EE 4CL4 – Control System Design

Homework Assignment #2

1. Consider an electronic amplifier with input voltage $v_i(t)$ and output voltage $v_o(t)$. Assume that:

$$v_o(t) = 8v_i(t) + 2\,. \tag{1}$$

   a. Show that the amplifier does not strictly satisfy the principle of superposition. Thus, this system is not strictly linear. (A better term for this system would be affine.)

   b. Note that the system can also be written as follows:

$$v_o(t) = 8v_i(t) + 2d_i(t), \tag{2}$$

   where $d_i(t)$ is a constant offset (equal to 1).
   Show that the principle of superposition does hold for the input vector $[v_i(t)\ \ d_i(t)]^{\mathrm{T}}$.

   c. Obtain an incremental model for $\Delta v_o(t) = v_o(t) - v_{oQ}$, $\Delta v_i(t) = v_i(t) - v_{iQ}$, where (v_{iQ}, v_{oQ}) is any point satisfying the model given by Eq. (1) above. Show that this incremental model is the same for all choices of the pair (v_{iQ}, v_{oQ}). **(25 pts)**

2. Consider the following nonlinear state space model:

$$\dot{x}_1(t) = -2x_1(t) + 0.1x_1(t)x_2(t) + u(t), \tag{3}$$

$$\dot{x}_2(t) = -x_1(t) - 2x_2(t)(x_1(t))^2, \tag{4}$$

$$y(t) = x_1(t) + (1 + x_2(t))^2\,. \tag{5}$$

   Build a linearized model around the operating point given by $u_Q = 1$. **(25 pts)**

3. A system transfer function is given by:

$$H(s) = \frac{-s+1}{(s+1)^2}\,. \tag{6}$$

   Compute the time instant, t_u, at which the step response exhibits maximum undershoot. **(25 pts)**

4. The unit step response of a system with zero initial conditions is given by:

$$y(t) = 3 - 2\,\mathrm{e}^{-2t} - \mathrm{e}^{-3t} \quad \forall t \geq 0\,. \tag{7}$$

   a. Compute the system transfer function.

   b. Compute the system response to a unit impulse. **(25 pts)**