# ECE 796: Models of the Neuron

## Slides for Lecture #9 Monday, March 16, 2009

#### Kinetic model of sodium channel

The number of channels in the state  $m_3h_1$  determines the sodium conductance.

#### Kinetic model of potassium channel

$$n_{0} \stackrel{4\alpha_{n}}{\underset{\beta_{n}}{\rightrightarrows}} n_{1} \stackrel{3\alpha_{n}}{\underset{2\beta_{n}}{\rightrightarrows}} n_{2} \stackrel{2\alpha_{n}}{\underset{3\beta_{n}}{\rightrightarrows}} n_{3} \stackrel{\alpha_{n}}{\underset{4\beta_{n}}{\rightrightarrows}} (n_{4})$$

The number of channels in the state  $n_4$  determines the potassium conductance.





FIG. 4. Figures 4-11 are for  $N_{\text{Na}}=300$ ,  $N_{\text{K}}=30$ , C=1  $\mu\text{F/cm}^2$ , and A=1  $\mu\text{m}^2$  with a time step of 5  $\mu$ sec. The initial conditions are the unstable steady-state values. This figure is the voltage for the Hodgkin-Huxley model.

FIG. 5. This figure is the voltage for the modified Hodgkin-Huxley model.



FIG. 10. This is a direct comparison of the voltage for the two models with the time axis expanded about 100-fold compared with Figs. 4 and 5. The dashed curve is for the Hodgkin-Huxley model and the solid curve is for the modified Hodgkin-Huxley model.



FIG. 11. This a direct comparison of  $n^4$  with  $x_4$ , and of  $m^3h$  with  $y_{31}$ , with the time axis expanded about 100-fold compared with Figs. 6, 7, 8, and 9. The dashed curve is for the Hodgkin-Huxley model and the solid curve is for the modified Hodgkin-Huxley model. The higher pair of curves are for potassium.

(from Fox & Lu, 1994)



FIG. 16. This shows variable m determined from the master equations. To get m from the master-equation simulation, one counts the total number of first m elements (out of three) in each channel that are open and divides by  $N_{\rm Na}$ .

FIG. 17. This shows variable m determined from the stochastic Hodgkin-Huxley model.

(from Fox & Lu, 1994)



FIGURE 1. Stimulus wave form: monophasic in (a) where  $a_1$  and  $pw_1$ , respectively, denote the stimulus current intensity and duration, and preconditioned monophasic in (b), where  $a_1$ ,  $pw_1$ ,  $a_2$ , and  $pw_2$  stand for the preconditioned current intensity and duration, and the suprathreshold current intensity and duration.



FIGURE 2. Transmembrane potentials in response to ten identical monophasic stimulus pulses with an amplitude of 6.2 pA and a duration of 100  $\mu$ s (left) and poststimulus time histograms generated from 1000 Monte Carlo runs (right), where FE, JT, and LT are shown in each inset. The sampling step was set at 1  $\mu$ s.



FIGURE 3. FE (top), latency (middle), and jitter (bottom) as a function of stimulus current intensity for four algorithms at  $\Delta t = 1 \,\mu$ s. Stimulus duration was 100  $\mu$ s. From the data shown in the top panel,  $I_{th}$  and RS were estimated for four algorithms:  $I_{th} = 5.658 \text{ pA}$  and RS= 0.0350 (CW),  $I_{th} = 5.610 \text{ pA}$  and RS= 0.0441 (SD),  $I_{th} = 5.657 \text{ pA}$  and RS= 0.0436 (R), and  $I_{th} = 5.931 \text{ pA}$  and RS= 0.0215 (F).



FIGURE 4. Transmembrane potentials in response to ten identical stimulus pulses conditioned (left) at  $\Delta t = 1 \ \mu$ s. Poststimulus time histograms given from 1000 Monte Carlo runs (right). The subthreshold stimulus current of 2.5 pA was applied initially for a duration of 500  $\mu$ s, followed by a stimulus with an amplitude of 3.5 pA and a duration of 100  $\mu$ s.



FIGURE 7. FE, latency, jitter vs. the sampling step [1, 2, 5, and 10 ( $\mu$ s)]. Those statistical parameters at each sampling step were estimated from 1000 Monte Carlo runs in which the preconditioned stimuli were presented. The data of CW, SD, R, and F algorithms are, respectively, plotted by the marks \*,  $\times$ , +, and  $\bigcirc$ .





FIGURE 1. Firing efficiency (top), mean latency (middle) and jitter (bottom) versus stimulus current for a monophasic pulse of duration 100  $\mu$ s for three different algorithms: the Chow & White algorithm (\*), the Fox algorithm with rounding down of  $N_{\text{Na}}(t)$  ( $\bigcirc$ ), and the Fox algorithm with rounding of  $N_{\text{Na}}(t)$  to the nearest integer ( $\square$ ).



(from Bruce, ABME 2007)

FIGURE 2. Mean latency (top) and jitter (bottom) versus firing efficiency for a monophasic pulse of duration 100  $\mu$ s for three different algorithms: the Chow & White algorithm (\*), the Fox algorithm with rounding down of  $N_{Na}(t)$  ( $\bigcirc$ ), and the Fox algorithm with rounding of  $N_{Na}(t)$  to the nearest integer ( $\Box$ ).



FIGURE 3. Example transmembrane potentials in response to 10 identical preconditioned monophasic pulse stimuli (left). Histograms of spike times for 1,000 trials (right). The insets to the histograms give the respective firing efficiency (FE), jitter (JT) and mean latency (LT) for the 1,000 trials. A preconditioning current of 9.434 pA was applied for 500  $\mu$ s, followed immediately by a current of 13.208 pA for 100  $\mu$ s.





FIGURE 4. Transmembrane potentials from the deterministic equivalent to the Fox model in response to the preconditioned monophasic pulse stimuli.









(from Bruce, ABME 2009)







#### Modified single m-particle sodium channel

$$\begin{array}{cccc} m_0 h_0 & \stackrel{\alpha_m}{\rightleftharpoons} & m_1 h_0 \\ \alpha_h & & \beta_h & & \alpha_h & \beta_h \\ m_0 h_1 & \stackrel{\alpha_m}{\rightleftharpoons} & & m_1 h_1 \end{array}$$



(from Bruce, ABME 2009)

### Standard sodium channel



### Modified single m-particle sodium channel



