

Homework 1

1. Evaluate (a) $(1 - j)^6$; (b) $(1 + j\sqrt{3})^{1/3}$; (c) $j^{1/5}$; (d) $(-j)^{1/3}$.
2. Find $u(x, y)$ and $v(x, y)$ if $f(z)$ is given by (a) $\cos z$; (b) $\cosh z$; (c) $\tan z$.
3. Integrate $\frac{z+1}{z^2(z+2)}$ around the circle $|z| = 1$.
4. Evaluate the integral $\int_0^{2\pi} \frac{\sin^2 \theta d\theta}{a+b \cos \theta}$ for a and b real and $a > b > 0$.
5. Find the Laurent expansion of $f(z)$ if $f(z)$ is given by (a) $\frac{z}{z^2-1}$ for $|z| > 1$; (b) $\frac{1}{(z-1)(2-z)}$ in $1 < |z| < 2$; (c) $\frac{1}{(z-1)(2-z)}$ for $|z| > 2$.
6. We have shown that $\mathcal{L}\{\sin \omega_0 t\} = \frac{\omega_0}{s^2 + \omega_0^2}$. From this fact, and by using properties of the Laplace transforms, find the transforms of the following functions: (a) $t \sin \omega_0 t$; (b) $\frac{\sin \omega_0 t}{t}$; (c) $e^{-at} \sin \omega_0 t$ ($a > 0$ and real).
7. Given $F(s) = \frac{-3s^3 + 2s^2 + 10s + 2}{(s^2 + 2s + 10)^2}$ without ever finding $f(t)$, calculate $f(0_-)$, $f(\infty)$, $f'(0_-)$, and $f'(\infty)$.
8. Let a system be described by $\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 10y = \frac{d^2 p}{dt^2} - 3\frac{dp}{dt} + 5p$. Find its impulse response.
9. Find the Laplace transform of the following functions: (a) $\frac{1}{2\sqrt{\pi t^3}}(e^{-at} - e^{-bt})u(t)$; (b) $\frac{1}{\sqrt{t}} \cos 2\sqrt{kt}u(t)$; (c) $\frac{1}{\sqrt{\pi t}} e^{-\lambda^2/4t}u(t)$.
10. Find the inverse Laplace transform of the following functions, by use of the Laplace inversion theorem: (a) $\frac{1}{s(s^2+a^2)}$; (b) $\frac{1}{s^4-a^4}$; (c) $\frac{1}{s(e^{sT}-a)}$.