

Q. 8.3)

NMP zero at $s = z_0$

From Lemma 8.3: $\int_0^\infty e(t)e^{-z_0 t} dt = \frac{1}{z_0}$

$\textcircled{1} [0, \infty) = [0, t_1] \cup [t_1, \infty]$

so $\frac{1}{z_0} = \int_0^{t_1} e(t)e^{-z_0 t} dt + \int_{t_1}^\infty e(t)e^{-z_0 t} dt$

since $|e(t)| \leq |e(t)|$

$$\frac{1}{z_0} = \int_0^{t_1} e(t)e^{-z_0 t} dt + \int_{t_1}^\infty e(t)e^{-z_0 t} dt$$

$$\leq \int_0^{t_1} |e(t)|e^{-z_0 t} dt + \int_{t_1}^\infty |e(t)|e^{-z_0 t} dt$$

Since $E_{\max} = \max_{t \in [0, t_1]} (e(t))$ and $|e(t)| \leq e^{-\alpha(t-t_1)}$

$$\leq \int_0^{t_1} E_{\max} e^{-z_0 t} dt + \int_{t_1}^\infty e^{-\alpha(t-t_1)} e^{-z_0 t} dt$$

$$\leq E_{\max} \left[-\frac{1}{z_0} e^{-z_0 t_1} \Big|_{t_0}^{t_1} \right] + e^{\alpha t_1} \int_{t_1}^\infty e^{-t(\alpha+z_0)} dt$$

$$\frac{1}{z_0} \leq E_{\max} \left[-\frac{1}{z_0} e^{-z_0 t_1} + \textcircled{1} \right] + \frac{e^{\alpha t_1}}{\alpha+z_0} \left[e^{-t(\alpha+z_0)} \Big|_{t_1}^\infty \right]$$

$$\frac{1}{z_0} \leq E_{\max} \left[-\frac{e^{-z_0 t_1}}{z_0} + 1 \right] + \frac{e^{-t_1 z_0}}{\alpha+z_0}$$

$$\frac{1}{z_0} - \frac{e^{-z_0 t_1}}{\alpha+z_0} \leq E_{\max} \left[1 - \frac{e^{-z_0 t_1}}{z_0} \right]$$

$$\frac{\frac{1}{z_0} - \frac{e^{-z_0 t_1}}{\alpha+z_0}}{1 - \frac{e^{-z_0 t_1}}{z_0}} \leq E_{\max} \quad ; \quad E_{\max} \geq \frac{(\alpha+z_0) - z_0 e^{-z_0 t_1}}{(\alpha+z_0)(z_0 - e^{-z_0 t_1})}$$

Lower bound for E_{\max} in
 $[0, t_1]$

Q 8.3) b)

$$|e(t)| \leq e^{-\alpha(t-t_1)} \quad \alpha > 0 \quad t > t_1 > 0$$

- when $t = t_1$; $e^{-\alpha(0)} = e^0 = 1$, so

$$|e(t)| \leq 1$$

- If $|e(t)| \leq 1$ then the plant output $y(t)$ is +ve.

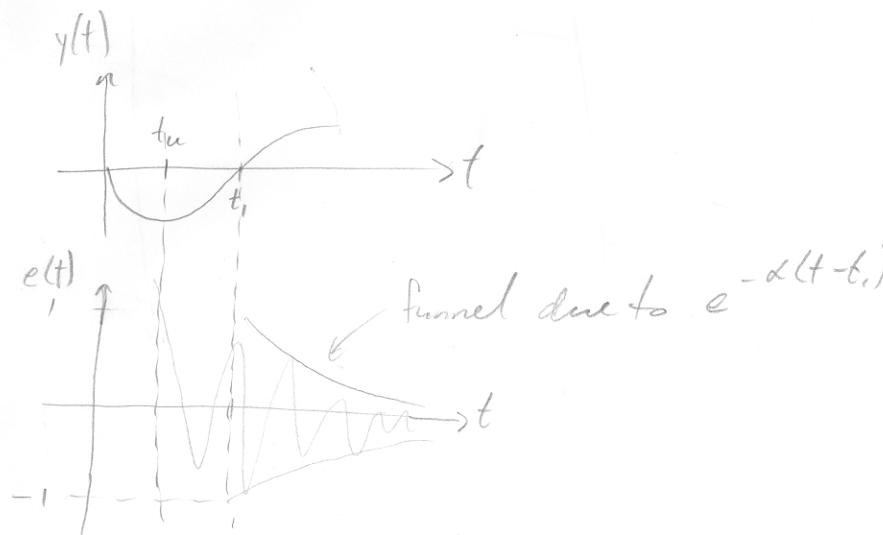
- A NMP zero creates an undershoot at $t = t_u$.

- So E_{max} must occur at $t = t_u$, and $|e(t_u)| = 1 + M_u > 1$

If $e^{-z_0 t_1} \approx z_0$ then E_{max} will be large,

and $e^{-z_0 t_1} \approx z_0$ at $t_1 = -\frac{\ln z_0}{z_0}$

i.e. when $0 < z_0 < 1$ E_{max} is large.



ANSWER
1. $\omega_n = 100 \text{ rad/s}$

2. z_0

3. t_u

4. M_u

5. E_{max}

6. α

7. t_1

8. t_u

9. E_{max}

10. z_0

11. t_1

12. t_u

13. E_{max}

14. α

15. t_u

16. E_{max}

17. t_1

18. z_0

19. t_u

20. E_{max}

21. α

22. t_1

23. t_u

24. E_{max}

25. z_0

26. t_1

27. t_u

28. E_{max}

29. α

30. t_1

31. t_u

32. E_{max}

33. z_0

34. t_1

35. t_u

36. E_{max}

37. α

38. t_1

39. t_u

40. E_{max}

41. z_0

42. t_1

43. t_u

44. E_{max}

45. α

46. t_1

47. t_u

48. E_{max}

49. z_0

50. t_1

51. t_u

52. E_{max}

53. α

54. t_1

55. t_u

56. E_{max}

57. z_0

58. t_1

59. t_u

60. E_{max}

61. α

62. t_1

63. t_u

64. E_{max}

65. z_0

66. t_1

67. t_u

68. E_{max}

69. α

70. t_1

71. t_u

72. E_{max}

73. z_0

74. t_1

75. t_u

76. E_{max}

77. α

78. t_1

79. t_u

80. E_{max}

81. z_0

82. t_1

83. t_u

84. E_{max}

85. α

86. t_1

87. t_u

88. E_{max}

89. z_0

90. t_1

91. t_u

92. E_{max}

93. α

94. t_1

95. t_u

96. E_{max}

97. z_0

98. t_1

99. t_u

100. E_{max}

Problem 8.4, Goodwin et al. p 238

$$\text{Given } G_o(s) = \frac{5(s-1)}{(s+1)(s-5)}$$

This plant has to be controlled in a feedback loop with one-degree-of-freedom.

8.4.1. Determine time-domain constraints for the plant input, the plant output, and the control error in the loop. Assume exact inversion at $w=0$ and step-like reference and disturbances.

$$G_o(s) = \frac{B_o(s)}{A_o(s)} \quad C(s) = \frac{P(s)}{L(s)}$$

Uncancelled plant poles & zeros impose algebraic / interpolation constraints on the sensitivity functions.

NB: zero @ $s=1 \rightarrow B_o(1)=0$
 pole @ $s=5 \rightarrow A_o(5)=0$
 exact inversion @ $w=0 \rightarrow L(0)=0$ in order to
 get $S_o(0)=0 \leftrightarrow T_o(0)=1$

Interpolation constraints:

$$S_o(1) = \frac{A_o(1)L(1)}{A_o(1)L(1) + B_o(1)P(1)} = 0$$

$$S_o(5) = \frac{A_o(5)L(5)}{A_o(5)L(5) + B_o(5)P(5)} = 0$$

$$T_o(1) = \frac{B_o(1)P(1)}{A_o(1)L(1) + B_o(1)P(1)} = 1$$

$$T_0(s) = \frac{B_0(s) P(s)}{A_0(s) L(s) + B_0(s) P(s)} = 1$$

$$S_{i_0}(1) = \frac{B_0(1) L(1)}{A_0(1) L(1) + B_0(1) P(1)} = 0$$

$$S_{u_0}(s) = \frac{A_0(s) P(s)}{A_0(s) L(s) + B_0(s) P(s)} = 0$$

• Effect of unit step reference:

$$Y(s) = T_0(s) \cdot \frac{1}{s} \xrightarrow[s=1]{} \int_0^\infty y(t) e^{-t} dt = \lim_{s \rightarrow 1} T_0(s) \cdot \frac{1}{s} = 1 \cdot 1 = 0$$

$$\xrightarrow[s=5]{} \int_0^\infty y(t) e^{-5t} dt = \lim_{s \rightarrow 5} T_0(s) \cdot \frac{1}{s} = 1 \cdot \frac{1}{5} = \frac{1}{5}$$

$$U(s) = S_{u_0}(s) \cdot \frac{1}{s} \xrightarrow[s=5]{} \int_0^\infty u(t) e^{-5t} dt = 0$$

$$E(s) = S_{i_0}(s) \cdot \frac{1}{s} \xrightarrow[s=1]{} \int_0^\infty e(t) e^{-t} dt = 1$$

$$\xrightarrow[s=5]{} \int_0^\infty e(t) e^{-5t} dt = 0$$

• Effect of unit step disturbance:

$$Y(s) = S_{i_0}(s) \cdot \frac{1}{s} \xrightarrow[s=1]{} \int_0^\infty y(t) e^{-t} dt = 0$$

$$U(s) = S_{u_0}(s) \cdot \frac{1}{s} \xrightarrow[s=5]{} \int_0^\infty u(t) e^{-5t} dt = 0$$

$$\xrightarrow[s=7]{} \int_0^\infty u(t) e^{-7t} dt = 1$$

$$E(s) = -S_{i_0}(s) \cdot \frac{1}{s} \xrightarrow[s=1]{} \int_0^\infty e(t) e^{-t} dt = 0$$

i Effect of unit step disturbance :

$$Y(s) = S_0(s) \cdot \frac{1}{s} \xrightarrow[s=5]{} \int_0^\infty y(t) e^{-5t} dt = 0$$

$$\xrightarrow[s=1]{} \int_0^\infty y(t) e^{-t} dt = 1$$

$$U(s) = -S_{u_0}(s) \cdot \frac{1}{s} \xrightarrow[s=5]{} \int_0^\infty u(t) e^{-5t} dt = 0$$

$$E(s) = -S_0(s) \cdot \frac{1}{s} \xrightarrow[s=5]{} \int_0^\infty e(t) e^{-5t} dt = 0$$

$$\xrightarrow[s=1]{} \int_0^\infty e(t) e^{-t} dt = 1$$

8.4.2 Why is the control of this nominal plant especially difficult?

→ contradicting requirements:

- the NMP zero sets an upper bound for the closed loop bandwidth since * indicates that if $y(t)$ settles much faster than e^{-t} , there will be a large undershoot.
- the unstable pole sets a lower bound for the closed loop bandwidth since * indicates that if $y(t)$ settles much slower than e^{-5t} , there will be a large overshoot.