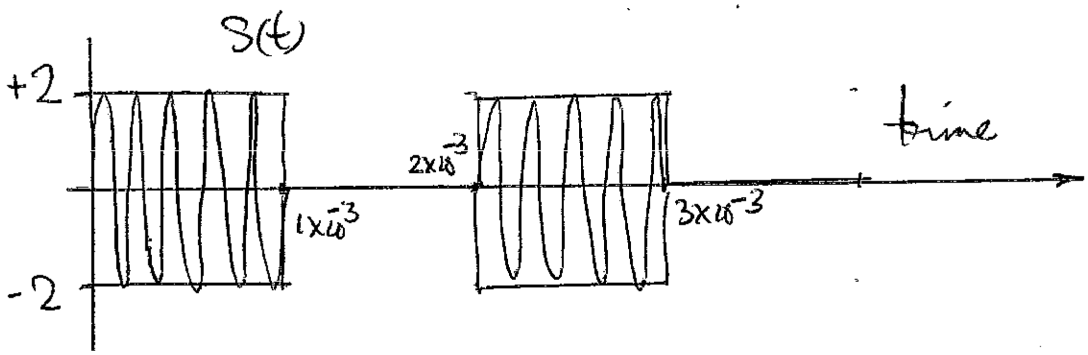
EE3TR4 Midterm Solutions 2013 J Reilly

1. (a) $s(t) = A_c\left[1 + k_a\, m(t)\right]\cos 2\pi f_c t$

for 100% modulation, $\max|k_a\, m(t)| = 1$.

$\therefore k_a = 2$. We are given $A_c = 1$.



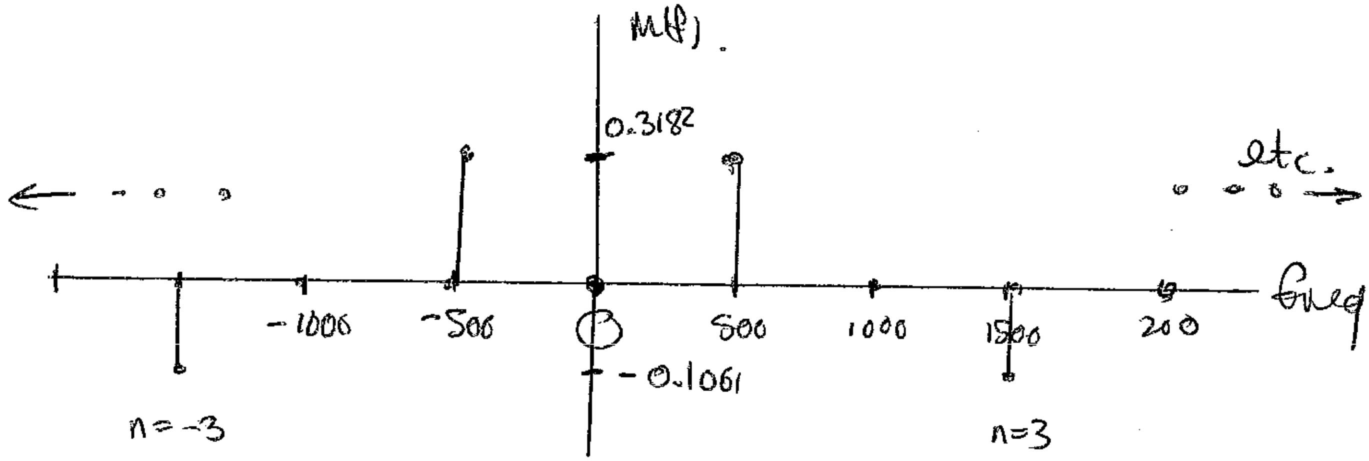
b. The spectrum of the wave m(t) is the same as that of the square wave discussed in class, except that the DC component is absent.

The period T_0 is 2 msec $\Rightarrow f_0 = 500$ Hz.
The wave is periodic, with 50% duty cycle, which implies the Fourier coefficients c_n are spaced every f_0 Hz with amplitude according to

$$c_n = \frac{A}{2} \operatorname{sinc} n/2 \quad , n = \pm 1, \pm 2, \ldots$$

For the msg signal here, the amplitude $A = 1$, but no DC component is present.

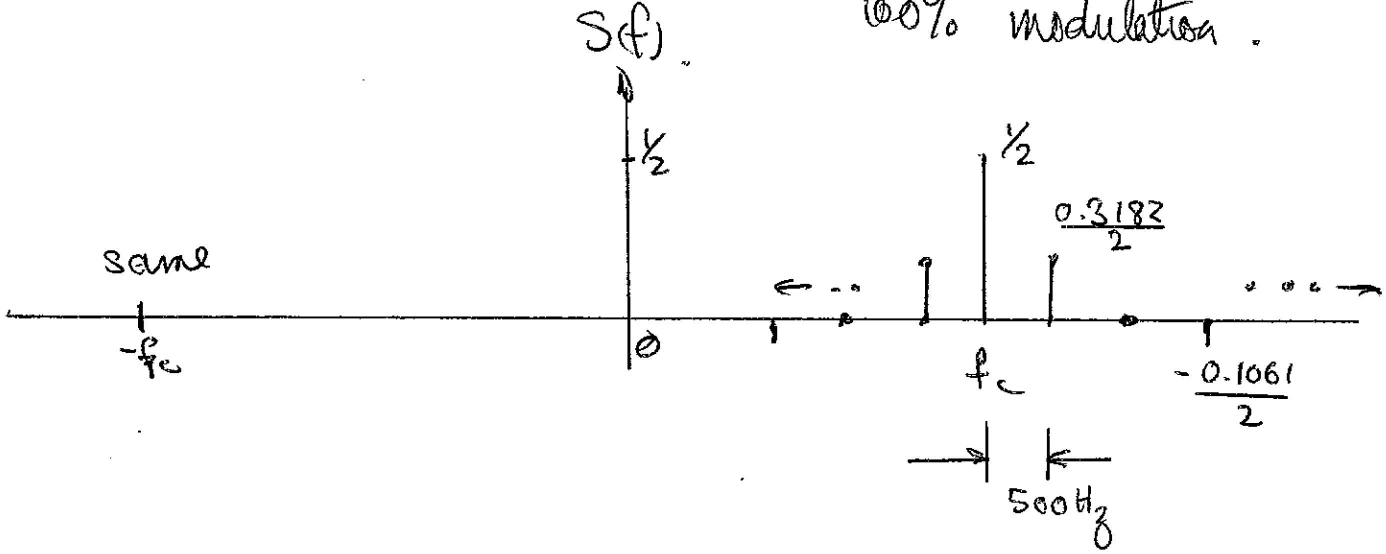
Therefore the spectrum $M(f)$ of $m(t)$ is given as



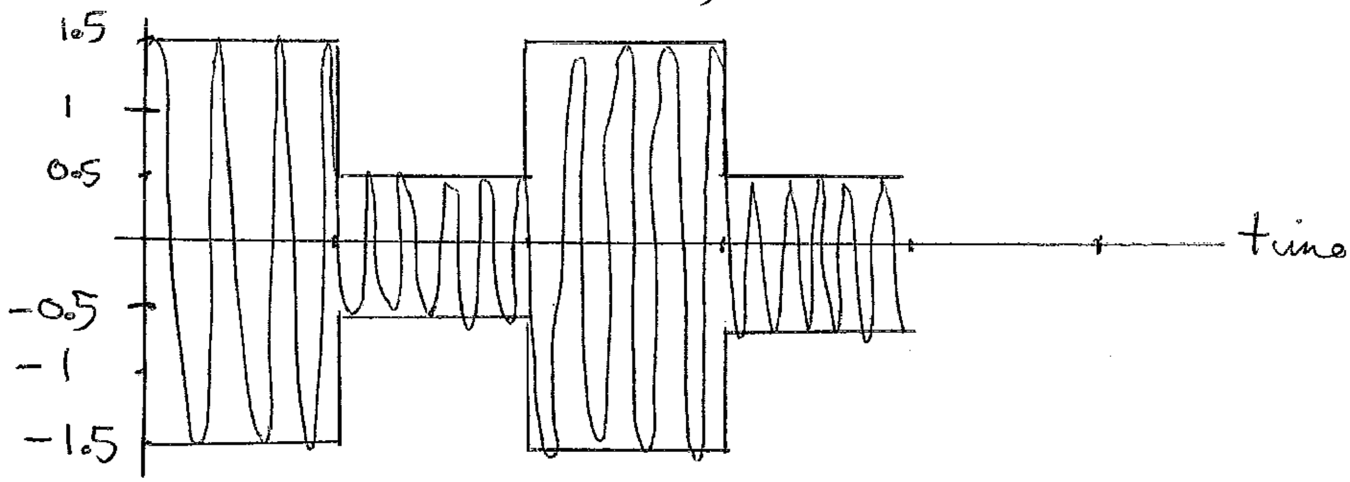
The modulated signal $s(t)$ maybe written as

$$s(t) = \underbrace{\cos 2\pi f_c t}_{\text{carrier term}} + 2\, m(t) \cos 2\pi f_c t$$

The spectrum $S(f)$ of $s(t)$ therefore consists of a spike of weight $\frac{1}{2}$ at $\pm f_c$ Hz, (carrier term) (first term) and the spectrum $M(f)$ above, weighted by $\frac{1}{2}$ and shifted up and down by f_c Hz.

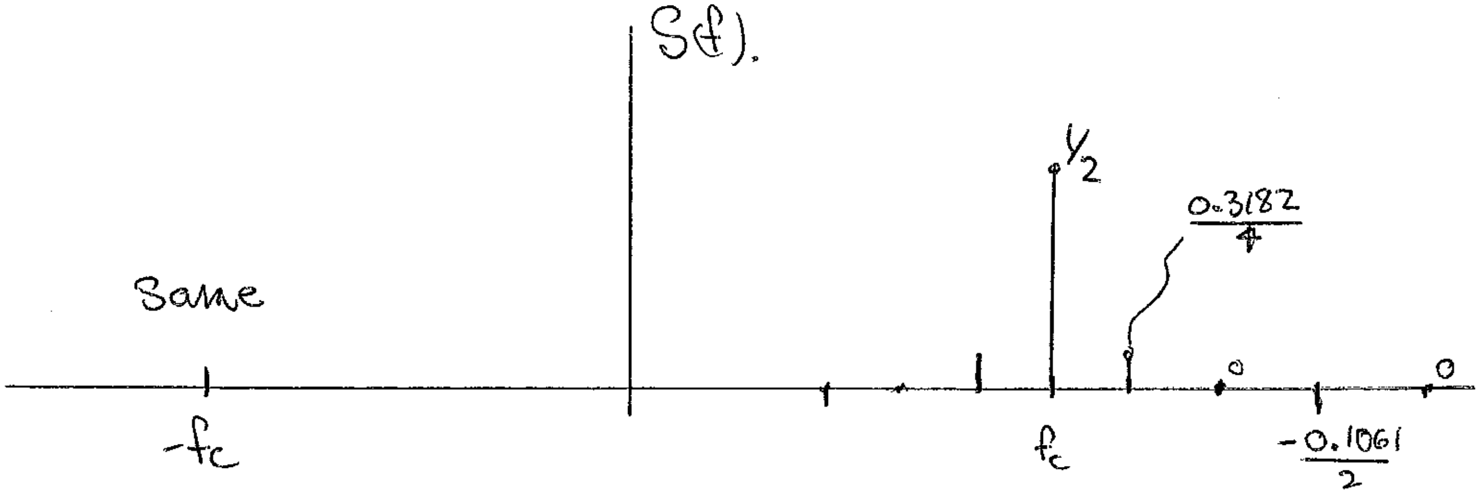


c. For 50% modulation, $k_a = 1$.



when the % modulation is 50%, $k_a = 1$, the spectrum M(f) is effectively reduced by a factor of 2. However, the carrier component remains the same.

50% modulation.



Question 2

c. This is a single sideband (SSB) modulation system

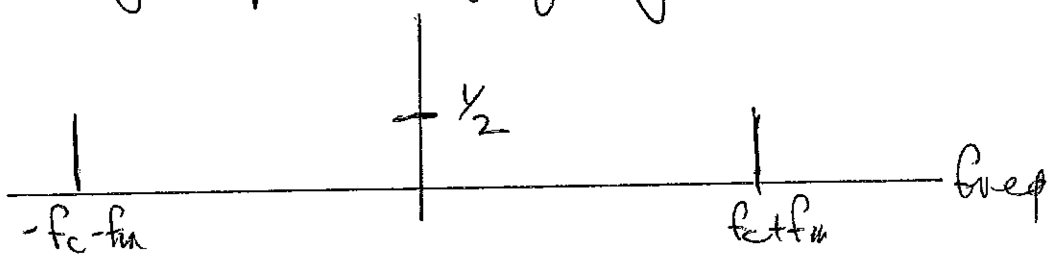
a. $s(t) = \cos(2\pi f_m t)\cos 2\pi f_c t - \sin 2\pi f_m t \sin 2\pi f_c t$

According to the tables, $[\alpha = 2\pi f_m t, \beta = 2\pi f_c t]$

$s(t) = \cos 2\pi(f_c + f_m)t$

(a single spike at frequency $\pm(f_c + f_m)$).

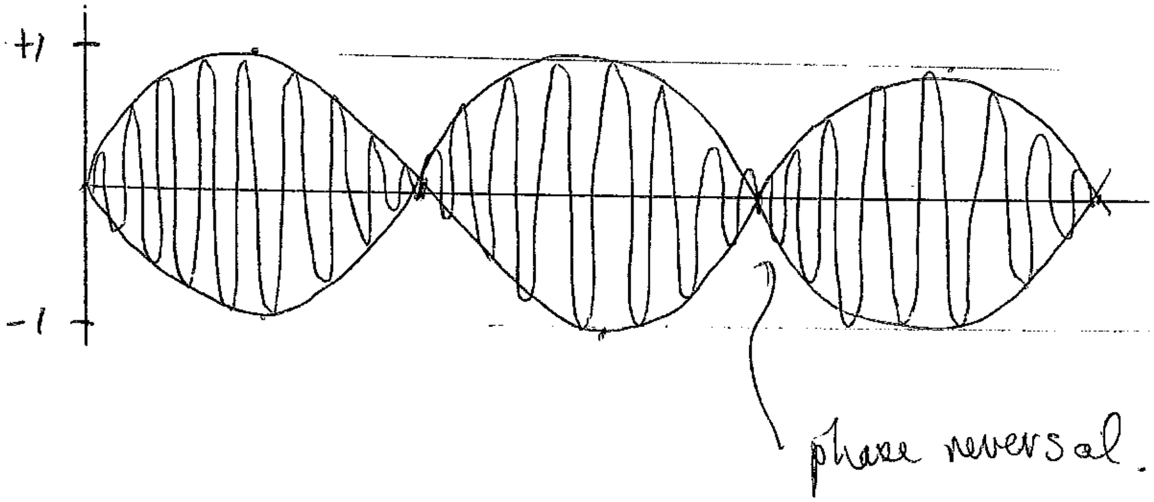
b.



d. The complex envelope in this case is

$$\hat{m}(t) = \cos(2\pi f_m t) + j \sin 2\pi f_m t$$
$$= \exp(j 2\pi f_m t).$$

e. This is a straightforward DSB/SC signal with a sinusoidal message:



3. $H(s)$ at $s = j2\pi(1.5)$ is evaluated to be $0.4061 e^{-j120.50^\circ}$

at $s = j2\pi(0.5)$ is $0.9701 e^{-j43.31}$

The system is linear and time invariant. The output due to the component at 1.5 Hz alone is

$$2 \times 0.4061 \cos\left(2\pi(1.5)t - 120.50^\circ\right)$$

That due to the component at 0.5 Hz is

$$1 \times 0.9701 \cos\left(2\pi(0.5)t - 43.31^\circ\right)$$

The total output is simply the sum of the two:

$$y(t) = 0.8122 \cos\left(2\pi(1.5)t - 120.50^\circ\right) + 0.9701 \cos\left(2\pi(0.5)t - 43.31^\circ\right)$$