EE4TK4 Project 2
Computer Simulation on Matched Filters for Detection of BPSK and BFSK Signals

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1 Introduction

As most of the communication channels are bandpass in nature [1], digital signals are modulated by a carrier wave of appropriate frequency before transmission. In order to distinguish the transmitted bits in the presence of noise with minimum probability of error, a receiver has to be carefully designed. For the detection of binary signals in spectrally white noise, the matched filter is optimum in the sense that the output signal-to-noise ratio (SNR) is maximized at the end of the bit period and thus the probability of error in the threshold detection is minimized.

In this project, we would like to design matched filters for the reception of binary Frequency Shift Keying (FSK) and Phase Shift Keying (PSK) signals and investigate the performance in each case.

2 Experiment

Consider the binary transmission system as shown in Figure 1. The input to the system is a binary sequence $\{b_k\}$ of bit duration T_b. Depending on the modulation scheme being employed, the modulated signal $s(t)$ takes on different waveforms. For BFSK systems, the modulated signals are given by

$$s(t) = \begin{cases} s_1(t) = A\cos(\omega_1 t) & \text{if } b_k = 1 \\ s_2(t) = A\cos(\omega_2 t) & \text{if } b_k = 0 \end{cases}$$

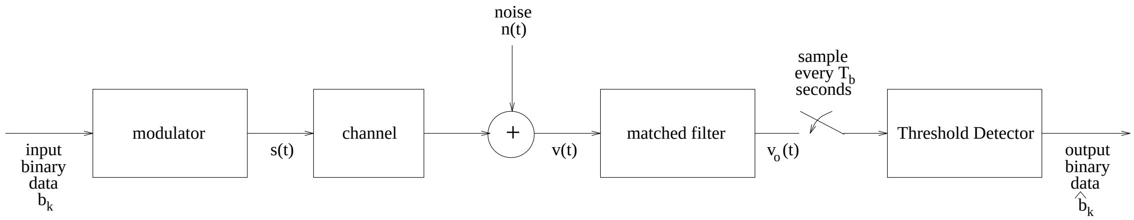

Figure 1: Block Diagram of a Binary Data Transmission System

where $0 < t < T_b$, A is the amplitude of the modulated waveform, and

$$\omega_1 = \frac{2\pi(n_0 + n_1)}{T_b},$$
$$\omega_2 = \frac{2\pi(n_0 - n_1)}{T_b}.$$

In this project, we choose

$$n_0 = 5,$$
$$n_1 = 2.$$

For BPSK modulation, we can write

$$s(t) = \begin{cases} s_1(t) = A\cos(\omega_c t) & \text{if } b_k = 1, \\ s_2(t) = -A\cos(\omega_c t) & \text{if } b_k = 0. \end{cases}$$

where $0 \le t \le T_b$, A is the amplitude of the modulated waveform, and we choose

$$\omega_c = 5\ \frac{2\pi}{T_b}.$$

The modulated signal will then be transmitted through the channel, and is corrupted by some additive noise $n(t)$. For simplicity, the channel is assumed to be ideal and the channel noise is assumed to be white and Gaussian with power spectral density

$$S_n(\omega) = \frac{\eta}{2}.$$

The optimum receiver consists of a matched filter, a sampler, and a threshold detector. The signal-plus-noise $v(t)$ is filtered, producing $v_o(t)$. The transmitted bit is obtained by comparing the sampled $v_o(t)$ at the end of the bit period with a pre-determined threshold V_T according to the following rule:

$$\hat{b}_k = \begin{cases} 1 & \text{if } v_o(T_b) > V_T, \\ 0 & \text{if } v_o(T_b) < V_T. \end{cases}$$

3 Procedures

In computer simulations, a very important issue is that the signals and noise have to be sampled. Since the bit period is T_b, and in the above theory, we assumed the transmitted sinusoidal pulses have rectangular envelope, therefore, to represent the continuous signal shapes adequately, we must sample at a substantially higher frequency than the bit frequency. Let the sampling frequency be

$$\omega_s = \frac{2\pi}{T_s} = N.\ \frac{2\pi}{T_b}$$

Here in this project, we choose T_s to be unity and $N = 30$. Using this sampling period, we generate the following:

1. Generate a random sequence of binary bits $\{b_k\}$ using MATLAB. Note that this sequence of binary digits are represented by rectangular pulses having amplitudes ± 1, each lasting for a period of T_b and sampled at T_s. What is the effective bandwidth of this sequence of rectangular pulses?
   This sequence of pulses will modulate different carriers of amplitude A in the transmission either by FSK or by PSK as described in Section 2.

2. Generate a zero-mean, white Gaussian random sequence $\nu(nT_s)$ sampled at T_s. This noise sequence has a two-sided power spectral density of $\frac{\eta}{2}$. In this project, we fix $\eta = 2$. Note that since $\nu(nT_s)$ is sampled, the power of this noise sequence depends on the sampling period T_s, the smaller is T_s, the larger is the noise power. Find an expression for the power of this noise sequence in terms of η and T_s.

3. Since the noise sequence $\nu(nT_s)$ has a bandwidth much larger than the effective bandwidth of the signal pulses, the noise outside the effective bandwidth is irrelevant to the transmission of the data. We therefore define an *effective* input signal-to-noise ratio ρ such that

   $$\rho = \frac{A^2 T_b}{2\eta}$$

   where A is the amplitude of the modulated pulse. When expressed in dB, we have

   $$SNR_e = 10\ \log_{10}[(A^2 T_b)/(2\eta)]$$

   In this project, we investigate the transmission and reception of FSK and PSK signals in an effective input signal-to-noise ratio range of -4 dB to 4 dB.

4. With a fixed value of $\eta = 2$, for each chosen value of SNR_e in the range of -4 db to 4 dB, generate a sequence of FSK modulated pulses and another sequence of PSK modulated pulses using the value of A calculated. This procedure yields the transmitted signal $s(t)$.

5. The sampled noise sequence is added to each of the $s(t)$ representing the FSK and PSK sequences. This mixture yields the noisy signal $v(t)$ resulted from transmitting through the channel.

6. Each of the noisy signal sequences is then passed through the matched filter designed respectively for the specific transmission scheme. Denote the output of the matched-filter at the end of each bit period by $v_0(kT_b)$.

7. Compare $v_0(kT_b)$ with the respective threshold in each of the cases and decide if the transmitted bit b_k is 1 or 0.

8. For a suitably selected number of bits transmitted in the sequence, evaluate the bit-error rate (experimental probability of error) of the each of the transmission schemes at the specific SNR_e. Bit-error rate is the number of erroneous bit at the output in comparison with the transmitted binary sequence, i.e.,

$$P_{\exp} = \frac{\text{total number of erroneous bits}}{\text{total number of transmitted bits}}$$

9. Since noise is a random process, repeat Step 1 through Step 8 for 20 realizations of noise and calculate the average bit-error rate, $\bar{P}_{\exp}$, for the same chosen SNR_e.

10. Repeat Step 1 through Step 9 for another value of effective signal-to-noise ratio (SNR_e ranges from -4 dB to 4 dB).

11. Plot the performance graphs of binary FSK and PSK transmissions (Bit-Error Rate against Signal-to-Noise Ratio). Compare these curves with the theoretical performance and comment on your results.

References

[1] K.M. Wong, "Notes on Digital Communication Systems", Custom Courseware, McMaster University, 2009.