Useful information

Speed of light in vacuum   c = 2.99793×10^{8} m/s

$\varepsilon_0 =$ free space permeability = $8.854 \times 10^{-12} H/m$

$\mu_0 =$ free space permeability = $4\pi \times 10^{-7} F/m$

Faraday's Law:

Emf = - $\frac{d\psi}{dt}$ where ψ = magnetic flux.

$\psi = \int_S \vec{B}.d\vec{S}$ Emf = $\oint_L \vec{E}.d\vec{L}$

Faraday's law in point form:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\vec{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\vec{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\vec{z}$$

Ampere's Law (Magnetostatics):

Integral form:

$$\oint_L \vec{H}.d\vec{L} = I$$

Differential form:

$$\nabla \times \vec{H} = \vec{J}$$

$$I = \int_s \vec{J}.\vec{dS}$$

Ampere-Maxwell Law

Integral form:

$$\oint_L \vec{H}.d\vec{L} = I + I_d$$

Differential form:

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

I_d =displacement current

Maxwell's equations:

Differential form:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla . \vec{D} = \rho$$

$$\nabla . \vec{B} = 0$$

Integral form:

$$\oint_L \vec{E}.d\vec{L} = -\int_s \frac{\partial \vec{B}}{\partial t}.d\vec{S}$$

$$\oint_L \vec{H}.d\vec{L} = I + I_d$$

$$\oint_s \vec{D}.\vec{ds} = q$$

$$\oint_s \vec{B}.\vec{ds} = 0$$

$\vec{B} = \mu\vec{H}$ $\vec{D} = \varepsilon\vec{E}$ $\vec{J} = \sigma\vec{E}$

Permittivity, $\varepsilon = \varepsilon_0 \varepsilon_r$

Permeability, $\mu = \mu_0 \mu_r$

Wave equation:

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

Speed of EM wave in a medium, $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$$v = \frac{c}{n}, \quad n = \sqrt{\varepsilon_r \mu_r}$$

Phasor Form:

$$E_x = \mathrm{Re}\{E_{xs} \exp(j\omega t)\}$$

$$H_y = \mathrm{Re}\{H_{ys} \exp(j\omega t)\}$$

E_{xs} and H_{ys} are phasors corresponding to E_x and H_y, respectively.

$$\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs}$$

$$k^2 = -j\omega\mu(\sigma + j\omega\varepsilon)E_{xs}$$

Propagation in dielectrics and conductors:

The plane wave in a perfect dielectric medium is given by

$$E_x = E_{x0} \cos(\omega t - \beta z)$$

$$H_y = H_{y0} \cos(\omega t - \beta z)$$

$$\frac{E_{x0}}{H_{y0}} = \eta = \left(\frac{\mu}{\varepsilon}\right)^{1/2}$$

$$\frac{\omega}{\beta} = v = \text{speed of EM wave}$$

$$\beta = \frac{2\pi}{\lambda} = \text{wave number}$$

$$v = \lambda f$$

$$\lambda_m = \lambda_0 / n$$

λ_0 is the wavelength in free space, λ_m = wavelength in a medium

The plane wave in a lossy dielectric medium is

$$E_x = E_{x0} \exp(-\alpha z) \cos(\omega t - \beta z)$$

$$H_y = H_{y0} \exp(-\alpha z) \cos(\omega t - \beta z)$$

$$\alpha = \mathrm{Re}(jk) = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2}$$

$$\beta = \mathrm{Im}(jk) = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right)^{1/2}$$

$$\frac{E_{x0}}{H_{y0}} = \eta = \left(\frac{\mu}{\varepsilon' - j\varepsilon''} \right)^{1/2}$$

The plane wave in a good conductor is

$$E_x = E_{x0} \exp(-\alpha z) \cos(\omega t - \beta z)$$

$$H_y = H_{y0} \exp(-\alpha z) \cos(\omega t - \beta z)$$

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

Skin depth

$$\delta = 1/\alpha$$

Loss tangent

$$\tan \theta = \frac{\sigma}{\omega \varepsilon'}$$

Transmission Lines:

$$\frac{d^2 V_s}{dz^2} = \gamma^2 V_s$$

$$\gamma^2 = ZY$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

For the loss less case, speed v and characteristic impedance Z_0 are

$$v = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

In general,

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

Power loss (dB) = 8.69 αz

Reflection coefficient and VSWR

$$\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad s = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Wave impedance,

$$Z_w(z) = Z_0 \frac{Z_L - jZ_0 \tan(\beta z)}{Z_0 - jZ_L \tan(\beta z)}$$

Plane wave reflection

Reflection coefficient

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Transmission coefficient

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

The wave impedance

$$\eta_w(z) = \eta_2 \frac{\eta_3 - j\eta_2 \tan(\beta_2 z)}{\eta_2 - j\eta_3 \tan(\beta_2 z)}$$

Trigonometric identities

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\exp(jA) = \cos A + j \sin A$$

THE END