ELEC ENG 4CL4 – Control System Design

Homework Assignment #1

Submission deadline: 12 noon on Friday, January 30, 2004, in the designated drop box in CRL-101B (the CRL photocopying room).

1. Consider a system that obeys the differential equation:

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+\cos x=0.$$

   a. Linearize this equation around the operating point $x=\pi/4$.

   b. Derive a state-space representation of the linear equation found in part a. **(25 pts)**

2. A two-phase (i.e., two-input) permanent magnet stepper motor can be described by the following set of differential equations:

$$\frac{d^2\theta}{dt^2}=-K_2 i_a\sin(K_3\theta)+K_2 i_b\cos(K_3\theta)-K_1\frac{d\theta}{dt},$$
$$\frac{di_a}{dt}=-K_5 i_a+K_4\frac{d\theta}{dt}\sin(K_3\theta)+K_6 v_a,$$
$$\frac{di_b}{dt}=-K_5 i_b-K_4\frac{d\theta}{dt}\cos(K_3\theta)+K_6 v_b,$$

   where θ is angular displacement of the rotor, i_a and i_b are the currents in the two phases, v_a and v_b are the voltages applied the two phases (i.e., the inputs), and $K_1,\ldots,K_6$ are constants.

   a. Derive a state-space representation of this system.

   b. Linearize the state-space model found in part a around the operating point $\theta=\text{constant}$. **(25 pts)**

3. Given the following differential equation, solve for $y(t)$ using the Laplace transform if all initial conditions are zero:

$$\frac{d^2y}{dt^2}+12\frac{dy}{dt}+32y=32\mu(t),$$

   where $\mu(t)$ is the unit step function. **(25 pts)**

4. A system has the transfer function:

$$H(s)=\frac{10}{(s+7)(s+8)}.$$

   a. Compute the system's response to the Dirac delta function (unit impulse) $\delta_{\mathrm{D}}(t)$.

   b. Compute the system's response to the unit step function $\mu(t)$. **(25 pts)**