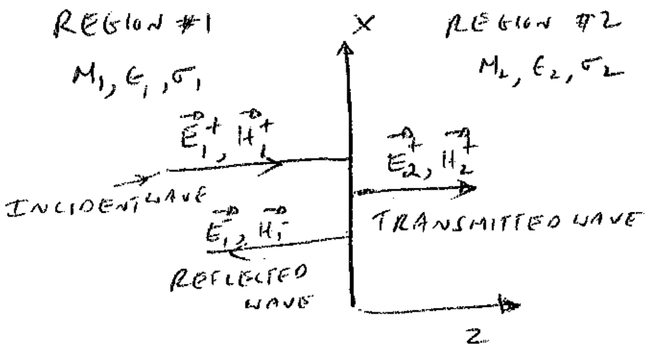
PLANE WAVE REFLECTION



CONSIDER THE INTERFACE BETWEEN TWO REGIONS. HERE, THE REGIONS COULD BE ANY TYPE OF MEDIUMS - DIELECTRIC or CONDUCTORS.. SUPPOSE THAT ~~INCIDENT~~ A WAVE IS INCIDENT AT THE BOUNDARY FROM LEFT ($z<0$). THE ELECTRIC & MAGNETIC FIELDS OF THE INCIDENT ~~WAVE ARE~~

$$\vec{E}_1^+ = E_{x1}^+ \vec{x} \quad ; \quad H_1^+ = H_{y1}^+ \vec{y}$$

HERE, THE SUPERSCRIPT '+' INDICATES IT IS TRAVELLING IN THE +VE z-DIRECTION & THE SUBSCRIPT '1' ~~INDICATES~~ CORRESPONDS TO THE REGION #1. LET

$$E_{x1}^+ = \text{Re}\{E_{xs1}^+ e^{j\omega t}\}$$

$$H_{y1}^+ = \text{Re}\{H_{ys1}^+ e^{j\omega t}\}$$

$$E_{xs1}^+(z) = A_1^+ e^{-jk_1 z}$$

$$E_{x1}^+ = A_1^+ e^{-\alpha_1 z} \cos(\omega t - \beta_1 z)$$

(1) REFER TO PREVIOUS NOTES.

$$H_{ys1}^{+}(z) = \frac{A_1^{+}}{\eta_1} e^{-jk_1 z} \quad \Big\} \rightarrow (2)$$

$$H_{y1}^{+} = \frac{A_1^{+}}{\eta_1} e^{-\alpha_1 z} \cos(\omega t - \beta_1 z)$$

HERE, WE TAKE A_1^+ AS REAL. k_1 & η_1 ARE COMPLEX IN GENERAL. THE WAVE IS TRANSMITTED TO REGION 2 AND ELECTRIC & MAGNETIC FIELDS IN REGION 2 ARE

$$E_{xs2}^{+}(z) = A_2^{+} e^{-jk_2 z}$$

$$H_{ys2}^{+}(z) = \frac{A_2^{+}}{\eta_2} e^{-jk_2 z} \quad \Big\} \rightarrow (3)$$

LET US SUPPOSE THE ENERGY OF THE INCIDENT WAVE IS TRANSMITTED TO REGION 2 WITHOUT REFLECTION AT THE BOUNDARY. WE MUST SATISFY THE BOUNDARY CONDITIONS AT $z=0$.

TANGENTIAL COMPONENTS OF $\vec{E}$ SHOULD BE CONTINUOUS AT THE BOUNDARY. SINCE $\vec{E}$ IS POLARIZED ALONG $\vec{x}$, THE FIELD IS TANGENT TO THE BOUNDARY SURFACE. AT $z=0$, FROM (1) & (3), WE HAVE

$$E_{xs1}^{+} = E_{xs2}^{+} \Rightarrow A_1^{+} = A_2^{+} \longrightarrow (4)$$

SIMILARLY, TANGENTIAL COMPONENTS OF $\vec{H}$ SHOULD BE CONTINUOUS. $\vec{H}$, BEING Y-DIRECTED, IS ALSO TANGENT TO THE BOUNDARY SURFACE. AT $z=0$,

$$H_{ys1}^{+} = H_{ys2}^{+} \Rightarrow \frac{A_1^{+}}{\eta_1} = \frac{A_2^{+}}{\eta_2} \Rightarrow \eta_1 = \eta_2 \Rightarrow (5)$$

THIS MEANS THAT THE MATERIALS ON BOTH SIDES OF THE BOUNDARY SHOULD BE THE SAME (i.e. NO BOUNDARY) TO HAVE NO REFLECTION. IN OTHERWORDS, WE ARE NOT ABLE TO SATISFY THE BOUNDARY CONDITIONS WHEN TWO DIFFERENT MATERIALS ($\eta_1 \neq \eta_2$) ARE PRESENT, & ~~NO~~ IN THE ~~REFLECTION~~ ABSENCE OF REFLECTION.

NEXT, LET US CONSIDER THE CASE IN WHICH THE INCIDENT WAVE IS PARTIALLY TRANSMITTED & THE REST IS REFLECTED. THE REFLECTED FIELD ~~IS~~

BACKWARD PROPAGATING WAVE

$$E^-_{xs1} = A_1^- e^{jk_1 z} \longrightarrow (6)$$

$$H^-_{ys1} = -\frac{A_1^-}{\eta_1} e^{jk_1 z} \longrightarrow (7)$$

($\because$ POYNTING VECTOR $\vec{E}^- \times \vec{H}^-$ SHOULD BE IN $-\vec{z}$ DIRECTION)

SO, THE TOTAL FIELD IN REGION 1 IS

$$\begin{aligned} E_{xs1} &= E^+_{xs1} + E^-_{xs1} \\ &= A_1^+ e^{-jk_1 z} + A_1^- e^{jk_1 z} \longrightarrow (6) \end{aligned}$$

$$\begin{aligned} H_{ys} &= H^+_{ys1} + H^-_{ys1} \\ &= \frac{A_1^+}{\eta_1} e^{-jk_1 z} - \frac{A_1^-}{\eta_1} e^{jk_1 z} \longrightarrow (7) \end{aligned}$$

CONTINUITY OF THE TANGENTIAL COMPONENT OF $\vec{E}$ & $\vec{H}$ AT THE BOUNDARY $(z=0)$ IMPLIES, (IN EQS. (3), (4), (6) & (7), SET $z=0$)

$$A_1^+ + A_1^- = A_2^+ \quad \rightarrow (8)$$

$$\frac{A_1^+ - A_1^-}{\eta_1} = \frac{A_2^+}{\eta_2} \quad \rightarrow (9)$$

$$(8) + \eta_1 \times (9) \Rightarrow \quad 2A_1^+ = A_2^+\left(1 + \frac{\eta_1}{\eta_2}\right) = A_2^+ \frac{(\eta_1 + \eta_2)}{\eta_2} \quad \rightarrow (10)$$

~~THE RATIO OF~~

THE RATIO OF THE AMPLITUDES OF THE TRANSMITTED & INCIDENT FIELDS DEFINES THE <u>TRANSMISSION COEFFICIENT</u>, τ

$$\boxed{\tau = \frac{A_2^+}{A_1^+} = \frac{2\eta_2}{\eta_1 + \eta_2} = |\tau| e^{j\varphi_\tau}} \quad \rightarrow (11)$$

ELIMINATING A_2^+ IN EQ. (8) BY USING EQ. (11), WE GET

$$A_1^+ + A_1^- = \frac{2\eta_2}{\eta_1 + \eta_2} A_1^+$$

$$A_1^- = \left(\frac{2\eta_2}{\eta_1 + \eta_2} - 1\right) A_1^+ = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} A_1^+$$

THE RATIO OF THE AMPLITUDE OF REFLECTED FIELD TO INCIDENT FIELD IS DEFINED AS REFLECTION COEFFICIENT, Γ,

$$\boxed{\Gamma = \frac{A_1^-}{A_1^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\varphi_\Gamma}} \rightarrow (12)$$

DIVIDING EQ. (8) BY A_1^+, WE HAVE

$$1 + \frac{A_1^-}{A_1^+} = \frac{A_2^+}{A_1^+} \Rightarrow \boxed{1 + \Gamma = \tau} \rightarrow (13)$$

SPECIAL CASES:

REGION 1 : PERFECT DIELECTRIC
REGION 2 : PERFECT CONDUCTOR

FOR A PERFECT CONDUCTOR, $\sigma = \infty$;

$$\therefore \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} = 0 \quad \text{SINCE } \sigma_2 = \infty.$$

NOW, EQ. (12) & (13) BECOME

$$\Gamma = -1 \rightarrow (14)$$

$$\tau = 0 \rightarrow (15)$$

FROM EQ. (11), $\tau = 0 \Rightarrow A_2^+ = 0 \Rightarrow$ EM WAVE IS NOT TRANSMITTED TO THE PERFECT CONDUCTOR & THE INCIDENT WAVE IS FULLY REFLECTED. ANOTHER WAY TO LOOK AT THIS

IS TO NOTE THAT THE SKIN DEPTH IS ZERO

$$\Gamma = -1 \Rightarrow A_1^- = -A_1^+ \rightarrow (16)$$

$|A_1^-| = |A_1^+| \Rightarrow$ THE AMPLITUDES OF INCIDENT & REFLECTED FIELDS ARE EQUAL. -VE SIGN IN EQ. (16) INDICATES THAT AT THE BOUNDARY, THE REFLECTED FIELD IS SHIFTED IN PHASE BY 180° RELATIVE TO THE INCIDENT FIELD. TOTAL FIELD IN REGION 1 IS

$$E_{xs1} = E_{xs1}^+ + E_{xs1}^-$$

$$= A_1^+ \left(\frac{e^{-jk_1 z} - e^{jk_1 z}}{2j} \right) 2j$$

$$= -2j A_1^+ \sin(k_1 z)$$

FOR A PERFECT (LOSSLESS) DIELECTRIC, ϵ IS REAL

$k_1 = \omega\sqrt{\mu_1 \epsilon_1}$ IS REAL.

due to $-j$

$$\therefore \quad E_{x1} = Re\{ E_{xs1}\, e^{j\omega t} \} = 2Re\{ A_1^+ e^{j(\omega t - j\pi/2)} \} \times \sin(k_1 z)$$

$$E_{x1} = 2A_1^+ \sin k_1 z \sin \omega t$$