Electrical Engineering 3TR4 Final Examination

Duration of Exam: 3 Hours

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This paper includes 3 pages and 6 questions **(including the fact sheet at the end)**. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted.**

(b) Attempt all questions.

(c) You must show your work to get full marks.

(d) All major questions are worth 10 marks.

(e) You may find the fact sheet at the end of this questionnaire to be useful.

1. Question 1.

a. A message signal $m(t)$ is given as $m(t) = \cos(2\pi f_m t)$ and the carrier is given as $A_c \cos(2\pi f_c t)$. Sketch the corresponding modulated wave $s(t)$ corresponding to

i) AM, for the case of 100% modulation

ii) double sideband suppressed carrier modulation. (4 marks)

Draw your sketches so that one is aligned directly above the other. Use identical scales for the time axes for both parts **i)** and **ii)**. Show values for all relevant quantities.

b. For each case write down the mathematical equation that describes the modulated wave $s(t)$. (2 marks)

c. Sketch the spectra corresponding to the modulated wave for both parts i) and ii) above. Again, show all relevant values. (4 marks)

2. **Question 2.**

   a. Draw a block diagram of a superheterodyne radio receiver, and briefly describe its operation. What is the advantage of the superheterodyne configuration over a more conventional approach? (8 marks)

   b. In commercial AM broadcasting, the receiver IF (intermediate frequency) filter is centered on 455 KHz. What is the range of local oscillator frequencies required to tune in radio frequency channels from 555 KHz to 1.555 MHz? (2 marks)

3. **Question 3.**
   We wish to design a digitally modulated communications system that will transmit 2 Mbits/sec over a channel with a bandwidth of 1.5 MHz.

   a. How many bits/symbol would you use and why? (4 marks)

   b. Draw the spectrum of the received signal due to a single isolated pulse at the transmitter, both before and after the sampler. Show all relevant values. (4 marks)

   c. What is the value of the parameter α? (2 marks)

4. **Question 4.**
   A signal component $s(t) = A\cos(2\pi f_o t)$ is added to a zero–mean white noise random process $w(t)$ with power spectral density level of $N_o/2$, and fed into a first–order low–pass filter whose transfer function is given as

$$H(f) = \frac{1}{1 + j2\pi f RC}.$$

   Calculate the signal–to–noise ratio (SNR) at the output of the filter. Assume $f_o = 1/(2\pi RC)$ and simplify results as far as possible.

5. **Question 5.**
   We have a digital communications system transmitting 1 Mbits/sec using 1 bit per symbol. The net loss from the transmitter to the receiver is 60dB and the value of N_o is 10^{-12} Watts/Hz. What is the required value of A at the transmitter for a bit error rate (i.e., probability of a bit error) of 10^{-6}? *Hint:* $E_b = A^2 T$ and the probability of a bit error is given by $P(\epsilon) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$. Values of the erfc$(\cdot)$ function are given at the end of this exam. If the value you need is not shown, use the closest one.

6. **Question 6.**
   A sample $x(t)$ from a random process is given by

$$x(t) = A\cos(2\pi f_o t) + \sigma w(t) \tag{1}$$

   where $w(t)$ is a zero–mean white noise process with a power spectral density value of $\frac{N_o}{2}$, and A, f_o and σ are constants.

   a. Find the autocorrelation function $R_x(\tau)$.

   b. Find the corresponding power spectral density (PSD) $S_x(f)$.

1 Table of Values of the Complementary Error Function erfc(·)

x	erfc(x)
2.0	4.7×10^{-3}
2.5	4.1×10^{-4}
3.0	2.2×10^{-5}
3.5	7.43×10^{-7}
4.0	1.54×10^{-8}

Fourier Transform Pairs

Time Function	**Fourier Transform**
$\text{rect}\left(\frac{t}{T}\right)$	$\text{Tsinc}(\text{fT})$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]$

Trigonometric Identities

$$\cos(\theta) = \frac{1}{2}\big[\exp(j\theta) + \exp(-j\theta)\big]$$
$$\sin(\theta) = \frac{1}{2j}\big[\exp(j\theta) - \exp(-j\theta)\big]$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$
$$\cos^2(\theta) = \frac{1}{2}\big[1 + \cos(2\theta)\big]$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\big[\cos(\alpha - \beta) - \cos(\alpha + \beta)$$
$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\big[\cos(\alpha - \beta) + \cos(\alpha + \beta)$$
$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\big[\sin(\alpha - \beta) + \sin(\alpha + \beta)$$

The End!