INFORMATION LOSSLESS FULL RATE FULL DIVERSITY CYCLOTOMIC LINEAR DISPERSION CODES

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ABSTRACT

It has recently been determined (by others) that one can construct full rate full diversity linear space-time block codes without a reduction in the achievable information rate. In this paper, we address two issues in those designs. One issue is whether the rotation matrix and the Diophantine number can be systematically and efficiently constructed for an arbitrary number of transmitter antennas and receiver antennas and whether the signal constellation is necessarily limited to PAM or QAM. The other issue is whether the currently available square designs can be generalized to a rectangular design and in particular, to a linear dispersion code design. This paper resolves these issues by proposing a trace-orthonormal linear space-time block code and linear dispersion code, and giving a systematic method to design such a code family. By carefully selecting V-structured matrices or conjugate V-structured matrices in this family, we can systematically and efficiently design information lossless full rate full diversity cyclotomic space-time codes.

1. INTRODUCTION

In this paper, we consider the discrete-time equivalent model of a baseband communication system equipped with M antennas at the transmitter and N antennas at the receiver, which can be represented in a compact matrix form as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \boldsymbol{\xi}, \tag{1}$$

where $\mathbf{r}$ is a $N \times 1$ receive signal vector, $\mathbf{H}$ is an $N \times M$ channel matrix which is known at the receiver but not known at the transmitter, $\mathbf{x}$ is an $M \times 1$ transmitted signal vector and $\boldsymbol{\xi}$ is a $N \times 1$ complex noise vector. Such multi-input and multi-output (MIMO) channels have recently attracted significant interests because they provide an important increase in capacity over single-input single-output channels [1]. A typical example is the V-BLAST scheme [2]. Recently, it has been shown how to construct linear space-time block codes for MIMO channels that achieve full rate full diversity without information loss [3], [4], [5]. In this paper, we will address two design issues in , [4], [5]. One is how to systematically and efficiently design a rotation matrix, the Diophantine number and the corresponding signal constellation for an arbitrary number of transmitter antennas and receiver antennas. The other issue is to generalize the square coding matrix designs [4], [5] to a rectangular design and in particular, to a linear dispersion code design. More specifically, motivated by [6], [7], [3], [4], [5], [8], we propose a trace-orthonormal space-time block code and linear dispersion code, and give a systematic method to design such code family. By carefully selecting so called V-structured matrices or conjugate V-structured matrices in this family, we can design information lossless full rate full diversity cyclotomic space-time codes systematically and efficiently. Due to the space limitation, we merely state our recent research results. Proofs will be provided elsewhere.

Notation: Throughout this paper we use the following notation: Matrices are denoted by uppercase boldface characters (e.g., $\mathbf{A}$), while column vectors are denoted by lowercase boldface characters (e.g., $\mathbf{b}$), The transpose of $\mathbf{A}$ is denoted by $\mathbf{A}^T$, and the conjugate and transpose of $\mathbf{A}$ by $\mathbf{A}^H$. The expression $\phi(n)$ denotes the Euler function; $\mathbb{Z}$ denotes the ring of integers; $\zeta_m = \exp\left(\frac{j2\pi}{m}\right)$; $\mathbb{Z}[\zeta_m]$ denotes the cyclotomic ring generated by $\mathbb{Z}$ and ζ_m; $\mathbb{Q}$ denotes the rational number field; $\mathbb{Q}(\zeta_m)$ denotes the cyclotomic field generated by $\mathbb{Q}$ and ζ_m; $\mathbf{W}_P$ denotes the $P \times P$ discrete Fourier transform matrix and $\mathbf{C}_P$ denotes the $P \times P$ circular generator matrix,

$$\mathbf{C}_P = \begin{pmatrix} \mathbf{0}_{1\times(P-1)} & 1 \\ \mathbf{I}_{P-1} & \mathbf{0}_{(P-1)\times 1} \end{pmatrix},$$

where $\mathbf{I}_{P-1}$ denotes the $(P-1) \times (P-1)$ identity matrix.

2. CHANNEL MODEL WITH SPACE-TIME CODES

2.1. Linear space-time block codes

First, let us consider the MIMO channel model (1) with an $M \times T$ linear space-time block code

$$\mathbf{X}(\mathbf{s}) = \sum_{k=1}^{Q} \mathbf{F}_k s_k. \tag{2}$$

Here, Q denotes the number of information symbols, each $\mathbf{F}_k$ is an $M \times T$ matrix, T is the number of channel uses, and $\mathbf{s}$ denotes a $Q \times 1$ transmission symbol vector $\mathbf{s} = [s_1, s_2, \cdots, s_Q]^T$. At the transmitter side, each column vector signal of $\mathbf{X}(\mathbf{s})$ is fed to the M transmitter antennas in channel model (1) for simultaneous transmission. At the receiver side, all these T received signal vectors can be organized in the more compact matrix form

$$\mathbf{R} = \mathbf{H}\mathbf{X}(\mathbf{s}) + \boldsymbol{\Xi}, \tag{3}$$

where $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_T]$ and $\boldsymbol{\Xi} = [\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_T]$. The channel model (3) can be equivalently rewritten by vectoring both side of (3) as [7], [3]

$$\mathrm{vec}(\mathbf{R}) = \mathcal{H}\mathcal{F}\mathbf{s} + \mathrm{vec}(\boldsymbol{\Xi}),$$

where $\mathcal{H} = \mathbf{I}_T \otimes \mathbf{H}$ and $\mathcal{F} = [\mathrm{vec}(\mathbf{F}_1), \cdots, \mathrm{vec}(\mathbf{F}_Q)]$.

2.2. Linear dispersion codes

For a linear dispersion code [6], a signal matrix is

$$\widetilde{\mathbf{X}}(\mathbf{s}) = \sum_{k=1}^{Q} \mathbf{A}_k s_k + \sum_{k=1}^{Q} \mathbf{B}_k s_k^\star, \tag{4}$$

where each of $\mathbf{A}_k$ and $\mathbf{B}_k$ denotes an $M \times T$ matrix. Similar to (3), the relationship between T input vector signals and T output vector signals can be described by

$$\widetilde{\mathbf{R}} = \mathbf{H}\widetilde{\mathbf{X}}(\mathbf{s}) + \mathbf{\Xi}. \tag{5}$$

It is desirable to express the channel model (5) as

$$\begin{pmatrix} \mathrm{vec}(\widetilde{\mathbf{R}}) \\ \mathrm{vec}(\widetilde{\mathbf{R}}^\star) \end{pmatrix} = \widetilde{\mathcal{H}}\widetilde{\mathcal{F}} \begin{pmatrix} \mathbf{s} \\ \mathbf{s}^\star \end{pmatrix} + \begin{pmatrix} \mathrm{vec}(\mathbf{\Xi}) \\ \mathrm{vec}(\mathbf{\Xi})^\star \end{pmatrix},$$

where

$$\widetilde{\mathcal{H}} = \begin{pmatrix} \mathcal{H} & \mathbf{0} \\ \mathbf{0} & \mathcal{H}^\star \end{pmatrix}, \ \widetilde{\mathcal{F}} = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^\star & \mathcal{A}^\star \end{pmatrix}$$

with $\mathcal{A} = [\mathrm{vec}(\mathbf{A}_1), \mathrm{vec}(\mathbf{A}_2), \cdots, \mathrm{vec}(\mathbf{A}_Q)]$, $\mathcal{B} = [\mathrm{vec}(\mathbf{B}_1), \mathrm{vec}(\mathbf{B}_2), \cdots, \mathrm{vec}(\mathbf{B}_Q)]$.

2.3. Design criterion

In order to obtain a good space-time code, one can use the pairwise error probability as a design criterion. For a quasi-static fading and perfect channel information at the receiver, minimizing the Chernoff bound of the average pairwise error probability of the maximum likelihood detector is equivalent to satisfying [9],

- The Rank Criterion: The minimum rank of $\mathbf{X}(\mathbf{e}) = \mathbf{X}(\mathbf{s}) - \mathbf{X}(\mathbf{s}')$ (or $\widetilde{\mathbf{X}}(\mathbf{e}) = \widetilde{\mathbf{X}}(\mathbf{s}) - \widetilde{\mathbf{X}}(\mathbf{s}')$) taken over all distinct pairs $\{\mathbf{s}, \mathbf{s}'\}$ is the diversity gain and should be maximized. Here, $\mathbf{e} = \mathbf{s} - \mathbf{s}'$. The maximum diversity or *full diversity* is MN if $T \geq M$.
- The Determinant Criterion: The minimum of $\left(\prod_{j=1}^{r} \lambda_j\right)^{1/r}$, taken over all distinct symbol vector pairs $\{\mathbf{s}, \mathbf{s}'\}$, is the coding gain and should be maximized. Here, the λ_i are the nonzero eigenvalues of $\mathbf{X}(\mathbf{e})\mathbf{X}^H(\mathbf{e})$ (or $\widetilde{\mathbf{X}}(\mathbf{e})\widetilde{\mathbf{X}}^H(\mathbf{e})$).

3. TRACE-ORTHONORMAL SPACE-TIME CODES

3.1. Trace-orthonormal linear space-time block codes

We first introduce the following definition.

Definition 1 *Let $T \geq M$. A sequence of $M \times T$ matrices $\mathbf{A}_k$ for $k = 1, \cdots, Q$ with $Q \leq MT$ is said to constitute a trace-orthonormal linear space-time block code if the following conditions are satisfied,*

$$\mathbf{F}_k \mathbf{F}_k^H = \mathbf{I}_M / M \tag{6a}$$

$$\mathrm{tr}\left(\mathbf{F}_k \mathbf{F}_{k'}^H\right) = \delta(k - k') \tag{6b}$$

for $k, k' = 1, \cdots, Q$. If $Q = TM$, it is said to constitute a trace-orthonormal full rate linear space-time block code.

In order to construct the codes, we further introduce the following definition.

Definition 2 *Let $T = KM$. An $T \times T$ unitary matrix $\mathbf{V}$ is said to be of V-structure if the entries of $\mathbf{V}$ satisfy $\sum_{n=0}^{K-1} |v_{k,Mn+r}|^2 = \frac{1}{M}$ for $k = 1, \cdots, T, r = 1, \cdots, M$.*

Theorem 1 *Let $T = KM$ and let $\mathbf{V}_1, \cdots, \mathbf{V}_M$ be the $T \times T$ V-structured matrices. Define the $M \times T$ matrices $\mathbf{E}_{Tm+n}$ as*

$$\mathbf{E}_{Tm+n} = [\mathrm{diag}(\mathbf{v}_n^{(m+1)}(1:M)), \cdots, \mathrm{diag}(\mathbf{v}_n^{(m+1)}((K-1)M+1:KM))]$$

for $m = 0, \cdots, M-1, n = 1, \cdots, T$, where $\mathbf{v}_n^{(m+1)}(kM+1:(k+1)M)$ denotes the $M \times 1$ column vector consisting of entries from $kM+1$ to $(k+1)M$ of the nth column of matrix $\mathbf{V}_{m+1}$. If the matrices $\mathbf{F}_k$ in (2) are defined by

$$\mathbf{F}_{mT+n} = \mathbf{C}_M^m \mathbf{E}_{Tm+n},$$

then, the matrix family $\{\mathbf{F}_k\}_{k=1}^{MT}$ constitutes a trace-orthonormal full rate linear space-time block code.

3.2. Trace-orthonormal linear dispersion codes

Definition 3 *Let $T \geq M$. A sequence of $M \times T$ matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ for $k = 1, \cdots, Q$ with $Q \leq MT$ is said to constitute a trace-orthonormal linear dispersion code if the following conditions are satisfied for $k, k' = 1, \cdots, Q$,*

$$\mathbf{A}_k \mathbf{A}_k^H + \mathbf{B}_k \mathbf{B}_k^H = \mathbf{I}_M / M \tag{7a}$$

$$\mathrm{tr}\left(\mathbf{A}_k \mathbf{A}_{k'}^H + \mathbf{B}_{k'} \mathbf{B}_k^H\right) = \delta(k - k') \tag{7b}$$

$$\mathrm{tr}\left(\mathbf{B}_k \mathbf{A}_{k'}^H + \mathbf{B}_{k'} \mathbf{A}_k^H\right) = 0. \tag{7c}$$

In particular, when $Q = MT$, it is said to constitute a trace-orthonormal full rate linear dispersion code.

We would like to make the following remarks.

1. We note that orthogonality conditions for the complex orthogonal space-time block codes $\mathbf{A}_k \mathbf{A}_{k'}^H + \mathbf{B}_{k'} \mathbf{B}_k^H = \delta(k-k')\mathbf{I}_M/M$ and $\mathbf{B}_k \mathbf{A}_{k'}^H + \mathbf{B}_{k'} \mathbf{A}_k^H = \mathbf{0}$ imply Conditions (7a), (7b) and (7c). Therefore, the trace-orthonormal linear dispersion codes are a generalization of complex orthogonal space-time block codes [10], [11], [12], [13], [14].
2. Conditions (7b) and (7c) result in $\widetilde{\mathcal{F}}$ being unitary. Therefore, a trace-orthonormal linear dispersion code is *information lossless* [6], [7].

We need the following definition to design a trace-orthonormal linear dispersion code.

Definition 4 *Let $T = KM$. A $2T \times 2T$ unitary matrix $\widetilde{\mathbf{V}}$ is said to be of conjugate V-structure if it satisfies the following two conditions for $k = 1, \cdots, T, r = 1, \cdots, M$;*

1) The matrix $\widetilde{\mathbf{V}}$ has the structure

$$\widetilde{\mathbf{V}} = \begin{pmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^\star & \mathbf{X}^\star \end{pmatrix}$$

2) $\sum_{n=0}^{K-1} \left(|x_{k,Mn+r}|^2 + |y_{k,Mn+r}|^2\right) = 1/M$, where $x_{m,n}$ and $y_{m,n}$ denote the entries of $T \times T$ matrices $\mathbf{X}$ and $\mathbf{Y}$, respectively.

Theorem 2 *Let $T = MK$ and $\widetilde{\mathbf{V}}_n$ for $n = 1, \cdots, T$ denote $2T \times 2T$ conjugate V-structured matrices. Let matrices $\mathbf{D}_{Tm+n}$ and $\mathbf{\Delta}_{Tm+n}$ be determined by*

$$\mathbf{D}_{Tm+n} = [\mathrm{diag}(\mathbf{x}_n^{(m+1)}(1:M)), \cdots, \mathrm{diag}(\mathbf{x}_n^{(m+1)}(K-1)M+1:KM))]$$
$$\mathbf{\Delta}_{Tm+n} = [\mathrm{diag}(\mathbf{y}_n^{(m+1)}(1:M)), \cdots, \mathrm{diag}(\mathbf{y}_n^{(m+1)}(K-1)M+1:KM))]$$

for $m = 0, \cdots, M-1, n = 1, \cdots, T$, where $\mathbf{x}_n^{(m+1)}(kM+1 : (k+1)M)$ and $\mathbf{y}_n^{(m+1)}(kM+1 : (k+1)M)$ denote the $M \times 1$ column vector consisting of entries from $kM+1$ to $(k+1)M$ of the nth column of matrices $\mathbf{X}_{m+1}$ and $\mathbf{Y}_{m+1}$ in the conjugate V-structured matrix $\mathbf{V}_m$, respectively. If the matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ for $k = 1, \cdots, MT$ are defined by

$$\begin{aligned} \mathbf{A}_{mT+n} &= \mathbf{C}^m \mathbf{D}_{Tm+n}, \\ \mathbf{B}_{mT+n} &= \mathbf{C}^m \mathbf{\Delta}_{Tm+n}, \end{aligned}$$

then, the matrix family $\{\mathbf{A}_k, \mathbf{B}_k\}_{k=1}^{MT}$ forms a trace-orthonormal full rate linear dispersion code.

4. DESIGN OF INFORMATION LOSSLESS FULL RATE FULL DIVERSITY SPACE-TIME CODES

In this section, we carefully select the V-structured matrices in the trace-orthonormal linear space-time code family and the conjugate V-structured matrices in the trace-orthonormal linear dispersion code family so that the resulting space-time codes provide full rate and full diversity without information loss.

4.1. Cyclotomic rotation matrix

Recently, Wang et al. [8] have obtained the following profound result on a cyclotomic linear diagonal space-time block code design.

Lemma 1 *Let $P = LJ$ and $L_t = \frac{\phi(P)}{\phi(L)}$. Then, all the L_t automorphisms of the cyclotomic field $\mathbb{Q}(\zeta_P)$, $\sigma_i, 1 \le i \le L_t$, that fix the cyclotomic subfield $\mathbb{Q}(\zeta_L)$ can be represented by $\sigma_i(\zeta_P) = \zeta_P^{1+P_iL}$ for $1 \le i \le L_t$, where $1 \le i \le L_t$ are integers that satisfy $0 = P_1 < P_2 < \cdots < P_{L_t} \le J-1$, and $1 + P_iL$ and P are coprime for $1 \le i \le L_t$.*

Using this lemma, we can prove the following theorem.

Theorem 3 *Let $M = \prod_{k=1}^{r} p_k^{\alpha_k}, L = \overline{L}\prod_{k=1}^{r} p_k^{\beta_k}$, where each p_k is a prime, $\alpha_k, \beta_k \ge 1$ and $\overline{L}$ is prime to M. The matrix $\mathcal{R}(LM, M) = \mathbf{W}_M^H \mathrm{diag}(1, \zeta_{LM}, \cdots, \zeta_{LM}^{M-1})$ is unitary and is of full diversity over the cyclotomic ring $\mathbb{Z}[\zeta_L]$; i.e., if $\mathbf{x} = [x_1, x_2, \cdots, x_M]^T = \mathcal{R}(LM, M)\mathbf{s}$, $\mathbf{s} \in \mathbb{Z}^M[\zeta_L]$, then $\prod_{k=1}^{M} x_k \neq 0$ for any nonzero symbol vector $\mathbf{s}$ belonging to $\mathbb{Z}^M[\zeta_L]$.*

Theorem 3 generalizes the result in [15], [16], which plays a core role in our design.

4.2. Design of linear space-time block codes

Theorem 4 *Let $T = KM$ and L be as defined in Theorem 3. Let the V-structured matrices in Theorem 1 be chosen as $\mathbf{V}_k = \zeta_{LM^3}^{k-1}(\mathbf{U} \otimes \mathcal{R}(LM, M))$ for $k = 1, \cdots, M$, where $\mathbf{U}$ is an arbitrary $K \times K$ unitary matrix. Then, the resulting signal matrix $\mathbf{X}(\mathbf{s})$ provides full rate and enables full diversity without information loss over any constellation set carved from $\mathbb{Z}^{TM}[\zeta_L]$.*

Theorem 4 can be proved by using Theorem 1, Theorem 3 and following similar steps in [4], [5]. Theorem 4 not only generalizes the square design in [4], [5] to a rectangular design, but also provides a much simpler way to design such codes.

4.3. Design of linear dispersion codes

The design of information lossless full rate full diversity linear dispersion codes is different from that of the analogous space-time block codes in that for linear dispersion codes, the Diophantine number [4], [5] and its conjugate are "hidden" in each rotation matrix.

Theorem 5 *Let the number of channel uses T and the conjugate V-structured matrices in Theorem 2 are chosen as follows:*

1. *For an even number of transmitter antennas, choose $T = M$ and*

$$\widetilde{\mathbf{V}}_1 = \begin{pmatrix} \mathbf{X}_{\mathrm{e}} & \mathbf{Y}_{\mathrm{e}} \\ \mathbf{Y}_{\mathrm{e}}^\star & \mathbf{X}_{\mathrm{e}}^\star \end{pmatrix} \mathrm{diag}(\zeta_{2LM}\mathbf{I}_M, \zeta_{2LM}^{-1}\mathbf{I}_M)$$
$$\widetilde{\mathbf{V}}_m = \begin{pmatrix} \mathbf{X}_{\mathrm{e}} & \mathbf{Y}_{\mathrm{e}} \\ \mathbf{Y}_{\mathrm{e}}^\star & \mathbf{X}_{\mathrm{e}}^\star \end{pmatrix} \mathrm{diag}(\zeta_{4LM^3}^{m-1}\mathbf{I}_M, \zeta_{4LM^3}^{-m+1}\mathbf{I}_M)$$

for $m = 2, \cdots, M$, where the matrices $\mathbf{X}_{\mathrm{e}}$ and $\mathbf{Y}_{\mathrm{e}}$ are determined by

$$\mathbf{X}_{\mathrm{e}} = \begin{pmatrix} \mathcal{R}(\frac{LM}{2}, \frac{M}{2}) & \mathcal{R}(\frac{LM}{2}, \frac{M}{2}) \\ \mathcal{R}(\frac{LM}{2}, \frac{M}{2}) & \mathcal{R}(\frac{LM}{2}, \frac{M}{2}) \end{pmatrix}$$
$$\mathbf{Y}_{\mathrm{e}} = \begin{pmatrix} \mathcal{R}^\star(\frac{LM}{2}, \frac{M}{2}) & -\mathcal{R}^\star(\frac{LM}{2}, \frac{M}{2}) \\ -\mathcal{R}^\star(\frac{LM}{2}, \frac{M}{2}) & \mathcal{R}^\star(\frac{LM}{2}, \frac{M}{2}) \end{pmatrix}.$$

2. *For an odd number of transmit antennas, choose $T = 2M$ and, for $m = 2, \cdots, M$,*

$$\widetilde{\mathbf{V}}_1 = \begin{pmatrix} \mathbf{X}_{\mathrm{o}} & \mathbf{Y}_{\mathrm{o}} \\ \mathbf{Y}_{\mathrm{o}}^\star & \mathbf{X}_{\mathrm{o}}^\star \end{pmatrix} \mathrm{diag}(\zeta_{2LM}\mathbf{I}_M, \zeta_{2LM}^{-1}\mathbf{I}_M)$$
$$\widetilde{\mathbf{V}}_m = \begin{pmatrix} \mathbf{X}_{\mathrm{o}} & \mathbf{Y}_{\mathrm{o}} \\ \mathbf{Y}_{\mathrm{o}}^\star & \mathbf{X}_{\mathrm{o}}^\star \end{pmatrix} \mathrm{diag}(\zeta_{4LM^3}^{m+1}\mathbf{I}_M, \zeta_{4LM^3}^{-m+1}\mathbf{I}_M),$$

where the matrices $\mathbf{X}_{\mathrm{o}}$ and $\mathbf{Y}_{\mathrm{o}}$ are defined by

$$\mathbf{X}_{\mathrm{o}} = \begin{pmatrix} \mathcal{R}(LM, M) & \mathcal{R}(LM, M) \\ \mathcal{R}(LM, M) & \mathcal{R}(LM, M) \end{pmatrix}$$
$$\mathbf{Y}_{\mathrm{o}} = \begin{pmatrix} \mathcal{R}^\star(LM, M) & -\mathcal{R}^\star(LM, M) \\ -\mathcal{R}^\star(LM, M) & \mathcal{R}^\star(LM, M) \end{pmatrix}.$$

Then, the resulting signal matrix $\widetilde{\mathbf{X}}(\mathbf{s})$ in (4) extracts full rate full diversity without information loss over any constellation set carved from $\mathbb{Z}^{TM}[\zeta_L]$.

Theorem 5 gives us a systematic, but simple, method to design an information lossless full rate full diversity linear dispersion code.

4.4. Design example and simulation

Example 1: For two transmitter antennas and two receiver antennas, using Theorem 2 and following steps similar to those in [17] we design an information lossless full rate full diversity linear dispersion code

$$\widetilde{\mathbf{X}}(\mathbf{s}) = \frac{1}{2}\begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix},$$

where

$$\begin{aligned} X_{11} &= s_1 e^{j\theta} + s_1^\star e^{-j\theta} + s_4 e^{j\theta} - s_4^\star e^{-j\theta} \\ X_{12} &= s_2 e^{j\varphi} + s_2^\star e^{-j\varphi} + s_3 e^{j\varphi} - s_3^\star e^{-j\varphi} \\ X_{21} &= s_2 e^{j\varphi} - s_2^\star e^{-j\varphi} + s_3 e^{j\varphi} + s_3^\star e^{-j\varphi} \\ X_{22} &= s_1 e^{j\theta} - s_1^\star e^{-j\theta} + s_4 e^{j\theta} + s_4^\star e^{-j\theta} \end{aligned}$$

with $\theta = \frac{2\pi}{3}$ and $\varphi = \frac{5\pi}{12}$. It can be verified that

$$\begin{aligned} \det(\widetilde{\mathbf{X}}(\mathbf{e})) = \quad & \tfrac{1}{4}((e_1 + e_4)^2 + j(e_2 + e_3)^2)e^{j2\theta} \\ & -\tfrac{1}{4}((e_1^\star - e_4^\star)^2 - j(e_2^\star - e_3^\star)^2)e^{-j2\theta}, \end{aligned}$$

and hence, $\det(\widetilde{\mathbf{X}}(\mathbf{e})) \neq 0$ for any constellation set carved from $\mathbb{Z}[j] \setminus 0$. A comparison of the error performance for QPSK signals of our code with that of the code in [17] (see Fig 1) shows that our code obtains a significant SNR gain (3 dB) at average bit error rate 10^{-6}

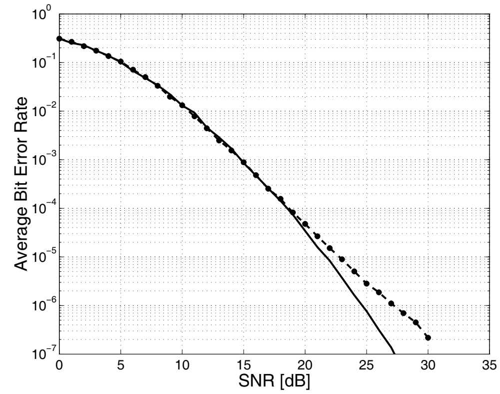

Fig. 1. Simulated average bit error rate performance comparison of the proposed code (solid) with the code [17](dash)

5. CONCLUSION

In this paper we have proposed a family of trace-orthonormal space-time code having a structure from which we can systematically and efficiently design information lossless full rate full diversity cyclotomic codes for the coherent receiver by carefully building cyclotomic rotation matrices.

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