4TL4 Tutorial Sept 30 + Oct 2/03

Time Domain Analysis

Ref.: Oppenheim + Schafer Chapter 2

2.1 For each of the following systems, determine whether the system is:

(1) Stable
(2) Causal
(3) Linear
(4) Time invariant
(5) Memoryless

a) $T(x[n]) = g[n]x[n]$, given $g[n]$

(1) stable if $g[n]$ is bounded
(2) always causal b/c it never looks into the future
(3) always linear: $T(ax_1[n] + bx_2[n]) = g[n](ax_1[n] + bx_2[n])$

$$= ag[n]x_1[n] + bg[n]x_2[n]$$
$$= aT(x_1[n]) + bT(x_2[n])$$

(4) not time invariant since $g[n]$ is not a constant
(5) always memoryless, doesn't depend on anything from the past i.e. n only

c) $T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]$

(1) always stable due to finite sum
(2) causal only if $n_0 = 0$, otherwise looks into the future
(3) always linear
(4) always time invariant, since it only depends on the input
(5) memoryless only if $n_0 = 0$, otherwise looks into the past

e) $T(x[n]) = e^{x[n]}$

- always stable, causal, time-invariant and memoryless

(3) Non-linear:

$$T(Ax_1[n] + Bx_2[n]) = e^{Ax_1[n] + Bx_2[n]}$$
$$= e^{Ax_1[n]} e^{Bx_2[n]} = T(Ax_1[n]) \cdot T(Bx_2[n])$$
$$\neq A\,T(x_1[n]) + B\,T(x_2[n])$$

g) $T(x[n]) = x[-n]$

(1) stable

(2) non-causal: $T(x[-n]) = x[n]$

(3) linear: $T(Ax_1[n] + Bx_2[n]) = Ax_1[-n] + Bx_2[-n]$
$= A\,T(x_1[n]) + B\,T(x_2[n])$

(4) not time invariant b/c you look back a different # of samples every time

(5) requires memory b/c you look into the past

2.4 Linear constant-coefficient difference (LCCD) eqn:

$$y[n] - \tfrac{3}{4}y[n-1] + \tfrac{1}{8}y[n-2] = 2x[n-1]$$

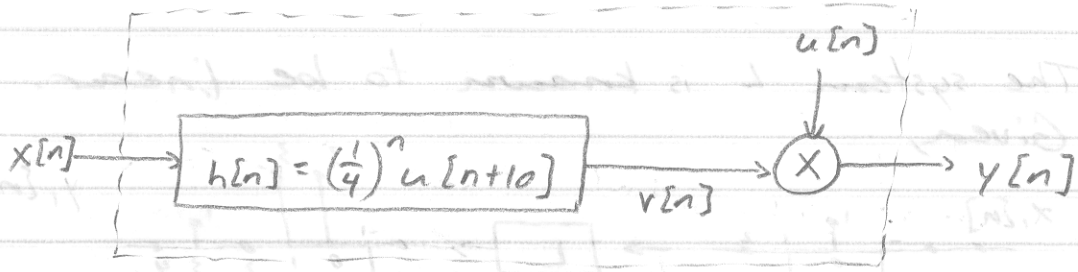
Determine $y[n]$ for $n \geq 0$ when $x[n] = \delta[n]$ and $y[n] = 0$ for $n < 0$

$$y[n] = \tfrac{3}{4}y[n-1] - \tfrac{1}{8}y[n-2] + 2\delta[n-1]$$

$$y[0] = 0$$
$$y[1] = 2$$
$$y[2] = \tfrac{3}{4} \cdot 2 = \tfrac{3}{2}$$
$$y[3] = \tfrac{3}{4} \cdot \tfrac{3}{4} \cdot 2 - \tfrac{1}{8} \cdot 2 = \tfrac{7}{8}$$
$$y[4] = \tfrac{3}{4} \cdot \tfrac{3}{4} \cdot \tfrac{3}{4} \cdot 2 - \tfrac{3}{4} \cdot \tfrac{1}{8} \cdot 2 - \tfrac{1}{8} \cdot \tfrac{3}{4} \cdot 2 = \tfrac{15}{32}$$
$$y[5] = \left(\tfrac{3}{4}\right)^4 \cdot 2 - \left(\tfrac{3}{4}\right)^2 \cdot \tfrac{1}{8} \cdot 2 - \left(\tfrac{3}{4}\right)^2 \tfrac{1}{8} \cdot 2 - \left(\tfrac{3}{4}\right)^2 \tfrac{1}{8} \cdot 2 + \left(\tfrac{1}{8}\right)^2 2$$
$$\vdots$$

$$y[n] = \frac{2^n - 1}{2^{2n-3}} = 8\left[\frac{2^n}{2^{2n}} - \frac{1}{2^{2n}}\right] = 8\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right]$$

2.15



a) Is the overall system LTI?

Test:

$$x_1[n] = \delta[n] \longrightarrow y_1[n] = \left(\tfrac{1}{4}\right)^n u[n]$$

$$x_2[n] = \delta[n-1] \longrightarrow y_2[n] = \left(\tfrac{1}{4}\right)^{n-1} u[n]$$

$$x_2[n] = x_1[n-1]$$

but $y_2[n] \neq y_1[n-1] = \left(\tfrac{1}{4}\right)^{n-1} u[n-1]$

$\therefore$ not LTI.

b) Is the overall system causal?

No. $x_2[n] = 0$ for $n < 1$

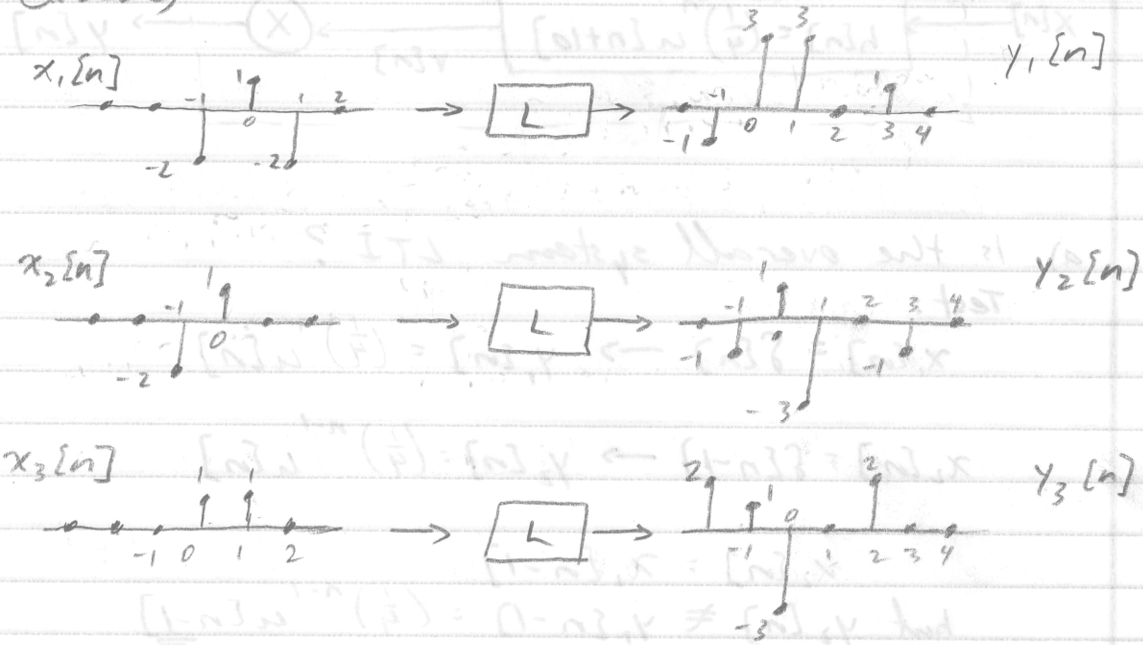
but $y_2[0] \neq 0$

c) Is the overall system stable in the BIBO sense?

Yes. since $h[n]$ is stable ~~and~~ $u[n]$ will not cause it to become unstable.

2.36 The system L is known to be linear.

Given,



a) Determine whether the system L could be time invariant.

$$\delta[n] = \tfrac{1}{2} x_1[n] - \tfrac{1}{2} x_2[n] + x_3[n]$$

since L is linear

$$L\{\delta[n]\} = \tfrac{1}{2} y_1[n] - \tfrac{1}{2} y_2[n] + y_3[n]$$

$$\delta[n-1] = -\tfrac{1}{2} x_1[n] + \tfrac{1}{2} x_2[n]$$

$$L\{\delta[n-1]\} = -\tfrac{1}{2} y_1[n] + \tfrac{1}{2} y_2[n]$$

since $L\{\delta[n]\} \neq L\{\delta[n-1]\}$ the system is not time invariant.

continued →

2.36

b) If the input $x[n]$ to the system L is $\delta[n]$, what is the system response $y[n]$?

As before, $\delta[n] = \frac{1}{2} x_1[n] - \frac{1}{2} x_2[n] + x_3[n]$

$$h[n] = L\{\delta[n]\} = \frac{1}{2} y_1[n] - \frac{1}{2} y_2[n] + y_3[n]$$

From the plots,

$$y_1[n] = -\delta[n+1] + 3\delta[n] + 3\delta[n-1] + \delta[n-3]$$

$$y_2[n] = -\delta[n+1] + \delta[n] - 3\delta[n-1] - \delta[n-3]$$

$$y_3[n] = 2\delta[n+2] + \delta[n+1] - 3\delta[n] + 2\delta[n-2]$$

$$\therefore h[n] = 2\delta[n+2] + \delta[n+1] - 2\delta[n] + 3\delta[n-1] + 2\delta[n-2] + \delta[n-3]$$