VOLTAGE STANDING WAVE RATIO (VSWR)

IN THIS SECTION, WE FOCUS ON LOSSLESS TRANSMISSION LINE. ($R = G = 0$). ~~THE ER~~ IN PRACTICE, TRANSMISSION LINES HAVE LOSSES. HOWEVER, IF THE LENGTH OF THE LINE IS SHORT AND/OR $R \ll \omega L$ & $G \ll \omega C$, LOSSES CAN BE IGNORED. IN THIS CASE, ANALYSIS BECOMES MUCH SIMPLER & IN FACT, THE ANALYSIS IS APPLICABLE TO MEASUREMENT OF CHARACTERISTICS OF TRANSMISSION LINES. IN SOME TRANSMISSION LINES, IT IS POSSIBLE TO MEASURE VOLTAGE AS A FUNCTION OF DISTANCE, Z.. BY MEASURING THE VOLTAGE MAXIMUM & VOLTAGE MINIMUM, IT IS POSSIBLE TO CALCULATE REFLECTION COEFFICIENT, Γ, & CHARACTERISTIC IMPEDANCE, Z_0 & OTHER IMPORTANT PARAMETERS OF THE TRANSMISSION ~~CO~~ SYSTEM. COAXIAL TRANSMISSION LINES HAVE A LONGITUDINAL GAP IN THE OUTER CONDUCTOR ALONG ITS ENTIRE LENGTH. A VOLTAGE ~~PLANE~~ PROBE COULD BE INSERTED BETWEEN THE INNER & OUTER CONDUCTOR THROUGH THE GAP.

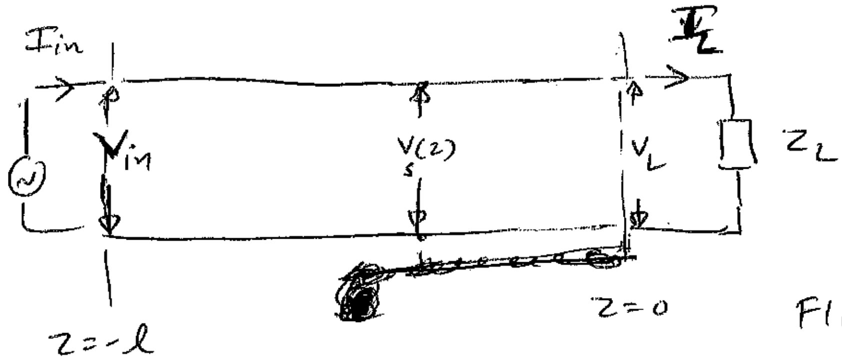


FIG. 1

FOR A LOSSLESS ~~TX~~ LINE, THE PHASOR VOLTAGE IS

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \rightarrow (1)$$

$$\left(\because \gamma = \alpha + j\beta z;\ \alpha = 0,\ \text{so } \gamma = j\beta \right)$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow (2)$$

SO, (1) BECOMES:

$$V_s(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z} \right) \rightarrow (3)$$

$$\text{LET } \Gamma = |\Gamma| e^{j\phi} \rightarrow (4)$$

MATCHED LOAD: $Z_L = Z_0 \Rightarrow \Gamma = 0$

LOAD IS A SHORT CIRCUIT: $Z_L = 0$, $\Gamma = -1 = 1 \cdot e^{j\pi}$

i.e. $|\Gamma| = 1$ & $\varphi = \pi$

LOAD IS AN OPEN CIRCUIT: $Z_L = \infty$; $\Gamma = \dfrac{Z_L(1 - Z_0/Z_L)}{Z_L(1 + Z_0/Z_L)} = 1$

$$|\Gamma| = 1 \quad \& \quad \phi = 0$$

<u>Z_L IS REAL & $Z_L < Z_0$:</u>

NOTE: Z_0 IS REAL $\left(= \sqrt{\frac{L}{C}}\right)$

~~WHEN~~ FOR LOSSLESS TR. LINE

$Z_L < Z_0 \Rightarrow \Gamma$ IS $-$VE

LET $\quad \Gamma = -|\Gamma|$

$\therefore \quad \phi = \pi$

<u>Z_L IS REAL & $Z_L > Z_0$:</u>

Γ IS +VE.

$$\Gamma = |\Gamma| e^{j0}$$

$$\phi = 0$$

<u>MATCHED LOAD:</u> $\quad \Gamma = 0 \Rightarrow$

$$V_s(z) = V_0^+ e^{-j\beta z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-j\beta z}$$

$$\frac{V_s(z)}{I_s(z)} = Z_0 \quad \text{AT ANY POINT IN THE TR. LINE.}$$

AT THE ~~INPUT~~ SOURCE, $z = -\ell$, $V_s(-\ell) = V_{in}$ &

$I_s(-\ell) = I_{in}$

$$\frac{V_{in}}{I_{in}} = Z_0 \Rightarrow$$ THE EFFECTIVE IMPEDANCE "SEEN" AT THE SOURCE TERMINALS IS $Z_0 = Z_L$.

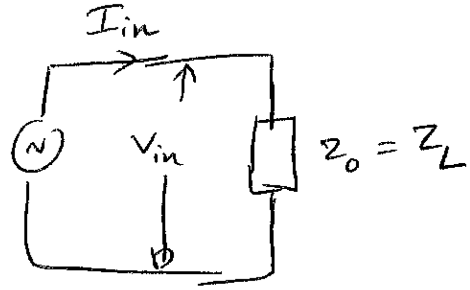


FIG. 2

FIG. 2 IS THE EQUIVALENT CIRCUIT OF THE TRANSMISSION SYSTEM SHOWN IN FIG. 1. WHEN THE LOAD IS MATCHED TO LINE, THE NET EFFECT IS THAT THE LOAD IS TRANSFERRED TO THE SOURCE TERMINAL AS IF THE TRANSMISSION LINE IS ABSENT.

~~LOAD IS A SHORT~~

$(Z_L = 0)$ SHORT CIRCUIT: $\quad \Gamma = -1 = 1 \cdot e^{j\pi}$

FROM (3), $\quad V_s(z) = V_0^+ \left(e^{-j\beta z} - e^{j\beta z} \right)$

$$= -2j\, V_0^+ \sin(\beta z)$$

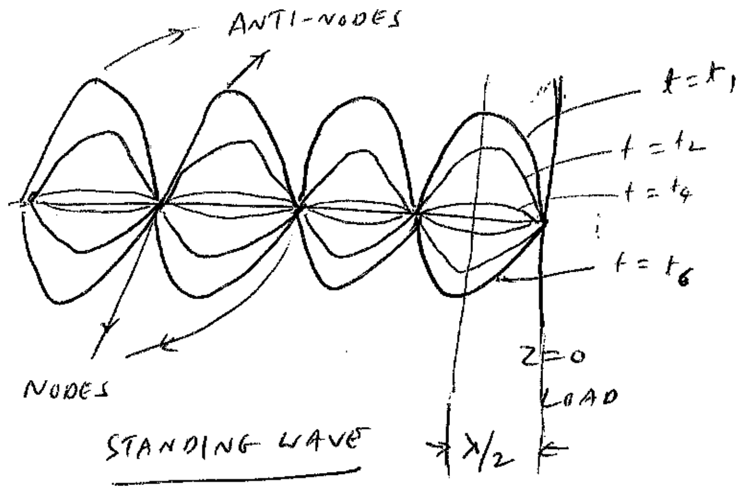
$$= -2 e^{j\pi/2}\, V_0^+ \sin(\beta z)$$

$v(z,t) = Re\{V_s(z) e^{j\omega t}\}$ — ASSUME IT TO BE REAL.

$= 2V_0^+ \sin(\beta z)\, Re\{ e^{j(\omega t + \pi/2)} \}$

$= -2V_0^+ \sin(\beta z) \cos(\omega t + \pi/2)$

$$\boxed{v(z,t) = 2V_0^+ \sin(\beta z) \sin(\omega t)} \longrightarrow 5$$



STANDING WAVE

AT $z=0$, $\sin(\beta z) = 0$. SO, AT LOAD, THE VOLTAGE IS ZERO ~~AT ALL~~ AT ANY TIME. IN FACT, IF

$$\beta z = 2m\pi, \quad m = 0, \pm 1, \pm 2, \ldots$$

or

$$z = \frac{2m\pi}{\beta},$$

$v(z,t)$ IS ZERO AT THESE LOCATIONS & THES ARE KNOWN AS NODES.

IF $\beta z = 2m\pi \pm \pi/2$, $m = 0, \pm 1, \pm 2, \ldots$

or

$$z = \frac{2m\pi \pm \pi/2}{\beta},$$

$$\sin(\beta z) = \pm 1 .$$

AT THESE LOCATIONS, THE VOLTAGE BECOMES MAXIMUM ($= 2V_0^+$) AT A CERTAIN TIME, THEN IT STARTS DECREASING & BECOMES MINIMUM ($-2V_0^+$). THESE POSITIONS ARE ANTI-NODES.

THE DISTANCE BETWEEN A NODE & AN ANTI-NODE IS $\lambda/4$. THIS CAN BE SHOWN AS FOLLOWS: AT $z=0$, WE HAVE A NODE. THE NEAREST ANTI-NODE OCCURS AT

$$\beta z = -\pi/2$$

or

$$\frac{2\pi}{\lambda} z = -\pi/2 \Rightarrow z = \frac{-\lambda}{4}$$

(NOTE: THE LINE IS IN $-z$ DIRECTION)

HERE, λ IS THE WAVELENGTH OF THE VOLTAGE WAVE,

$$\lambda = v/f$$

$f \to$ frequency

YOU CAN EASILY VERIFY THAT THE DISTANCE BETWEEN NODES (or ANTI-NODES) IS $\lambda/2$.

PARTIAL REFLECTION,

FROM EQS. (3) & (4), WE HAVE

$$V_s(z) = V_0^+ \left[e^{-j\beta z} + |\Gamma| e^{j(\beta z + \phi)} \right]$$

$$= V_0^+ e^{j\phi/2} \left[e^{-j\overbrace{(\beta z + \phi/2)}^{\theta}} + |\Gamma| \cdot e^{j\overbrace{(\beta z + \phi/2)}^{\theta}} \right] \quad \text{— (STEP 2)}$$

$$= V_0^+ e^{j\phi/2} \left[(|\Gamma| + (1-|\Gamma|)) e^{-j\theta} + |\Gamma| e^{j\theta} \right] \rightarrow \text{(STEP 3)}$$

$$= V_0^+ e^{j\phi/2} \left[2|\Gamma| \cos\theta + (1-|\Gamma|) e^{-j(\beta z + \phi/2)} \right] \rightarrow \text{(STEP 4)}$$

$$= V_0^+ \left[e^{j\phi/2} 2|\Gamma| \cdot \cos\theta + (1-|\Gamma|) e^{-j\beta z} \right] \quad \text{— (STEP 5)}$$

IN STEP 3, THE INCIDENT WAVE IS SPLIT INTO TWO PARTS, $|\Gamma|$ & $(1-|\Gamma|)$. THE FRACTION $|\Gamma| V_0^+$ OF THE INCIDENT WAVE INTERFERES WITH THE REFLECTED WAVE TO FORM A STANDING WAVE. THE REST OF THE INCIDENT WAVE REMAINS AS A TRAVELLING ~~STANDING~~ WAVE. THIS CAN BE SEEN BY ~~TAKING~~ CONVERTING ~~TO~~ IT INTO REAL INSTANTANEOUS FORM:

$$v(z,t) = \cancel{\mathrm{Re}\{V_s(z)\}} = \mathrm{Re}\{V_s(z) e^{j\omega t}\}$$

$$= V_0^+ \mathrm{Re}\left\{ 2|\Gamma| \cos\theta \cdot e^{j(\omega t + \phi/2)} + (1-|\Gamma|) e^{j(\omega t - \beta z)} \right\}$$

$$= \underbrace{2|\Gamma| \cdot V_0^+ \cos(\beta z + \phi/2) \cdot \cos(\omega t + \phi/2)}_{\text{STANDING WAVE}} + \underbrace{V_0^+ (1-|\Gamma|) \cos(\omega t - \beta z)}_{\text{TRAVELLING WAVE.}}$$

VOLTAGE MAXIMUM & MINIMUM:

$$V_s(z) = V_0^+\left[e^{-j\beta z} + |\Gamma|\, e^{j(\beta z+\varphi)}\right]$$

$$= V_0^+ e^{-j\beta z}\left[1 + |\Gamma|\, e^{j(2\beta z+\varphi)}\right] \longrightarrow (6)$$

THE MAXIMUM VOLTAGE ~~OCCUR~~ IS OBTAINED WHEN THE FIRST & SECOND TERMS ADD IN PHASE, i.e.

$$2\beta z + \varphi = 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

or

$$z_{max} = \frac{2n\pi - \varphi}{2\beta} \longrightarrow (7a)$$

AT THESE LOCATIONS, THE VOLTAGE IS

$$V_s(z) = V_0^+ e^{-j\beta z}(1+|\Gamma|)$$

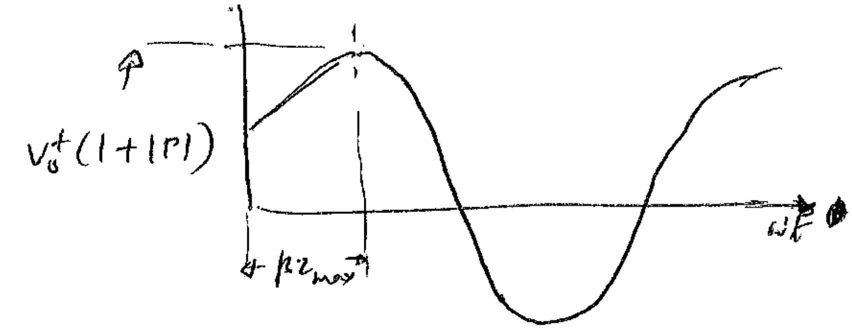
$$v(z,t) = \operatorname{Re}\{V_s(z)\, e^{j\omega t}\}$$

$$= V_0^+[1+|\Gamma|]\cos(\omega t - \beta z) \longrightarrow (7)$$

FROM (7), WE SEE THAT THE AMPLITUDE OF THE VOLTAGE WAVE IS $V_0^+(1+|\Gamma|)$. THE INSTANTANEOUS VOLTAGE AT THESE LOCATIONS IS $V_0^+(1+|\Gamma|$ AT A CERTAIN TIME t_1; IT WILL BECOME 0 AT ~~SOME~~ TIME t_2 & BECOMES $-V_0^+(1+\Gamma)$ AT TIME t_3.

THE RMS VOLTAGE AT THESE LOCATIONS IS $V_0^+[1+|\Gamma|]/\sqrt{2}$.

(a) $z = z_{max}$



THE MINIMUM VOLTAGE IS OBTAINED WHEN THE FIRST & SECOND TERMS ADD UP DESTRUCTIVELY, i.e.

$$2\beta z + \varphi = 2n\pi + \pi \quad . \quad n = 0, \pm 1, \pm 2$$

$$e^{j(2\beta z + \varphi)} = e^{j(2n\pi + \pi)} = -1$$

$$z_{min} = \frac{(2n+1)\pi - \varphi}{2\beta}$$

&

AT THESE LOCATIONS, THE VOLTAGE IS

$$V_s(z) = V_0^+ e^{-j\beta z}(1 - |\Gamma|)$$

$$v(z,t) = Re\{V_s(z) e^{j\omega t}\}$$

$$= V_0(1 - |\Gamma|)\cos(\omega t - \beta z) \longrightarrow (8)$$

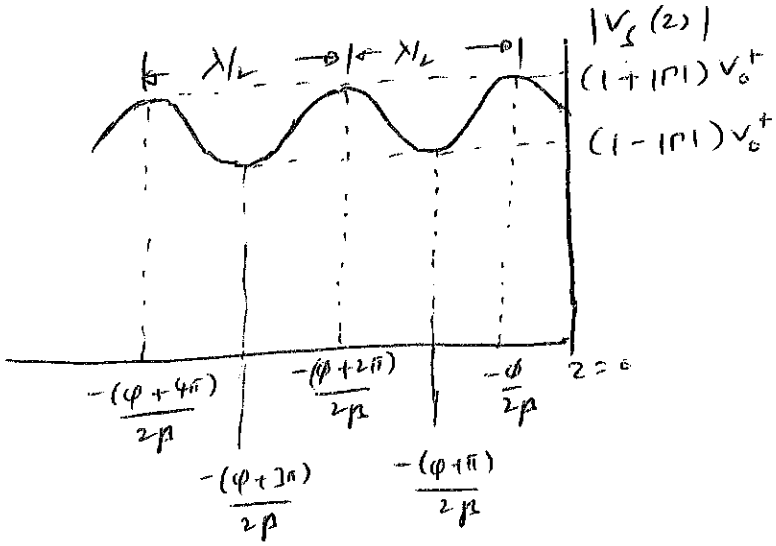
FROM (8), WE SEE THAT THE AMPLITUDE (i.e. PEAK) OF THE VOLTAGE WAVE IS $V_0^+(1-|\Gamma|)$. THE RMS VOLTAGE IS $V_0^+(1-|\Gamma|)/\sqrt{2}$.

FROM EQ. (6), WE FIND

$$|V_s| = |V_0^+| \, \underbrace{|e^{-j\beta z}|}_{=1} \, \left| \left(1 + |\Gamma| e^{j(2\beta z + \varphi)} \right) \right|$$

$$|V_s|_{max} = V_0^+ (1 + |\Gamma|) \rightarrow (9)$$

$$|V_s|_{min} = V_0^+ (1 - |\Gamma|) \rightarrow (10)$$



THE FIRST MAXIMUM OCCURS AT

$$z_{max,1} = \frac{2m\pi - \varphi}{2\beta} \quad \text{when } m = 0$$

$$= \frac{-\varphi}{2\beta} \rightarrow (11)$$

THE SECOND MAXIMUM OCCURS AT $(m = -1)$

$$z_{max,2} = \frac{-2\pi - \varphi}{2\beta} \rightarrow (12)$$

NOTE, IF m IS +VE, $z > 0$. HOWEVER, THE LINE IS LOCATED IN THE −VE z AXIS.

DISTANCE BETWEEN SUCCESSIVE VOLTAGE MAXIMA IS

$$\left| z_{max,1} - z_{max,2} \right| = \frac{2\pi}{2\beta} = \lambda/2 \quad \left(\because \beta = \frac{2\pi}{\lambda} \right) \longrightarrow (13)$$

THE VOLTAGE STANDING WAVE RATIO (VSWR) IS DEFINED AS

$$VSWR = S = \frac{|V_s|_{,max}}{|V_s|_{,min}} = \frac{V_0^+ (1 + |\Gamma|)}{V_0^- (1 - |\Gamma|)}$$

$$\boxed{S = \frac{1 + |\Gamma|}{1 - |\Gamma|}} \longrightarrow (14)$$

~~FOR A SHORT CIRCUIT, $|\Gamma| =$~~

FOR A SHOR CIRCUIT or OPEN CIRCUIT, $|\Gamma| = 1$ & $S = \infty$. FOR A MATCHED LOAD, $\Gamma = 0$ & $S = 1$.

IN PRACTICE, VOLTAGE MAXIMUM & MINIMUM ARE MEASURED & S IS CALCULATED. ~~USING EQ. (14)~~ USING EQ. (14), $|\Gamma|$ CAN BE CALCULATED. THE PHASE OF Γ IS THEN FOUND BY MEASURING THE LOCATION OF THE FIRST MAXIMUM or MINIMUM (SEE EQ. 11). ONCE Γ IS KNOWN, LOAD IMPEDANCE CAN BE FOUND IF Z_0 IS ~~KNOWN~~ KNOWN.

EXAMPLE: THE VSWR IN A TRANSMISSION LINE IS 4; THE SPACING BETWEEN SUCCESSIVE VOLTAGE MAXIMA IS 20 CM & THE FIRST MAXIMUM AT A DISTANCE OF 8 CM IN FRONT OF THE LOAD. CALCULATE (i) REFLECTION COEFFICIENT, Γ (ii) Z_0, IF THE LOAD IMPEDANCE IS ~~30.68∠2.386~~ $30.64\angle 0.8339$

THE SPACING BETWEEN SUCCESSIVE VOLTAGE MAXIMA $= \lambda/2 = 20$ cm

$$\therefore \quad \lambda = 40 \text{ cm}.$$

$$\beta = \frac{2\pi}{\lambda} = 15.7 \text{ m}^{-1}$$

THE FIRST MAXIMUM IS LOCATED AT

$$-\frac{\phi}{2\beta} = -8 \text{ cm}$$

$$\therefore \quad \phi = 2 \times 15.7 \times 8 \times 10^{-2} \text{ rad}$$

$$= 2.512 \text{ rad}.$$

$$\text{VSWR} = S = \frac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow S(1-|\Gamma|) = 1+|\Gamma|$$

$$S-1 = |\Gamma|(S+1)$$

$$\therefore \quad \cancel{|\Gamma| = \frac{S+1}{S-1} = \frac{4+1}{4+}}$$

(i) $\therefore |\Gamma| = \dfrac{S-1}{S+1} = \dfrac{3}{5} = 0.6$

$$\Gamma = |\Gamma| e^{j\varphi} = 0.6 \angle 2.512$$

(ii)
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow \Gamma(Z_L + Z_0) = Z_L - Z_0$$

$$(\Gamma + 1) Z_0 = Z_L (1 - \Gamma)$$

$$Z_0 = Z_L \frac{(1-\Gamma)}{1+\Gamma}$$

$$\cancel{= 30.64}$$

$$\therefore Z_0 = 30.64 \angle 0.8339 \frac{(1 - 0.6 \angle 2.512)}{1 + 0.6 \angle 2.512}$$

$$= 75 \Omega$$