problem. One way to do that is to model the actual channel as lying in a compact set around the channel estimate at the BS, and to design the precoder so that the transmission power is minimized subject to the SINR constraints holding for all channels in the set; e.g., [5–8]. An alternative to that "worst-case" approach is to model the uncertainties probabilistically, and to seek to minimize the transmission power subject to the SINR constraints holding with a given probability; i.e. minimizing the power subject to outage constraints [9–11]. Although the techniques that we develop herein are directly applicable to that problem, we will focus on the related problem of minimizing the outage probability subject to a constraint on the transmission power.

Many of the existing design techniques for probabilistic uncertainty models are based on conservative convex restrictions of the design problem that replace the outage constraint by a set of channels over which zero outage is to be guaranteed [9, 10, 12, 13]. Doing so enables the application of existing techniques for the worst-case problem, but these techniques are inherently conservative and their performance degrades significantly in the presence of larger uncertainties or aggressive SINR targets. Furthermore, although the resulting design problems are convex, they typically involve linear matrix inequalities (LMIs), and the computational cost of solving them can be quite substantial. Finally, most of the existing approaches to this "zero-outage" conversion of outage-based problems have been developed for cases in which the uncertainty can be modelled as being additive and Gaussian. In time-division duplexing (TDD) systems that exploit reciprocity, the Gaussian assumption is often quite reasonable (e.g., [11]), but in frequency division duplexing (FDD) systems that employ structured vector quantization schemes to feed CSI back to the BS, the Gaussian model is not appropriate.

In this paper we address each of these issues. Although our approach can be applied to the case of Gaussian uncertainties, it is developed specifically for the case of limited feedback. Our approach includes a variant of the "zero-outage" region approximation of the outage constraints, but we also provide an alternative approach that provides better performance outside the region for which zero outage can be obtained. That approach is particularly attractive as it leads to a problem that has a quasi-closed form solution.

2. SYSTEM MODEL AND DESIGN APPROACH

We consider a K-user unicast MISO downlink in which a BS equipped with N_t antennas sends independent messages to K single antenna users. The BS employs linear beamforming to construct the transmitted signal at each channel use, $\mathbf{x} = \sum_i^K \mathbf{w}_i s_i$, where s_i is the normalized symbol intended for user i, and $\mathbf{w}_i$ is the associated beamformer. The signal received by the i^{th} user is

$$y_i = \mathbf{h}_i^H \mathbf{w}_i s_i + \textstyle\sum_{j \neq i} \mathbf{h}_i^H \mathbf{w}_j s_j + n_i, \qquad (1)$$

where $\mathbf{h}_i^H$ denotes the channel between the BS and receiver i, and n_i represents the additive zero-mean circular complex Gaussian noise at that user. If receiver i has obtained $\mathbf{h}_i^H \mathbf{w}_i$ through a dedicated training step [14], then it can perform coherent detection. If, in addition, the additive noise is uncorrelated in time and the interference term in (1) is approximately Gaussian, then the key performance metric of the link of user i is the SINR,

$$\text{SINR}_i = \frac{\mathbf{h}_i^H \mathbf{W}_i \mathbf{h}_i}{\mathbf{h}_i^H (\sum_{j \neq i} \mathbf{W}_j) \mathbf{h}_i + \sigma_i^2}, \tag{2}$$

where σ_i^2 is the noise variance and $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H$. If the BS can be provided with accurate estimates of the channels then, given SINR targets γ_i for each user, the QoS problem can be written as:

$$\begin{aligned} \min_{\{\mathbf{w}_i\}_{i=1}^K} \quad & \textstyle\sum_i \|\mathbf{w}_i\|^2 \\ \text{subject to} \quad & \text{SINR}_i \geq \gamma_i, \quad i = 1, 2, \ldots, K. \end{aligned} \tag{3}$$

In subsequent formulations we will leave it implicit that the constraints must be satisfied for all $i \in \{1, 2, \ldots, K\}$ and that the optimization is over the design variables associated with all users. The formulation in (3) is not convex, but there are several efficient algorithms for finding globally optimum solutions [2], [4].

In practice, the BS only has an estimate $\hat{\mathbf{h}}_i$ of $\mathbf{h}_i$. Although one could attempt a mismatched design in which the estimates $\hat{\mathbf{h}}_i$ are substituted for $\mathbf{h}_i$ in (3), the sensitivity of the SINR to the channel estimates suggests that it may be more effective to develop a model for the uncertainty and incorporate that model into the design. One way to do so is to postulate a conditional distribution $p_i(\mathbf{h}_i|\hat{\mathbf{h}}_i)$, select an outage probability δ_i, then seek to minimize the transmission power subject to the i^{th} SINR constraint holding with probability $1 - \delta_i$, or show that those specifications cannot be achieved; i.e.,

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \textstyle\sum_i \|\mathbf{w}_i\|^2 \\ \text{subject to} \quad & \text{Prob}(\text{SINR}_i \geq \gamma_i) \geq 1 - \delta_i. \end{aligned} \tag{4}$$

Although some of the techniques we will develop here apply to that problem, we will focus on the related problem of minimizing the outage probability subject to a power constraint P; i.e.,

$$\min_{\mathbf{w}_i} \max_i \quad \delta_i \tag{5a}$$

$$\text{subject to} \quad \text{Prob}(\text{SINR}_i \geq \gamma_i) \geq 1 - \delta_i, \tag{5b}$$

$$\textstyle\sum_i \|\mathbf{w}_i\|^2 \leq P. \tag{5c}$$

In general, the problem in (5) is difficult to solve even for simple distributions. This is mainly due to the fact that even when deterministic expressions for the probabilistic constraints can be obtained, they are typically non-convex in the design parameters; e.g., [11].

One strategy for finding good solutions to the problem in (4) is to find a region R_i in which the channel $\mathbf{h}_i$ will lie with a probability of at least $1 - \delta_i$ and ensure zero-outage in this region [9], [10], [12], [13]. Although this method can be quite effective when the uncertainties are small, it is, by its very nature, conservative and may have no solution even when (4) does. The concept of the zero-outage region can also be applied to the problem in (5). Rather than designing $\mathbf{w}_i$ to minimize the maximum outage, we can seek to maximize the volume of the zero outage region; i.e.,

$$\begin{aligned} \max_{\mathbf{w}_i} \min_i \quad & \text{vol}(R_i) \\ \text{subject to} \quad & \text{SINR}_i \geq \gamma_i, \quad \forall \mathbf{h}_i \in R_i, \\ & \textstyle\sum_i \|\mathbf{w}_i\|^2 \leq P. \end{aligned} \tag{6}$$

A weakness of this formulation is that it simply attempts to maximize the size of the zero-outage region without regard for the behaviour outside this region. One of the main contributions of this paper will be to identify alternative strategies that enable the designer to balance the performance obtained when the uncertainties are small with the performance when the uncertainty is larger.

The specific techniques that we will develop in this paper are for the case of an FDD downlink system with structured vector quantization [15]. In these systems the receiver estimates the channel based on training signals sent by the BS and separately quantizes the gain and direction of the channel. More specifically, we let $\tilde{\mathbf{h}}_i$ denote the receiver's estimate, the receiver quantizes $\alpha_i = \|\tilde{\mathbf{h}}_i\|^2$ using a scalar quantizer and quantizes $\tilde{\mathbf{h}}_{n_i} = \tilde{\mathbf{h}}_i / \|\tilde{\mathbf{h}}_i\|$ using memoryless vector quantization over a Grassmannian codebook [15]; i.e., if $\mathcal{C} = \{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_M\}$ denotes a Grassmannian codebook of M elements in $\mathbb{C}^{N_t}$, the subspace that characterizes the direction of the channel is represented by $\mathbf{h}_{q_i} = \arg\min_{\mathbf{v} \in \mathcal{C}} 1 - |\tilde{\mathbf{h}}_{n_i}^H \mathbf{v}|^2$. If we assume that $\tilde{\mathbf{h}}_i$ is estimated accurately, α_i is quantized at a high resolution, and there are no errors in the feedback path, the transmitter's estimate is related to the actual channel by:

$$\mathbf{h}_i = \sqrt{\alpha_i}(\mathbf{h}_{q_i} + \mathbf{e}_i), \tag{7}$$

where the statistics of the error $\mathbf{e}_i$ are dependent on the codebook and the statistics of the channel. Given the intricate nature of the statistics of $\mathbf{e}_i$, even in the context of randomized codebooks [16], our initial development will be based on a model in which $\mathbf{e}_i$ lies in a region R_i that is a spherical cap on the Grassmannian manifold of radius ε, centered at h_{q_i}. This region is characterized by:

$$\|\mathbf{e}_i\| \leq \varepsilon \tag{8a}$$

$$\|\mathbf{h}_{q_i} + \mathbf{e}_i\| = 1. \tag{8b}$$

We will assume that the scalars α_i are perfectly known at the BS.

In the next section we will develop a robust downlink precoder design for the above model that seeks to maximize the zero-outage region; i.e. maximize ε, subject to a power constraint. That design will form the basis of an alternative design, developed in Section 4, in which we seek to improve the overall outage performance by allowing a non-zero outage probability for small errors and seeking improved performance in the presence of larger errors.

3. ZERO OUTAGE REGION MAXIMIZATION

Under the uncertainty model in (8), the problem in (6) of maximizing the zero-outage region subject to a power constraint is

$$\max_{\mathbf{W}_i, \varepsilon} \quad \varepsilon \tag{9a}$$

$$\text{subject to} \quad \text{SINR}_i(\mathbf{e}_i) \geq \gamma_i, \quad \forall \mathbf{e}_i \text{ satisfying (8)}, \tag{9b}$$

$$\textstyle\sum_i \text{tr}(\mathbf{W}_i) \leq P, \tag{9c}$$

$$\mathbf{W}_i \succeq \mathbf{0}, \ \text{rank}(\mathbf{W}_i) = 1. \tag{9d}$$

This problem is difficult to solve for two reasons. First the rank constraint is non-convex. Second, the SINR constraints are non-convex and there is an infinite number of them. We will address the first difficulty by removing the rank constraint, which results in a semidefinite relaxation of the problem. To address the second difficulty we rewrite the SINR constraint as

$$(\mathbf{h}_{q_i} + \mathbf{e}_i)^H \mathbf{Q}_i (\mathbf{h}_{q_i} + \mathbf{e}_i) - \sigma_i^2/\alpha_i \geq 0, \tag{10}$$

where $\mathbf{Q}_i = \mathbf{W}_i/\gamma_i - \sum_{j \neq i} \mathbf{W}_j$, and we rewrite (8b) as $\|\mathbf{e}_i\|^2 + 2Re\{\mathbf{e}_i^H \mathbf{h}_i\} = 0$. These reformulations enable us to obtain the following precise finite representation of the infinite number of constraints in (9b): the constraint in (9b) holds for all admissible $\mathbf{e}_i$ if and only if there exist scalars $\lambda_i \geq 0$ and μ_i such that

$$\begin{pmatrix} \mathbf{Q}_i & \mathbf{Q}_i \mathbf{h}_{q_i} \\ \mathbf{h}_{q_i}^H \mathbf{Q}_i & \mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} - \sigma_i^2/\alpha_i \end{pmatrix} + \lambda_i \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\varepsilon^2 \end{pmatrix} + \mu_i \begin{pmatrix} \mathbf{I} & \mathbf{h}_{q_i} \\ \mathbf{h}_{q_i}^H & 0 \end{pmatrix} \succeq \mathbf{0}. \quad (11)$$

This result can be viewed as an extension of the S-Lemma [17]. It can be established using techniques analogous to those used in [18] to prove that strong duality holds for a class of complex quadratically-constrained quadratic optimization problems with at most two constraints.

Using this precise reformulation of (9b), the semidefinite relaxation of (9) becomes

$$\begin{aligned} \max_{\mathbf{W}_i, \lambda_i, \mu_i, \varepsilon} \quad & \varepsilon \\ \text{subject to} \quad & \text{equation (11)}, \\ & \textstyle\sum_i \operatorname{tr}(\mathbf{W}_i) \leq P, \\ & \mathbf{W}_i \succeq \mathbf{0},\ \lambda_i \geq 0. \end{aligned} \quad (12)$$

Since (11) contains the bilinear term $\lambda_i \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\varepsilon^2 \end{pmatrix}$, the problem in (12) is not convex in ε. However, it is convex in the other variables and it is quasi-convex in ε. Therefore, an optimal solution can be efficiently found by performing a bisection search on ε in which the problem solved at each step is the feasibility problem that arises when ε is fixed in (12). Once a solution to (12) has been obtained, the matrices $\mathbf{W}_i$ are used to generate vectors $\mathbf{w}_i$ that correspond to good solutions to (9); cf. [19]. When each $\mathbf{W}_i$ has rank one, the vectors $\mathbf{w}_i$ are optimal. Numerical evidence, and theoretical results for closely related problems with small uncertainty sets [8], suggest that such a solution almost always exists.

As mentioned earlier, a weakness in the formulation in (9) and its relaxation in (12) is that it only attempts to maximize the zero-outage region and does so conservatively without regard for the behaviour outside this region. In the case of Gaussian uncertainties, the size of the zero-outage region is relatively small (e.g., [9], [10]), and as we will see in Section 6, that is also true in the limited feedback case. In the next section we will develop an approximation of (12) that provides better performance for larger uncertainties.

4. OFFSET MAXIMIZATION ALGORITHM

In terms of the original chance constraints in (5b), what we are seeking is a set of beamformers $\mathbf{w}_i$ that ensure that $\text{SINR}_i \geq \gamma_i$ with high probability. Using the notation in (10), we can express that as

$$\mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} + 2\text{Re}(\mathbf{e}_i^H \mathbf{Q}_i \mathbf{h}_{q_i}) + \mathbf{e}_i^H \mathbf{Q}_i \mathbf{e}_i - \sigma_i^2/\alpha_i \geq 0 \quad (13)$$

holding with high probability. To gain some insight into how this is achieved in (12), consider the "south east" block of the LMI in (11). This ensures that any feasible point for (12) satisfies

$$\mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} - \sigma_i^2/\alpha_i - \lambda_i \varepsilon^2 \geq 0. \quad (14)$$

A comparison between (13) and (14) shows that for a zero-outage region of a given size ε, the feasible points of (12) with larger values of λ_i have larger values for $\mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} - \sigma_i^2/\alpha_i$ and hence greater robustness to uncertainties of size larger than ε. In this section we will use that insight to develop an alternate ad-hoc "offset maximization" algorithm that may not have a zero-outage region, but provides greater robustness against larger uncertainties.

The proposed approach is based on maximizing the offset values $r_i = \lambda_i \varepsilon^2$ for some fixed small ε rather than maximizing ε. Using this substitution in (12), the middle block of the LMI in (11) becomes $\begin{pmatrix} (r_i/\varepsilon^2)\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -r_i \end{pmatrix}$. By analyzing the Schur complement of the "north west" block of this variant of the LMI, it can be shown that for a given ε there is a $\delta_i > 0$ such that satisfying the scalar constraint $\mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} - \sigma_i^2/\alpha_i - r_i - \delta_i \geq 0$ is sufficient for the LMI to be satisfiable. Furthermore, as $\varepsilon \to 0$, $\delta_i \to 0$, which expands the feasible set for r_i. By exploiting that observation, we can consider the following problem in which we seek to maximize the minimum offset, r_i, subject to (14),

$$\begin{aligned} \max_{\mathbf{W}_i, r_i, t} \quad & t && (15a) \\ \text{subject to} \quad & \textstyle\sum_i \operatorname{tr}(\mathbf{W}_i) \leq P, && (15b) \\ & \mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} - \sigma_i^2/\alpha_i - r_i \geq 0, && (15c) \\ & \mathbf{W}_i \succeq \mathbf{0},\ r_i \geq t. && (15d) \end{aligned}$$

Analysis of this problem reveals that at optimality (15c) holds with equality. Indeed if that were not the case, then we could choose a larger r_i and maintain feasibility. A larger r_i would not decrease the objective. Furthermore, at optimality all r_is are equal. To show that we observe that if, at optimality, r_1 was smaller than the other r_is then we could decrease the power of any of $\mathbf{W}_k, k \geq 2$, which would retain feasibility and yet allow for a larger value for r_1. That would contradict the assumed optimality. Having made those observations, we can write the following problem, which is equivalent to (15), but is simpler:

$$\begin{aligned} \max_{\mathbf{W}_i \succeq \mathbf{0}, r} \quad & r && (16a) \\ \text{subject to} \quad & \textstyle\sum_i \operatorname{tr}(\mathbf{W}_i) \leq P, && (16b) \\ & \mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} - \sigma_i^2/\alpha_i - r \geq 0. && (16c) \end{aligned}$$

The SINR constraint in (16c) has an interesting interpretation. If we recall that the BS's estimate of $\mathbf{h}_i$ is $\hat{\mathbf{h}}_i = \sqrt{\alpha_i}\mathbf{h}_{q_i}$, the SINR condition can be rewritten as

$$\frac{\hat{\mathbf{h}}_i^H \mathbf{W}_i \hat{\mathbf{h}}_i}{\hat{\mathbf{h}}_i^H (\sum_{j \neq i} \mathbf{W}_j) \hat{\mathbf{h}}_i + \sigma_i^2 + \alpha_i r} \geq \gamma_i. \quad (17)$$

That is, the problem in (16) obtains robustness to uncertainties in the CSI by seeking, with a particular affine scaling, the largest noise variances for which a mismatched design can satisfy the original SINR constraints.

The connection between the problem in (16) and the mismatched design problem also enables us to prove that the problem in (16) has an optimal solution in which all $\mathbf{W}_i$ are rank 1, and hence the semidefinite relaxation is tight. This can be shown by letting r_{max} denote the optimal value for r in (16). The set of $\{\mathbf{W}_i\}_{i=1}^K$ for which r_{max} is achieved is also an optimal solution for the problem

$$\begin{aligned} \min_{\mathbf{W}_i \succeq \mathbf{0}} \quad & \textstyle\sum_i \operatorname{tr}(\mathbf{W}_i) \\ \text{subject to} \quad & \mathbf{h}_{q_i}^H \mathbf{Q}_i \mathbf{h}_{q_i} - \sigma_i^2/\alpha_i - r_{max} \geq 0. \end{aligned} \quad (18)$$

This problem takes the same form as the QoS problem in the perfect CSI case; cf. (3). It has been shown that that problem has an optimal solution in which all the $\mathbf{W}_i s$ are rank one [2].

5. CLOSED FORM EXPRESSIONS

The connection of the problem in (16) to that in (18), and the fact that (18) has a rank-1 solution for any r, enables us to construct a quasi closed-form solution to (16). To derive that solution, we select a value for $r > 0$, denoted by r_0, and rewrite (18) as

$$\min_{\mathbf{w}_i} \quad \sum_i \mathbf{w}_i^H \mathbf{w}_i \tag{19a}$$

$$\text{s.t.} \quad \mathbf{h}_{q_i}^H \Big(\mathbf{w}_i \mathbf{w}_i^H / \gamma_i - \sum_{j \neq i} \mathbf{w}_j \mathbf{w}_j^H \Big) \mathbf{h}_{q_i} - \sigma_i^2 / \alpha_i \geq r_0. \tag{19b}$$

Motivated by the analysis in [2], [4] of the perfect CSI problem, we study the KKT conditions for (19). If we let ν_i denote the Lagrange multiplier for the i^{th} constraint in (19b), then a necessary condition for optimality is

$$\mathbf{w}_i = \Big(\frac{\nu_i}{\gamma_i} \mathbf{h}_{q_i} \mathbf{h}_{q_i}^H - \sum_{j \neq i} \nu_j \mathbf{h}_{q_j} \mathbf{h}_{q_j}^H \Big) \mathbf{w}_i, \tag{20}$$

which is an eigen equation. Using (20), it can be shown that each Lagrange multiplier ν_i satisfies a fixed point equation

$$\nu_i^{-1} = \mathbf{h}_{q_i}^H \Big(\mathbf{I} + \sum_j \nu_j \mathbf{h}_{q_j} \mathbf{h}_{q_j}^H \Big)^{-1} \mathbf{h}_{q_i} \Big(1 + \tfrac{1}{\gamma_i} \Big). \tag{21}$$

To obtain an optimal solution to (19), we first find the values for ν_i in (21) using the algorithm in [4]. Then, the normalized directions $\bar{\mathbf{w}}_i = \mathbf{w}_i / \|\mathbf{w}_i\|$ can be obtained from (20). What remains is to determine the squared norm of $\mathbf{w}_i$, and the optimal value for r_0. We will denote the squared norm by β_i; i.e., $\mathbf{w}_i = \sqrt{\beta_i}\bar{\mathbf{w}}_i$. Using the fact that at optimality the constraints (19b) hold with equality, we obtain a set of linear equations for the vector $\boldsymbol{\beta} = [\beta_1, \beta_2, ..., \beta_K]$ that takes the form $\mathbf{A}\boldsymbol{\beta} = \mathbf{b} + r_0 \mathbf{1}$, where $\mathbf{1}$ is a vector of all ones. (We have left the definitions of $\mathbf{A}$ and $\mathbf{b}$ implicit.) Since the transmitted power is $\sum_i \beta_i$, we obtain the following relationship between the transmitted power and r_0: $\sum_i \mathbf{w}_i^H \mathbf{w}_i = \mathbf{1}^T \mathbf{A}^{-1} \mathbf{b} + r_0 \mathbf{1}^T \mathbf{A}^{-1} \mathbf{1}$. Using the fact that the power constraint holds with equality at optimality, this provides a relationship between P and the optimal value of r, namely $r_{max} = (P - \mathbf{1}^T \mathbf{A}^{-1} \mathbf{b}) / (\mathbf{1}^T \mathbf{A}^{-1} \mathbf{1})$. The inverse of this relation, namely $P = \mathbf{1}^T \mathbf{A}^{-1} \mathbf{b} + r_{max} \mathbf{1}^T \mathbf{A}^{-1} \mathbf{1}$ explicitly states the additional power required to provide the robustness specified by r_{max}; the non-robustified case corresponds to $r = 0$, cf. (17).

6. SIMULATIONS

We consider a downlink system consisting of a BS with 4 antennas and 3 single antenna receivers, each with a noise variance $\sigma^2 = 0.01$, The channel vectors $\mathbf{h}_i$ are modelled using standard i.i.d. Rayleigh fading model, and the experiments are based on a 13-bit randomly generated Grassmannian codebook. To illustrate the features of the proposed approaches to the outage minimization problem, we generated many channel realizations, and quantized each realization to an element in the codebook. We then randomly selected three elements of the codebook to be the quantized channel directions for the receivers. For each of the three elements we had about 5000 channels that were quantized to that element. The scalers α_i were assumed to be perfectly known at the BS. We then constructed the zero-outage region precoder design in (12) and the quasi-closed form solution to the offset maximization problem (in Section 5) for the case of a power constraint $P = 1$. Finally we evaluated whether the designs achieved the required SINRs, γ_i, of

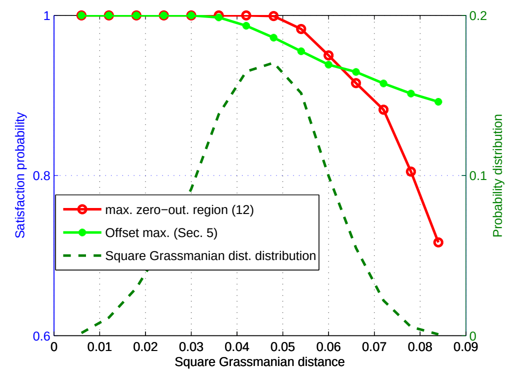

Fig. 1: Probability of satisfying the SINR constraints versus squared Grassmannian distance for a BS with 4 antennas, 3 users, $\gamma = 8$dB, $\sigma = 0.1$ and $P = 1$.

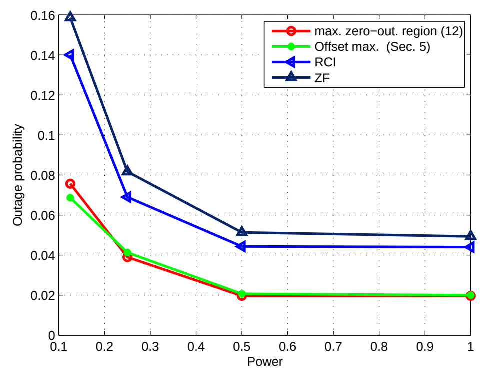

Fig. 2: Outage probability versus the power constraint for a BS with 4 antennas, 3 users, $\gamma = 3$dB, $\sigma = 0.1$.

8dB on the actual channels that were quantized to the chosen codebook elements. In Fig. 1 we plot the probability that $\text{SINR}_i \geq \gamma_i$ against the size of the quantization error, which is measured as the squared Grassmannian distance between the quantized and actual channels $d_i^2 = \min_\theta \| \mathbf{h}_i e^{j\theta} / \sqrt{\alpha_i} - \mathbf{h}_{q_i} \|^2$. As expected, for small uncertainties the design based on the zero-outage region does not incur an outage. However for larger uncertainties, the performance degrades quite rapidly. In contrast, the offset maximization algorithm may incur outages for uncertainties of intermediate size, but provides better performance in the presence of larger uncertainties. For reference, Fig. 1 also includes the distribution of the size of the quantization error.

In Fig. 2 we compare the performance of the proposed designs against schemes that employ zero-forcing (ZF) [20] or regularized channel inversion (RCI) [21] beamforming, with uniform power loading. For a target SINR of 3dB, we plot the outage probability of each method against the power constraint. The probability is calculated over 1000 channel realizations, each of which is quantized using the randomly generated codebook. As can be seen from the figure, the proposed approaches provide significantly lower outage rates than the ZF and RCI designs.

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