COMP ENG 4TL4:

Digital Signal Processing

Notes for Lecture #23

Friday, October 31, 2003

Modern window types (have been derived based on optimality criteria):

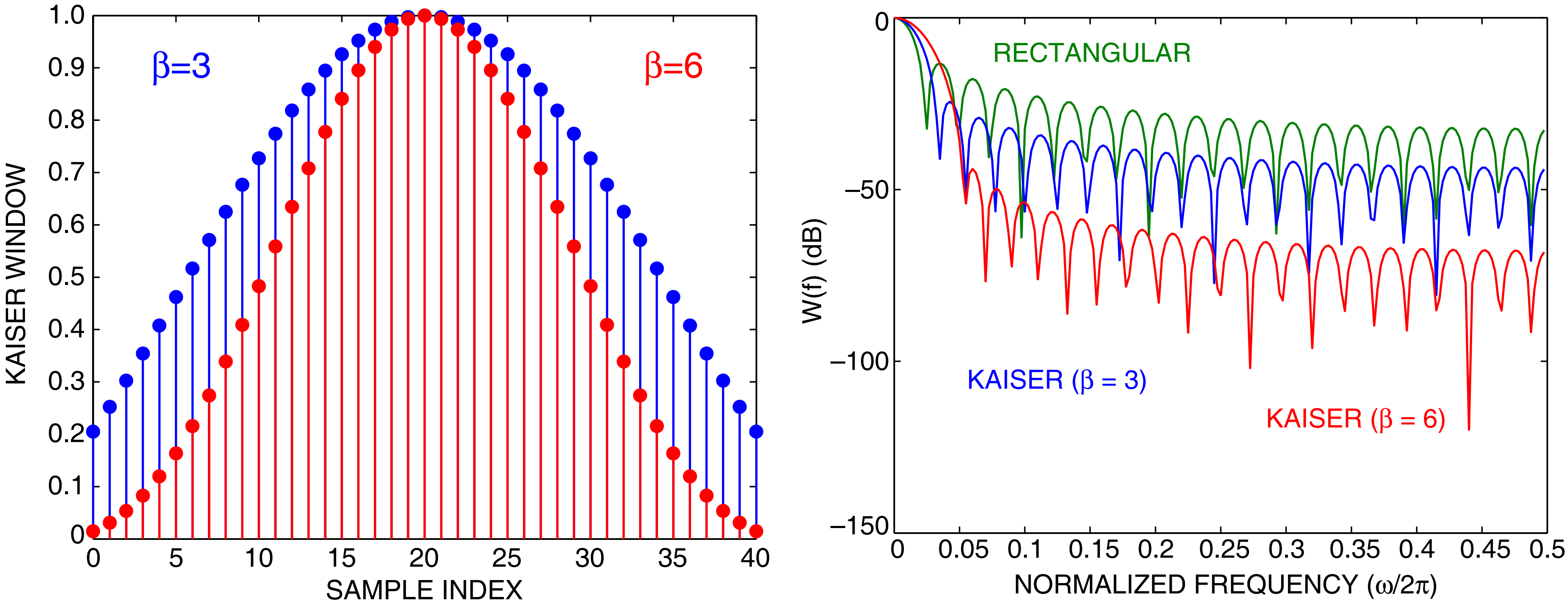
Kaiser window and Dolph-Chebyshev window:

minimize the width of the mainlobe under the constraints that:

1. the *window length be fixed* and
2. the *sidelobe levels not exceed a given value*.

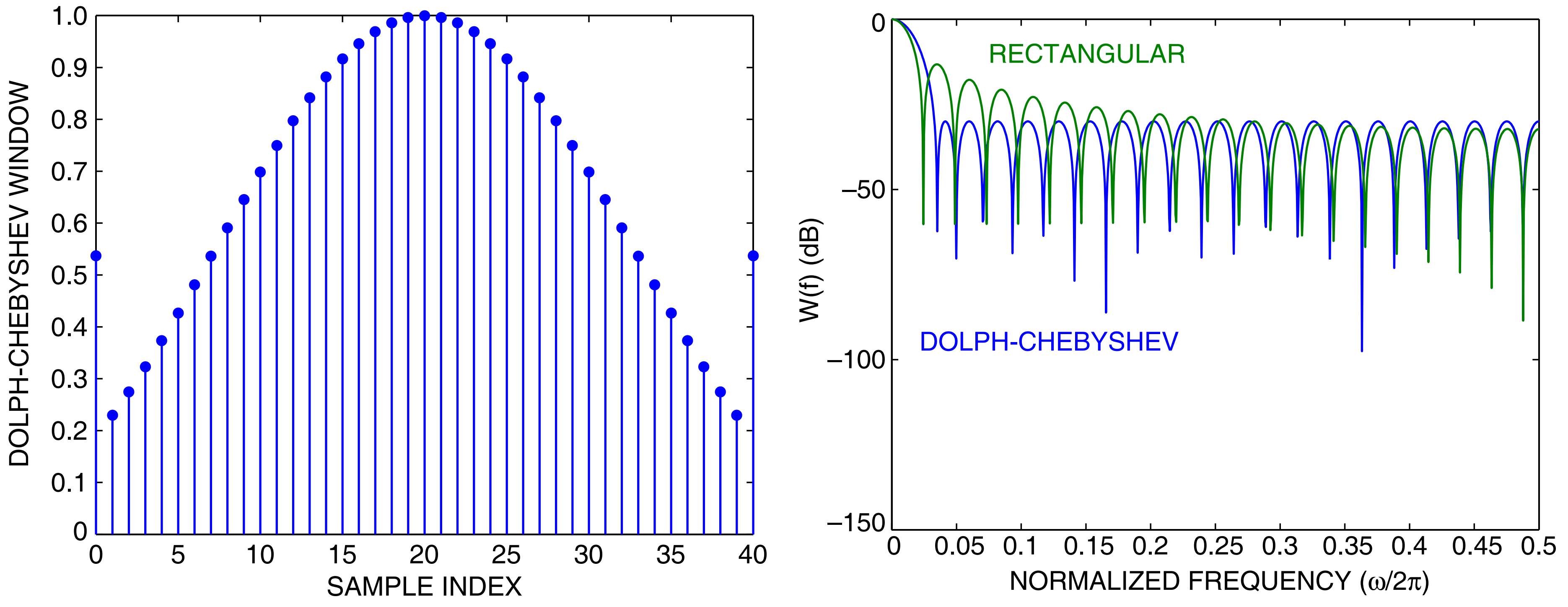
These windows provide more flexibility than the classical windows because a desired tradeoff between mainlobe width and sidelobe levels can be achieved!

Kaiser window compared to the rectangular window in the frequency domain: $(M = 40)$



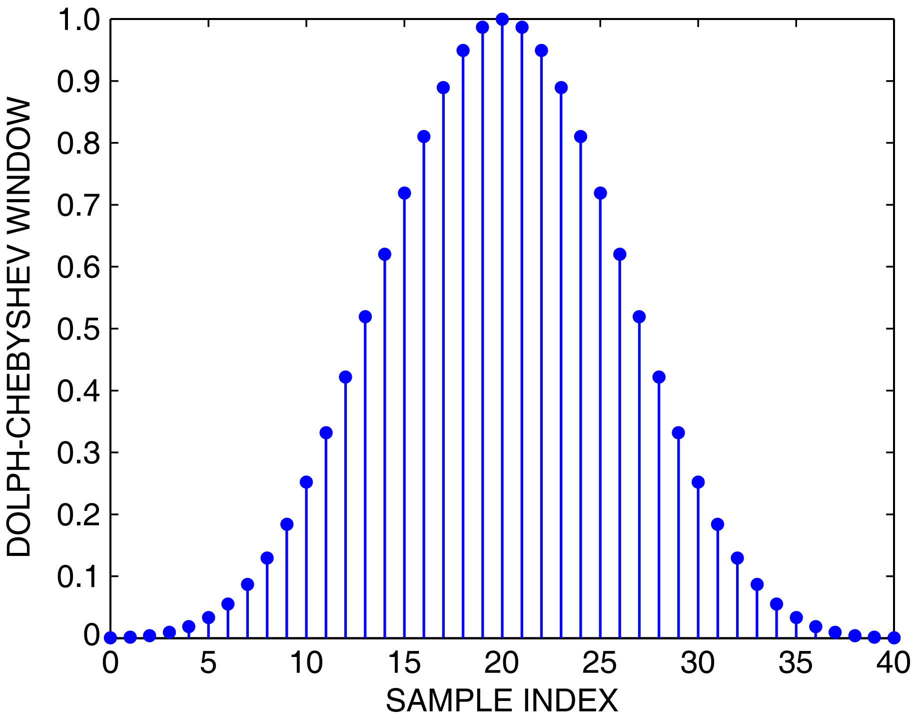
The Kaiser window has two parameters: the length $M+1$ and a shape parameter β. The value of β required to achieve a particular maximal sidelobe level can be obtained from an empirically-derived formula—see *Oppenheim and Schafer* pp. 474–485 for more details.

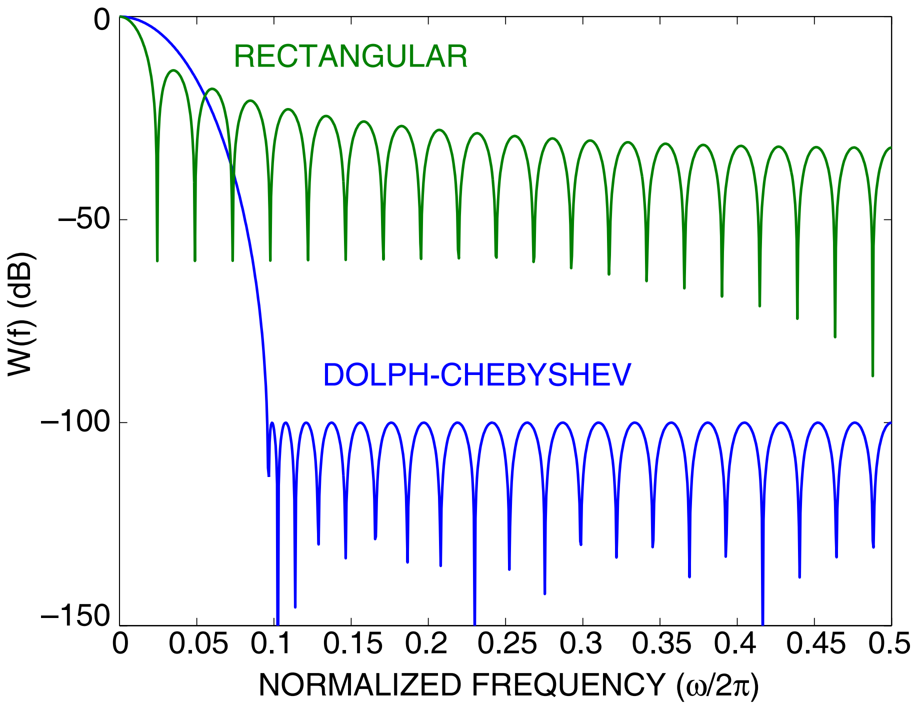
Dolph-Chebyshev window with –30 dB of ripple compared to the rectangular window in the frequency domain: $(M = 40)$



The Dolph-Chebyshev window has two parameters: the length $M+1$ and the desired sidelobe level. Note the equal levels of the sidelobes.

Dolph-Chebyshev window with –100 dB of ripple compared to the rectangular window in the frequency domain: $(M = 40)$





Now, let us take the Dolph-Chebyshev window and design our example lowpass filter using the window method with $M = 10$, $M = 40$, and $M = 160$.

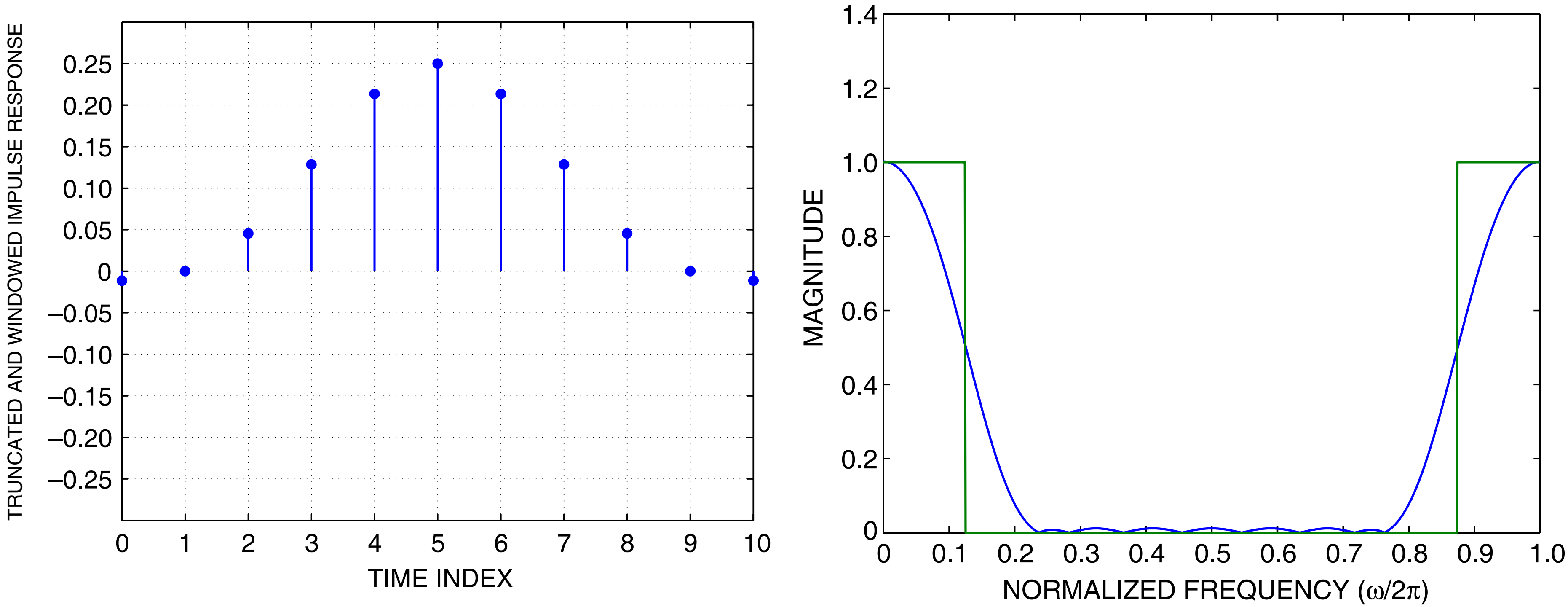
Remember that:

$$h_{\mathrm{id}}[n] = \frac{\sin\left(\pi\left(n - M/2\right)/4\right)}{\pi\left(n - M/2\right)}.$$

We apply a Dolph-Chebyshev window of length $M+1$.

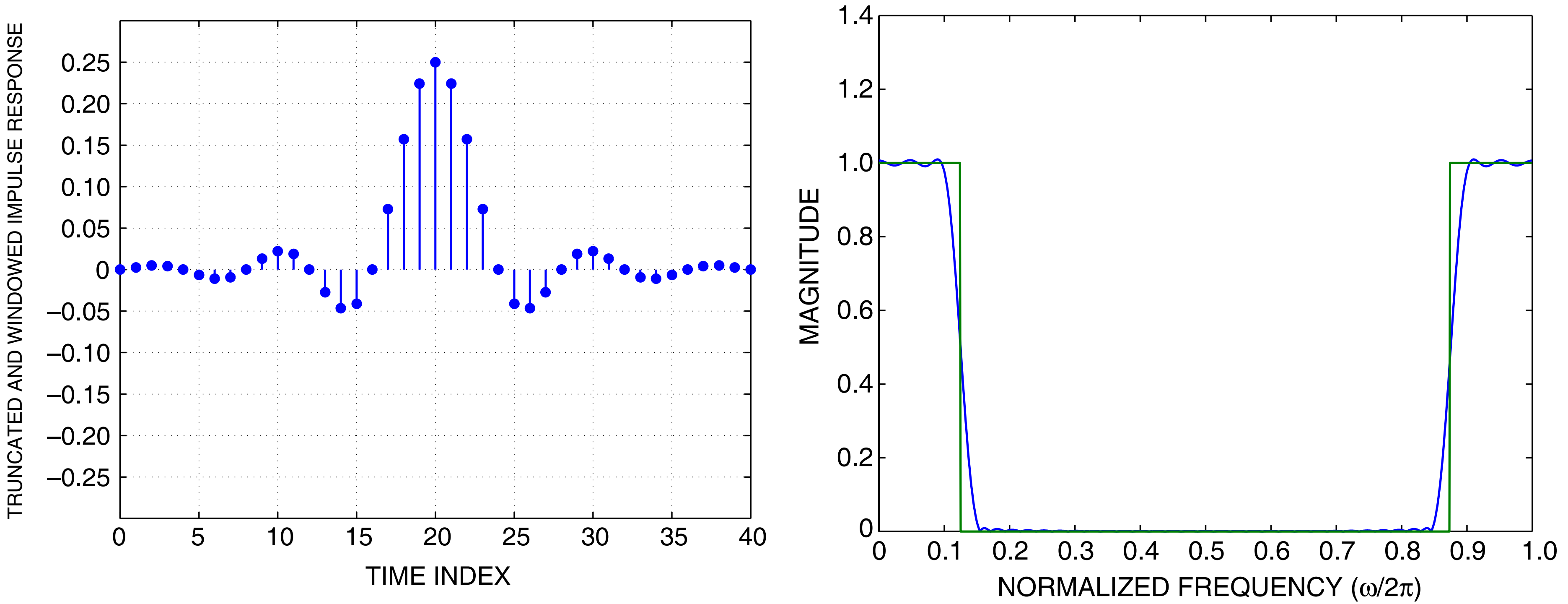
Truncated and windowed impulse response and corresponding approximation of lowpass frequency response:

(window ripple –30 dB; $M = 10$)



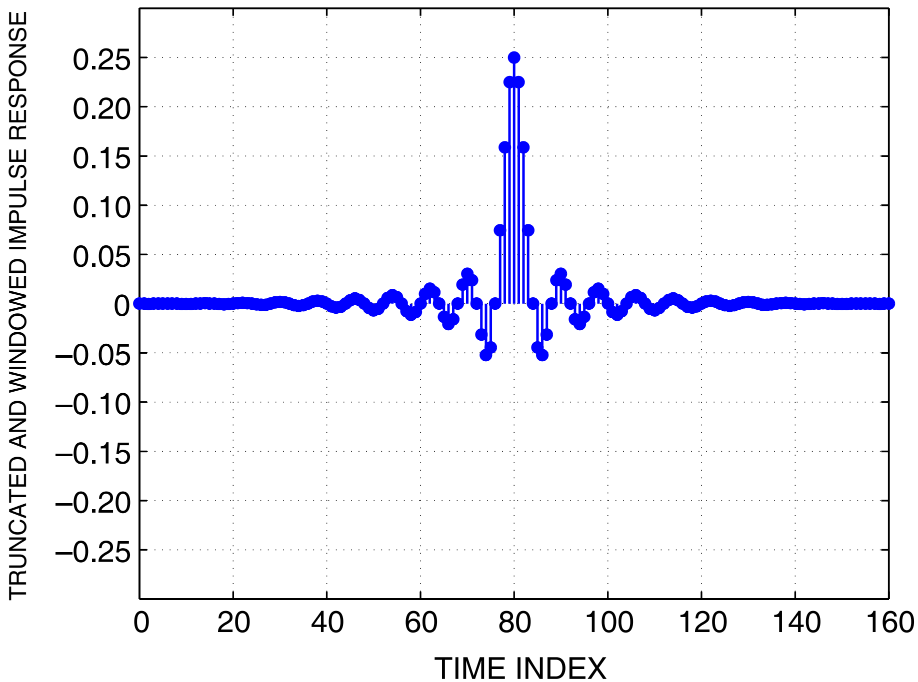
Truncated and windowed impulse response and corresponding approximation of lowpass frequency response:

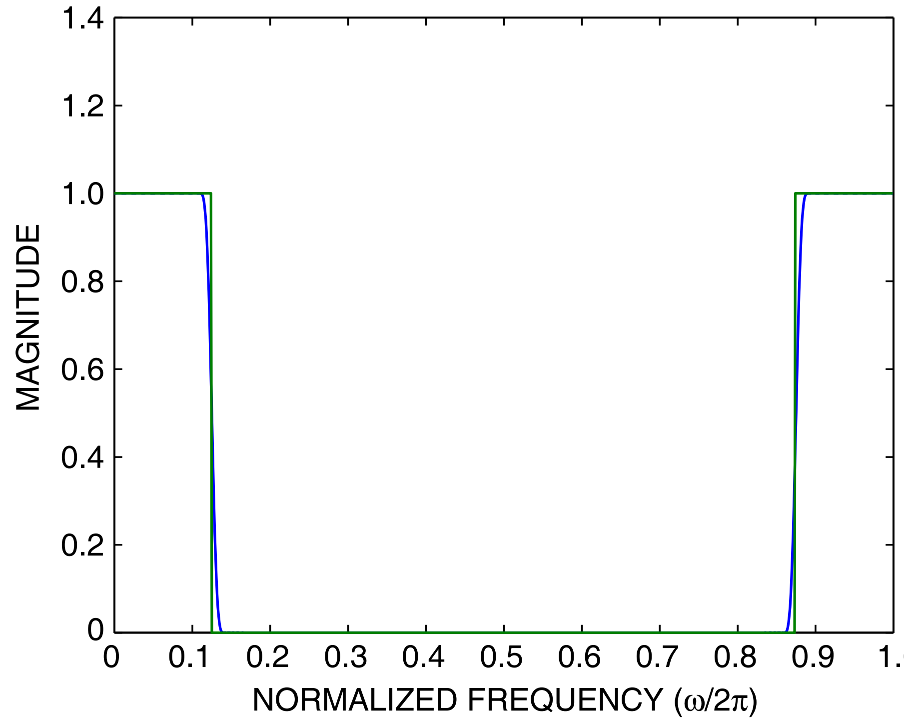
(window ripple –30 dB; $M = 40$)



Truncated and windowed impulse response and corresponding approximation of lowpass frequency response:

(window ripple –60 dB; $M = 160$)





There is no Gibbs phenomenon with the Dolph-Chebyshev window!

Optimization-based methods: the idea is to find the best approximation to the ideal frequency response for a given fixed M.

Question: What should be our criterion for the "best" approximation?

Answer #1: How about minimizing the mean-square error?

$$\epsilon^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left| H_{\text{id}}\left(e^{j\omega}\right) - H\left(e^{j\omega}\right)\right|^2 \, d\omega.$$

Unfortunately, the solution to this minimization problem is:

$$h[n] = \begin{cases} h_{\text{id}}[n]\,, & 0 \le n \le M, \\ 0\,, & n > M, \end{cases}$$

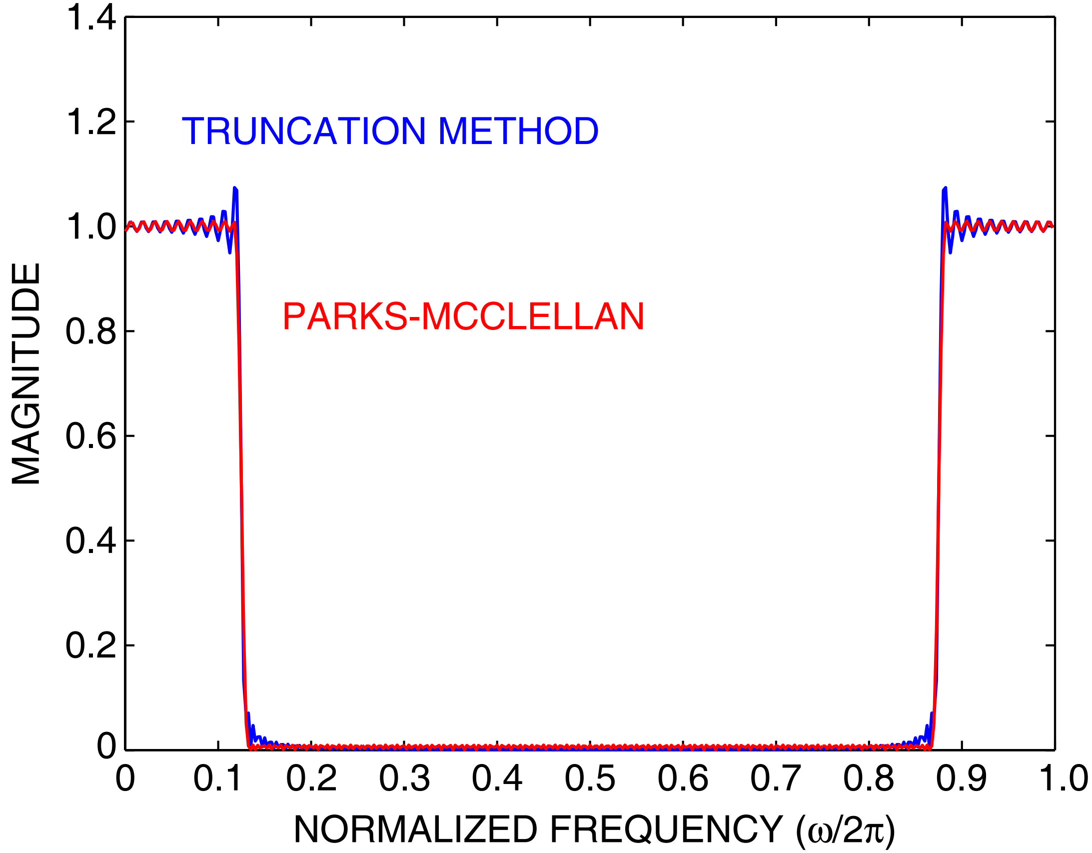
the truncation method, which we know suffers from the Gibbs phenomenon!

Answer #2: How about minimizing the maximum error?

This is referred to as the $\mathrm{minimax}$ strategy, and several algorithms have been developed to solve this problem.

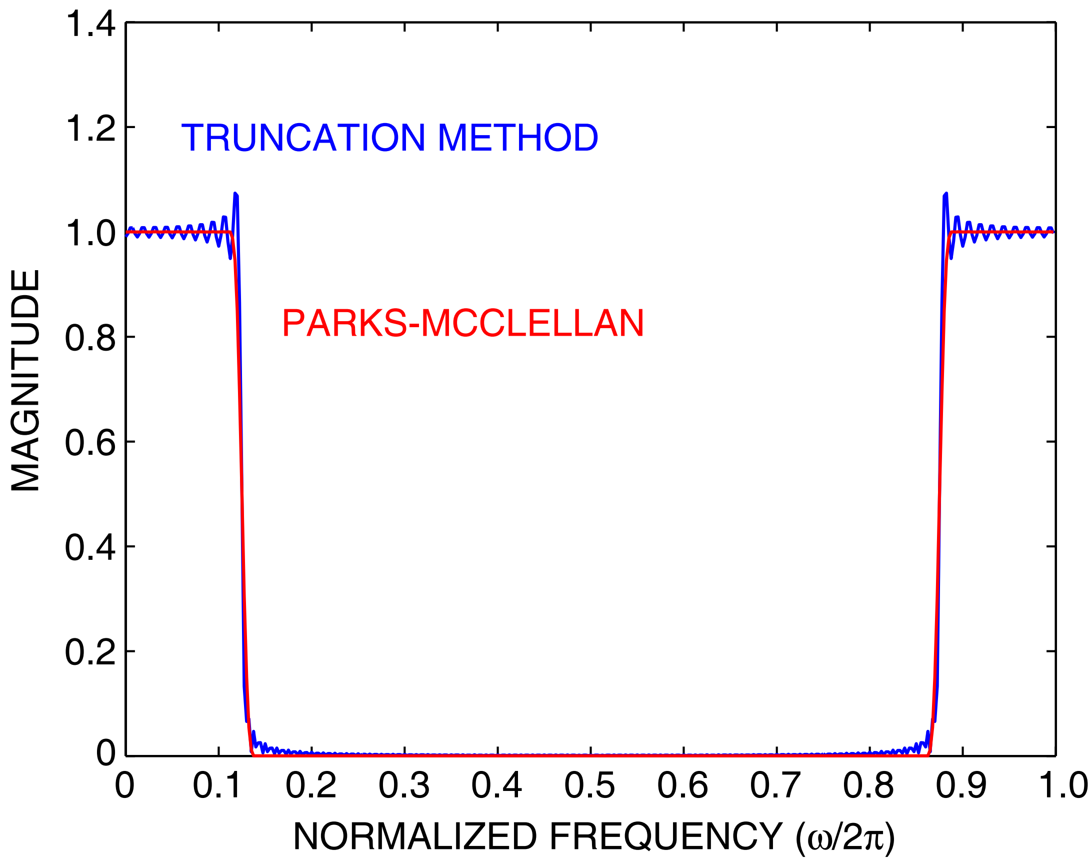
The most widely used of these is the Parks-McClellan algorithm, often referred to (mistakenly) as the remez algorithm, which determines the optimal (in the $\mathrm{minimax}$ sense) equiripple FIR filter for a given desired frequency response.

Parks-McClellan optimal $(\mathrm{minimax})$ equiripple FIR filter lowpass frequency response: $(M = 160)$



If some nonzero passband ripple and stopband attenuation is permissible, then a very sharp transition region matching that of the truncation method can be obtained.

Parks-McClellan optimal $(\mathrm{minimax})$ equiripple FIR filter lowpass frequency response: $(M = 160)$



With just a slight relaxation of the slope of the transition region, the passband ripple and stopband attenuation can be dramatically reduced.