PROPAGATION IN GOOD CONDUCTORS

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_\sigma + \frac{\partial \vec{D}}{\partial t}$$

$\vec{J}_\sigma = \sigma \vec{E}$

= CONDUCTION CURRENT DENSITY

WITH $\vec{E} = E_x \vec{x}$

$\vec{H} = H_y \vec{y}$

AND $E_x = Re\{E_{xs}\, e^{j\omega t}\}$

$H_y = Re\{H_{ys}\, e^{j\omega t}\}$

WE HAVE FOUND THAT

$$\nabla \times H_{ys} = (\sigma + j\omega\epsilon) E_{xs} \quad \rightarrow (1)$$

$$J_{\sigma s} = \sigma E_{xs} = \text{CONDUCTION CURRENT DENSITY} \quad \rightarrow (2)$$

$$J_{ds} = j\omega\epsilon E_{xs} = \text{DISPLACEMENT CURRENT DENSITY.} \quad \rightarrow (3)$$

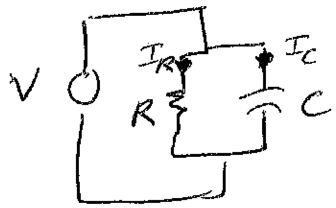
THE RATIO OF CONDUCTION CURRENT DENSITY TO DISPLACEMENT CURRENT DENSITY IS

$$\frac{J_{\sigma s}}{J_{ds}} = \frac{\sigma}{j\omega\epsilon}$$

FROM (2) & (3), WE SEE THAT DISPLACEMENT CURRENT DENSITY LEADS CONDUCTION CURRENT DENSITY BY 90°. THIS IS SIMILAR TO THE CURRENT THROUGH A CAPACITOR LEADS THE CURRENT THROUGH A RESISTOR IN PARALLEL BY 90° IN AN

$\vec{B}$

ELECTRIC CIRCUIT.

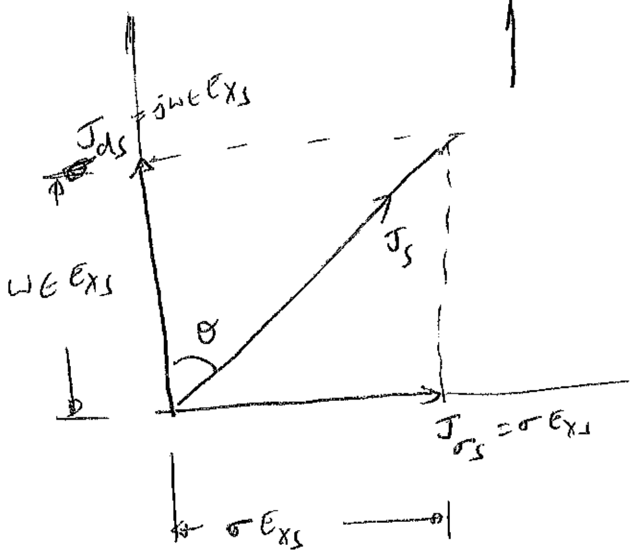


$$I_R = V/R$$

$$I_C = \frac{V}{1/j\omega C} = j\omega C V$$

$$\frac{I_R}{I_C} = \frac{1/R}{j\omega C}$$

CIRCUIT		FIELD
V	$\rightarrow$	E_{xs}
R	$\rightarrow$	$1/\sigma$
C	$\rightarrow$	ϵ



$$\boxed{\tan\theta = \frac{\sigma E_{xs}}{\omega\epsilon E_{xs}} = \frac{\sigma}{\omega\epsilon}} \longrightarrow (4)$$

IS KNOWN AS LOSS TANGENT. 'θ' IS THE ANGLE BY WHICH THE DISPLACEMENT CURRENT DENSITY LEADS THE TOTAL CURRENT DENSITY.

FROM MAXWELL'S EQUATIONS, WE FOUND THAT

$$\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs} \longrightarrow (5)$$

$$k^2 = -j\omega\mu(\sigma + j\omega\epsilon) \longrightarrow (6)$$

$$E_{xs} = \underbrace{A e^{jkz}}_{\text{BACKWARD}} + \underbrace{B e^{-jkz}}_{\text{FORWARD}}$$

CONSIDER THE FORWARD PROPAGATION:

$$E_{xs} = B e^{-jkz} \longrightarrow (7)$$

$$jk = \alpha + j\beta \longrightarrow (8)$$

$$E_{xs} = B e^{-\alpha z} e^{-j\beta z}$$

$$E_x = Re\{E_{xs} e^{j\omega t}\}$$

$$= B e^{-\alpha z} \cos(\omega t - \beta z) \longrightarrow (9)$$

FROM (6)

$$k = \sqrt{-j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= \sqrt{\omega^2\mu\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}$$

$$jk = j\omega\sqrt{\mu\epsilon}\left(1 - \frac{j\sigma}{\omega\epsilon}\right)^{1/2} \longrightarrow (10)$$

FOR A GOOD CONDUCTOR, THE LOSS TANGENT $\sigma/\omega\epsilon$ IS MUCH LARGER THAN UNITY. FOR EXAMPLE, FOR COPPER $\sigma = 5.8 \times 10^7$ S/m at 100 MHz, $\sigma/\omega\epsilon = 1.04 \times 10^{10} \gg 1$. HENCE, I could be ignored in Eq. (10) for a good conductor.

$$jk = j\omega\sqrt{\mu\epsilon}\left(\frac{-j\sigma}{\omega\epsilon}\right)^{1/2}$$

$$= j\sqrt{\omega\mu\sigma}\,(-j)^{1/2}$$

$$-j = 1\angle{-\pi/2}$$

$$\sqrt{-j} = 1^{1/2}\angle{-\pi/4} = \frac{1}{\sqrt{2}}(1-j)$$

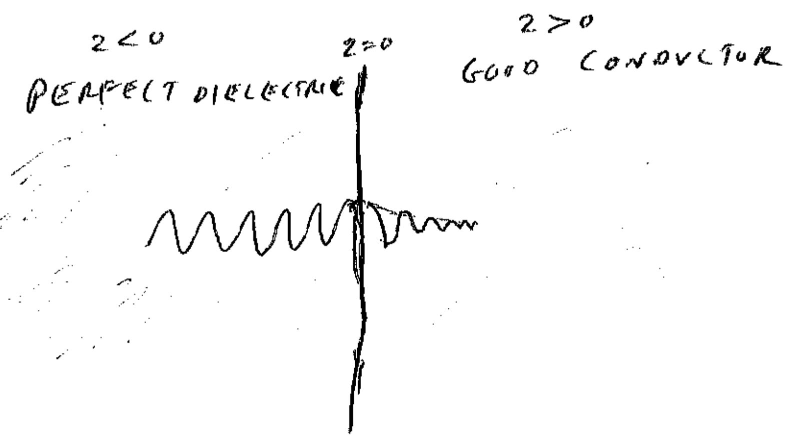
$$jk = \sqrt{\frac{\omega\mu\sigma}{2}}\,(1+j) = (1+j)\sqrt{\pi f\mu\sigma}$$

$$= \alpha + j\beta$$

$$\therefore \boxed{\alpha = \sqrt{\pi f\mu\sigma} = \beta} \longrightarrow (11)$$

Substituting Eq. (11) in (9), we find

$$E_x = B\,e^{-z\sqrt{\pi f\mu\sigma}}\cos\left(\omega t - z\sqrt{\pi f\mu\sigma}\right) \longrightarrow (12)$$



Consider an EM wave impinging on ~~the~~ a good conductor from a perfect dielectric. Let the boundary surface

BE AT $z=0$. AT $z=0$, EQ. (12) BECOMES

$$\mathcal{E}_x = B\cos(\omega t)$$

THIS IS THE SOURCE FIELD THAT ESTABLISHES THE FIELDS WITHIN THE CONDUCTOR. THE CONDUCTION CURRENT DENSITY IS

$$J_x = \sigma \mathcal{E}_x$$
$$= \sigma \mathcal{E}_{x_0} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma}) \rightarrow (13)$$

FROM (12) & (13), WE SEE THAT THE AMPLITUDES OF ELECTRIC FIELD INTENSITY, $\mathcal{E}_x$ & CONDUCTION CURRENT DENSITY DECREASE EXPONENTIALLY WITH PENETRATION INTO THE CONDUCTOR.

WHEN $z\sqrt{\pi f \mu \sigma} = 1$, THE AMPLITUDE BECOMES $B e^{-1}$.

AT

$$z = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

THE AMPLITUDE DROPS TO $e^{-1} = 0.368$ OF THE INITIAL VALUE AT THE BOUNDARY. THIS DISTANCE IS DENOTED BY δ & IS TERMED THE <u>SKIN DEPTH</u>.

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha}}$$

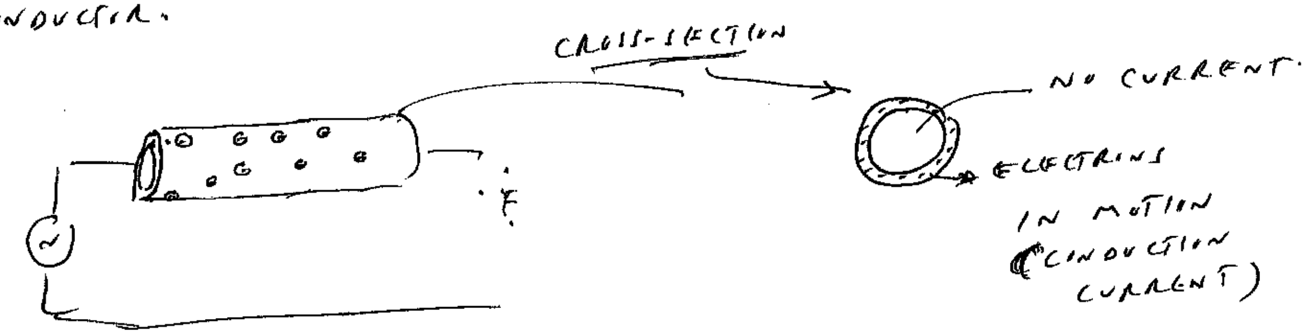
FOR COPPER, $\sigma = 5.8 \times 10^{7}$ S/m, $\mu = 4\pi \times 10^{-7}$ H/m

$$\delta_{cu} = \frac{1}{\sqrt{f}} \cdot \frac{1}{\sqrt{\pi \times 5.8 \times 10^{7} \times 4\pi \times 10^{-7}}} = \frac{0.066}{\sqrt{f}}$$

AT A POWER FREQUENCY, $(f = 60\text{ Hz})$, $\delta_{cu} = 8.53$ mm..

AT THE MICROWAVE FREQUENCY $(f = 10\text{ GHz})$ $\delta_{cu} = 6.61 \times 10^{-4}$ mm. THE FIELD INSIDE A GOOD CONDUCTOR SUCH AS COPPER IS ESSENTIALLY ZERO AT DISTANCES GREATER THAN ~~THE~~ A FEW SKIN DEPTHS.

ELECTROMAGNETIC ENERGY IS NOT TRANSMITTED IN THE INTERIOR OF THE CONDUCTOR; IT TRAVELS IN THE REGION SURROUNDING THE CONDUCTOR.



NOTE THAT THE ELECTRIC FIELD INTENSITY DECAYS EXPONENTIALLY WITH DISTANCE (FROM THE DIELECTRIC - CONDUCTOR BOUNDARY). THIS LOSS OF ENERGY OF THE EM WAVE CORRESPONDS TO THE GAIN OF ENERGY OF ELECTRONS (KINETIC ENERGY). THE ELECTRIC FIELD INTENSITY IS THE FORCE PER +VE UNIT CHARGE.

So, ~~ELECTRONS~~ THE FREE ELECTRONS EXPERIENCE A FORCE DUE TO E_X & OSCILLATE. ELECTROMAGNETIC ENERGY IS USED TO SET UP ELECTRON MOTION (i.e. CURRENT). IF YOU PUT A METAL PLATE IN A MICROWAVE, YOU WILL FIND SPARKS WHICH INDICATES THAT ELECTRONS ARE IN MOTION.

SUPPOSE WE HAVE A COPPER BUS BAR IN THE SUBSTATION OF AN ELECTRIC UTILITY COMPANY (OPERATING AT 60 HZ). THE COPPER BUS SHOULD CARRY LARGE CURRENTS. IF YOU PICK THE DIMENSIONS OF 5CM × 5CM, THEN MUCH OF THE COPPER IS WASTED, SINCE THE FIELDS EXPONENTIALLY DECAY IN ONE SKIN DEPTH OF 8.5 MM. A HOLLOW CONDUCTOR WITH A THICKNESS OF 1.5CM WOULD BE A MUCH BETTER DESIGN.