Electrical Engineering EE3TR4

Midterm test: 1.5 Hours
Feb. 25, 2019

Instructors:
Dr. S. Kumar
Dr. J. Reilly

This examination paper includes 4 pages and 3 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) **If you want your paper to be considered for re-marking, then answer in pen and do not use white-out.**

(b) Any version of the Casio FX991 are the only calculators approved for this exam. **No other aids are permitted**.

(c) There are 3 questions. Attempt all three.

(d) You must show your work for full marks.

(e) **Make sure you read the entire paper over in its entirety before you start!**

1. In commercial AM radio systems, the spectra for all stations (i.e. channels) are arranged side–by–side (contiguously) in the frequency domain. In AM systems, the bandwidth of each channel is 10 KHz. In a typical environment, there are many channels available, each with its own unique centre frequency.

a) Explain fully an efficient technique for selecting the channel of interest. (7 marks)

b) Explain the advantages of this system. (3 marks)

2. Consider the block diagram shown in Fig. 2. The input waveform $x(t)$ is a square wave with 50% duty cycle, amplitude = 1V, at a frequency of 1 KHz. The left-hand block $H_1(f)$ is an ideal low-pass filter with a

Continued on Page 2

cutoff frequency of 2 KHz, and whose phase response is zero. The cutoff frequency f_o in the right-hand filter $H_2(f)$ is 1 KHz. Provide a closed-form expression (i.e. a formula) describing the output waveform $y(t)$ showing all values. *Hint:* The crib sheet at the end will be helpful.

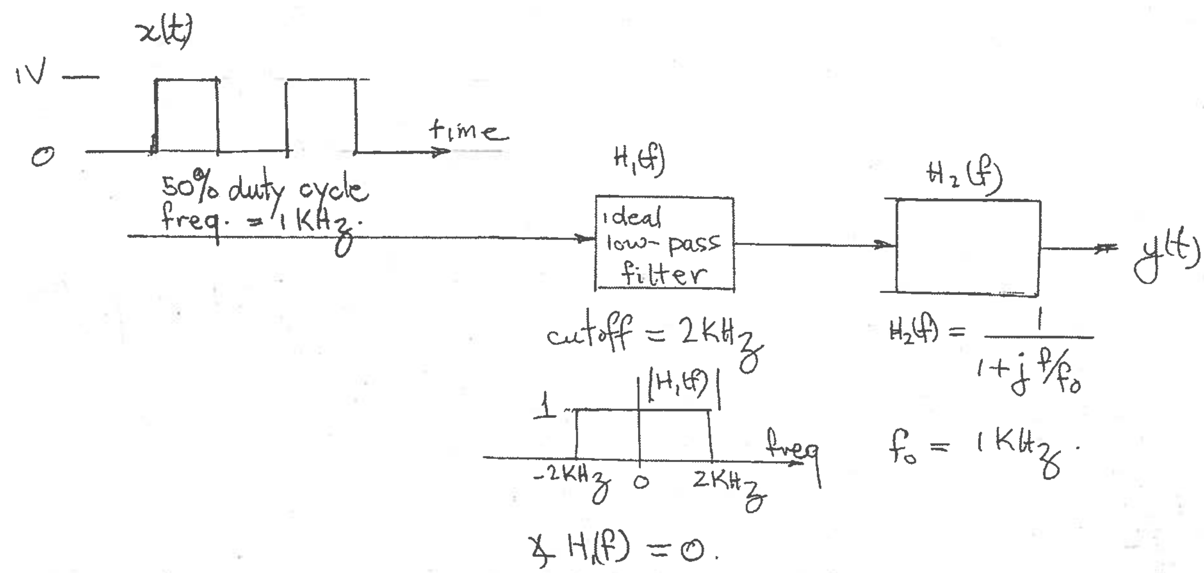


Figure 2: Block Diagram for Q2.

Continued on Page 3

3. The triangular waveform shown below is multiplied by a square wave of frequency 100 Hz with 50% duty cycle and amplitude 1, to produce the wave $s(t)$.

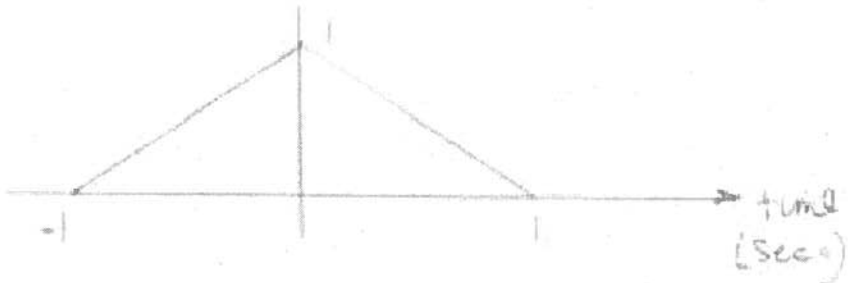


Figure 3: Waveform for Q3.

a) Sketch the spectrum $S(f)$ of $s(t)$ and explain how you got it. (8 marks)

b) Explain how to convert $s(t)$ into a different type of wave, which has effectively been multiplied by a sinusoid (i.e., instead of a square wave) also at frequency 100 Hz. (2 marks)

Continued on Page 4

Fourier Transform Pairs

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	Tsinc(fT)
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$

For a periodic square wave of amplitude A, the Fourier coefficients c_n are given as $c_n = A\delta \ \text{sinc}(\delta n)$, where δ is the duty cycle. Recall $\text{sinc}(0) = 1, \text{sinc}(1/2) = 0.6366, \text{sinc}(1) = 0, \text{sinc}(1.5) = -0.2033, \ldots$

Trigonometric Identities

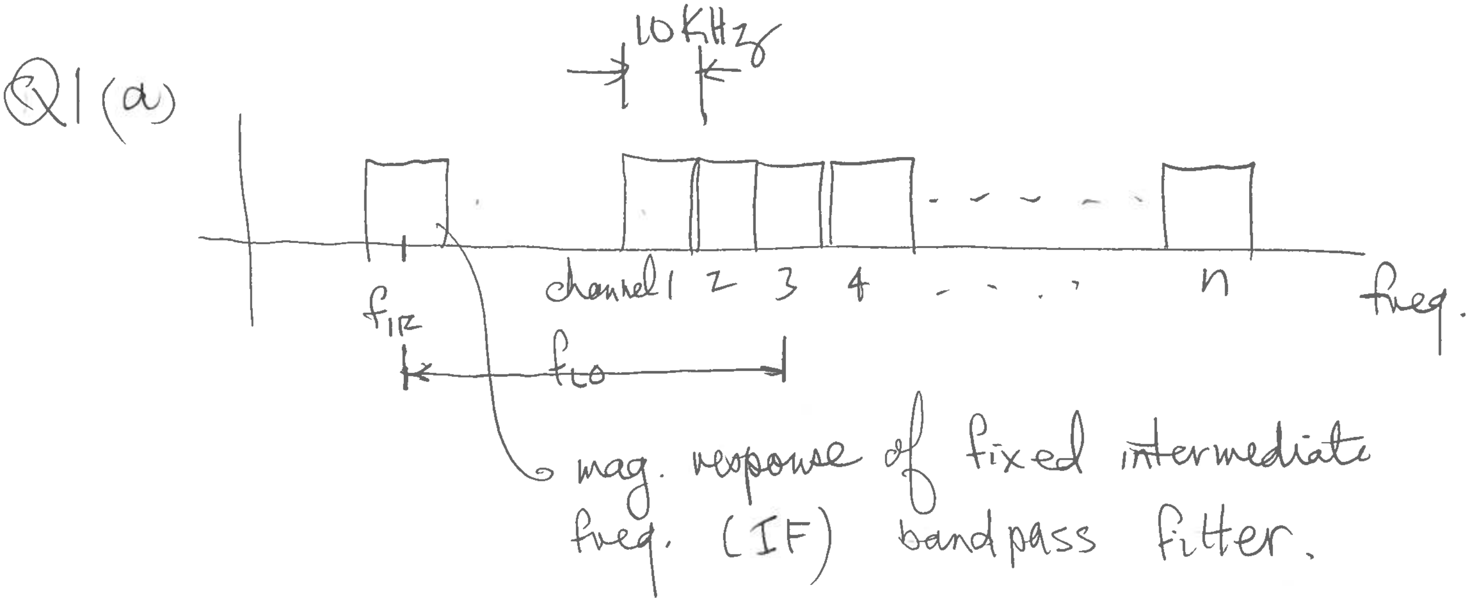
$$\cos(\theta) = \frac{1}{2}\left[\exp(j\theta) + \exp(-j\theta)\right]$$
$$\sin(\theta) = \frac{1}{2j}\left[\exp(j\theta) - \exp(-j\theta)\right]$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$
$$\cos^2(\theta) = \frac{1}{2}\left[1 + \cos(2\theta)\right]$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right)$$
$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right)$$
$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

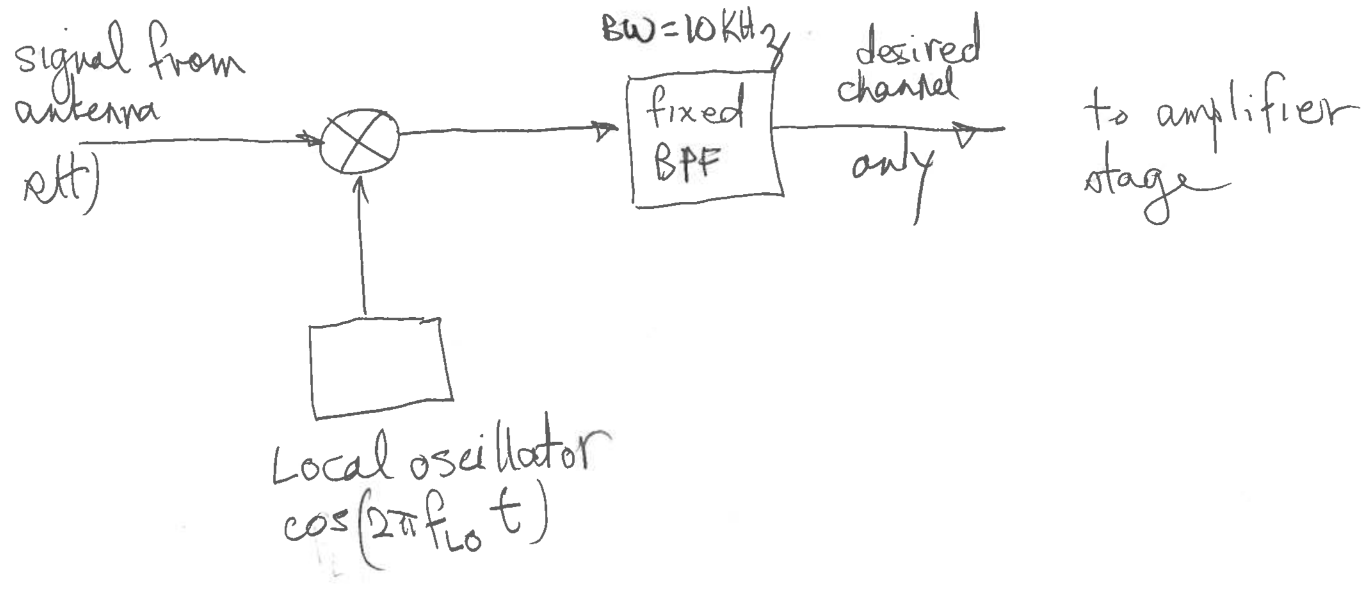
The End.

EE 3TR4 Midterm Solutions 2019

Q1 (a)



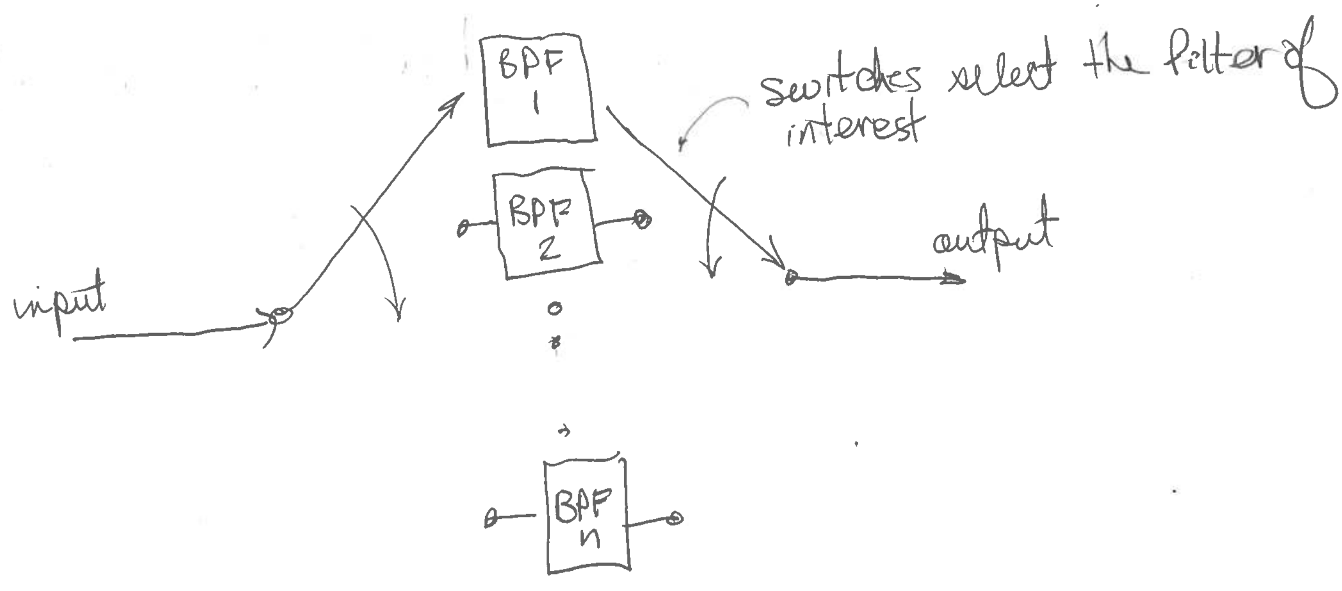
The perferred method is the super-heterodyne system. The block diagram configuration is as follows



The desired channel is selected by setting f_{LO} to be $f_i - f_{IF}$, where f_i is the centre freq.

of the channel of interest. For example, in the top diagram last page, ch3 is selected. In that case, f_{LO} is the difference between f_3 & f_{IF}, as shown. When s(t) is mult'd by f_{LO}, the spectrum S(f) moves up & down by f_{LO}. The image that is shifted down passes thru the BPF & the one shifted up "disappears".

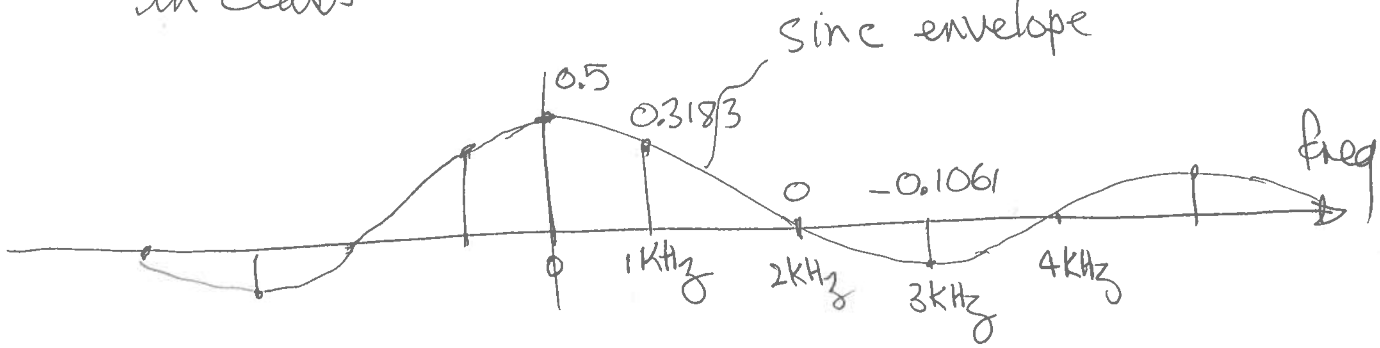
b) The reason why this method is effective is that it is difficult to build a bandpass filter whose centre freq. can vary, or a bank of filters from which only one such filter is switched into use, as below



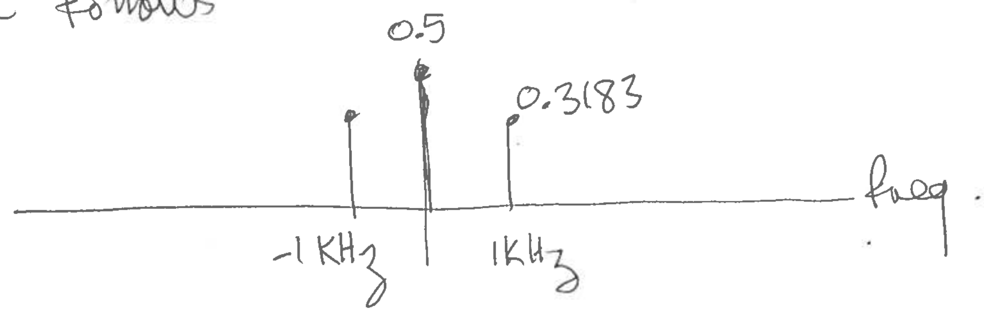
switches change to select the desired channel. This configuration is very expensive.

On the other hand, it is very straightforward (3).
to build sharp, fixed bandpass filter and
a variable-freq. oscillator.

Q2 The input square wave has the following
spectrum, as in the labs and as discussed
in class

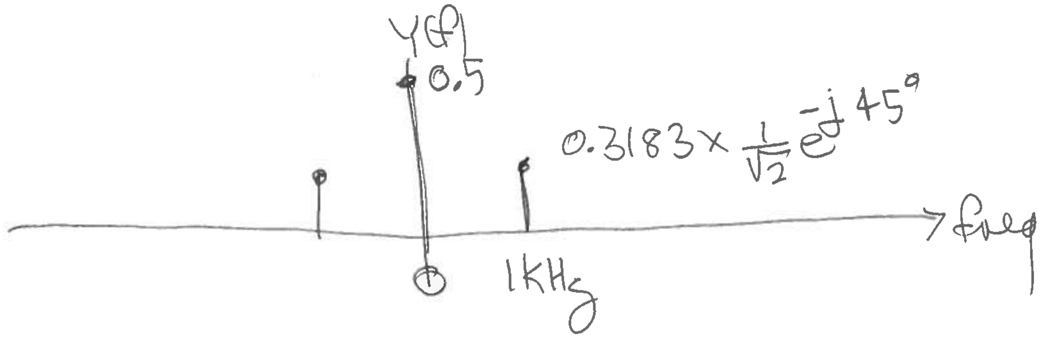


Therefore the output spectrum from $H_1(f)$ is
as follows



At $f = 1KHz$, $H_2(f) = \frac{1}{1+j} = \frac{1}{\sqrt{2}} e^{-j45^\circ}$.

$\therefore$ Y(f) has the form



Now we transform Y(f) into the time domain. The spike at 0 Hz is a DC signal with value 0.5V. The spikes at ± 1KHz transform into a cos(.) with amplitude

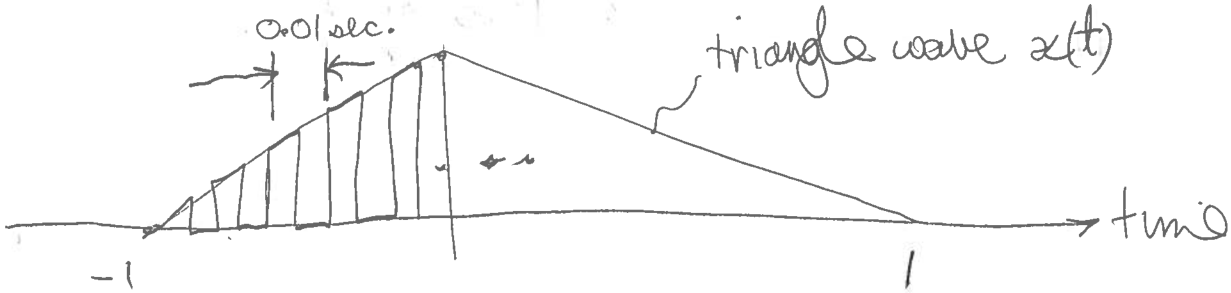
$0.3183 \times \frac{1}{\sqrt{2}} \times 2$ and a phase of $-45°$

$$\therefore \quad y(t) = 0.5 + 0.4501 \cos(2\pi \times 1 \times 10^{3} t - 45°)$$

Q3

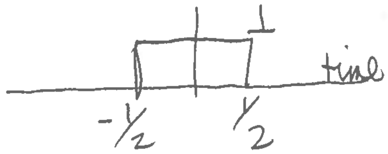
Assuming that the periodic square wave has a DC component ie

then $s(t)$ has the form



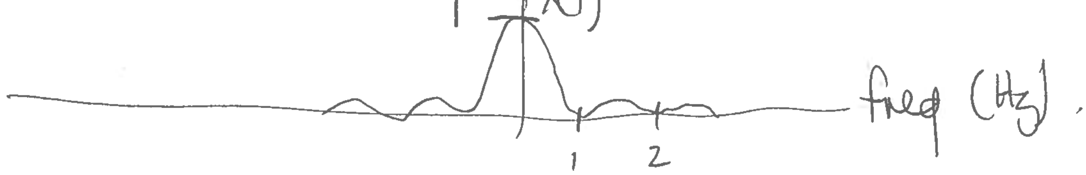
$s(t)$ is formed in the time domain by multiplying $sq(t)$ with the triangle wave $x(t)$. Therefore the spectrum $S(f)$ is the convolution in freq. of $X(f)$ with $Sq(f)$.

$x(t)$ is the convolution in time of the rectangle wave



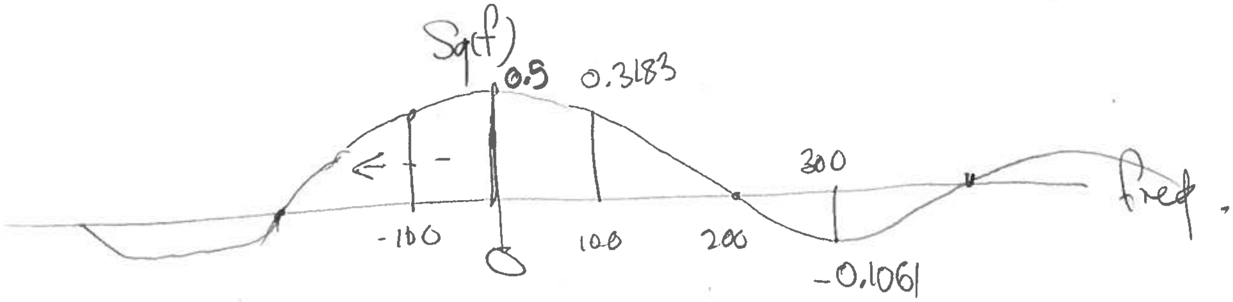
$\rightleftharpoons$ sinc f

with itself. $\therefore$ $X(f) = \text{sinc}^2(f)$.



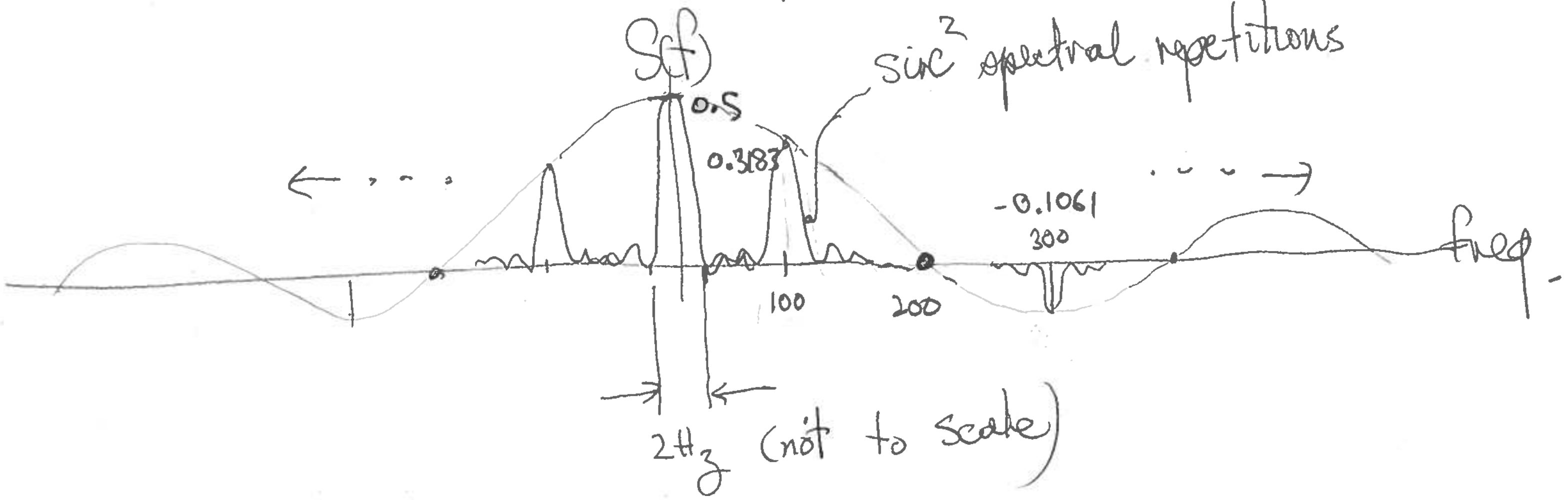
As is evident from the lab, the spectrum $S_q(f)$ is discrete, with components at multiples of $f_0 = 100\,Hz$. The weighting of each component C_n is given as

$$C_n = \delta A \operatorname{sinc} n\delta \qquad \delta = \text{duty factor} = 1/2.$$

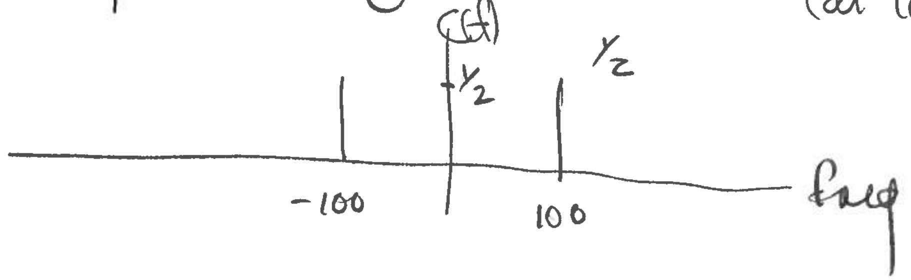


$\therefore$ the spectrum $S(f) = S_q(f) \circledast X(f)$.

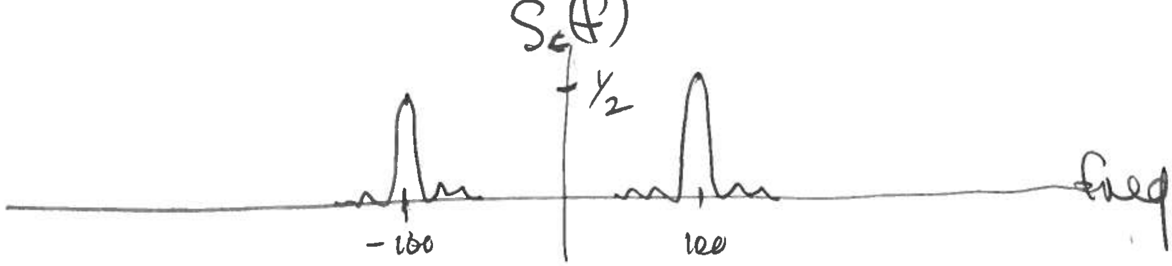
This will have the appearance of the following –



b) The spectrum of a sinusoid c(t) is simply (at 100 Hz)



∴ $S_c(f)$ when x(t) is multiplied by a sinusoid (instead of a square wave) will look as follows



Thus $S_c(f)$ may be obtained from S(f) by low-pass filtering (with a cutoff eg. at 200 Hz)

⊛ Note: The presence of the DC component in this situation is ambiguous. We will accept answers for any reasonable configuration of the DC component.