EE3TR4 Midterm Solutions 2017 J. Reilly

Q1. For 100% AM modulation,

$\max |k_a m(t)| = 1.$ Thus $k_a = 1$.

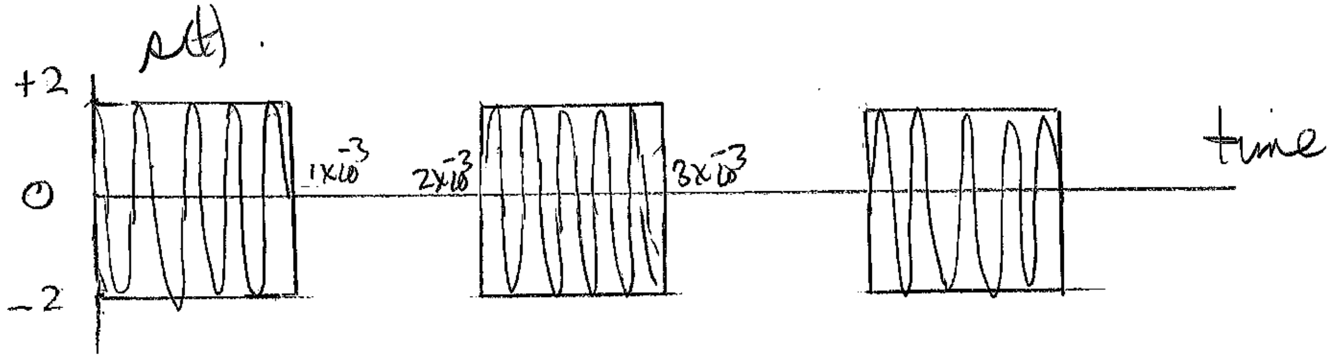
The AM waveform in general is given as

$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$

in this case, $A_c = k_a = 1$

$\therefore s(t) = [1 + m(t)] \cos 2\pi f_c t$

a)



b) $s(t) = \cos 2\pi f_c t + m(t) \cos 2\pi f_c t$ (1)

The spectrum of m(t) (square wave, 50% duty cycle) by a series of impulses at freq f_0, where $f_0 = 1/T$ where T is the period (500 Hz),

(cont'd)

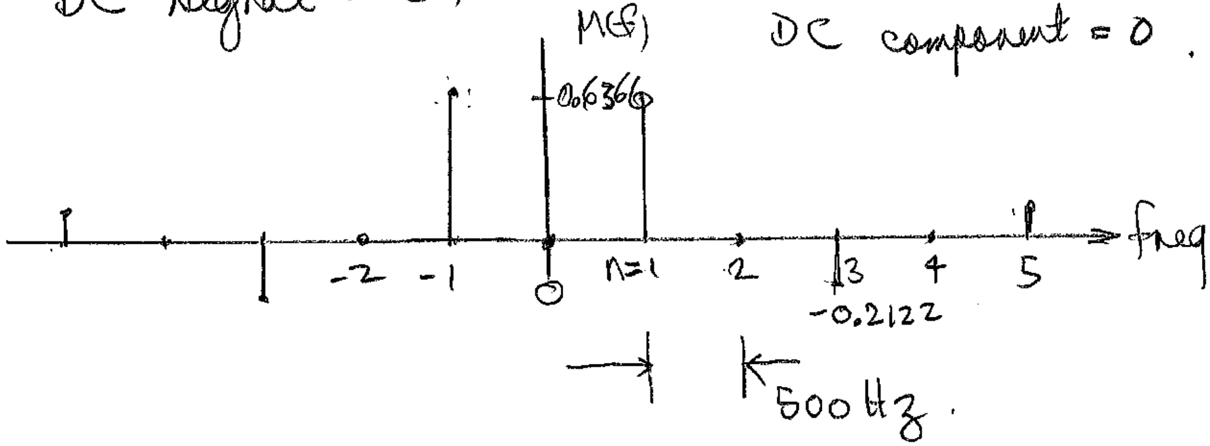
with weights $c_n = A\delta \,\mathrm{sinc}(n\delta)$ where δ = duty cycle $= 1/2$ $A = 2$ DC component $= c_0 = 0$.

n	c_n
0	0
1	0.6366
2	0
3	−.2122
4	0
5	0.1274

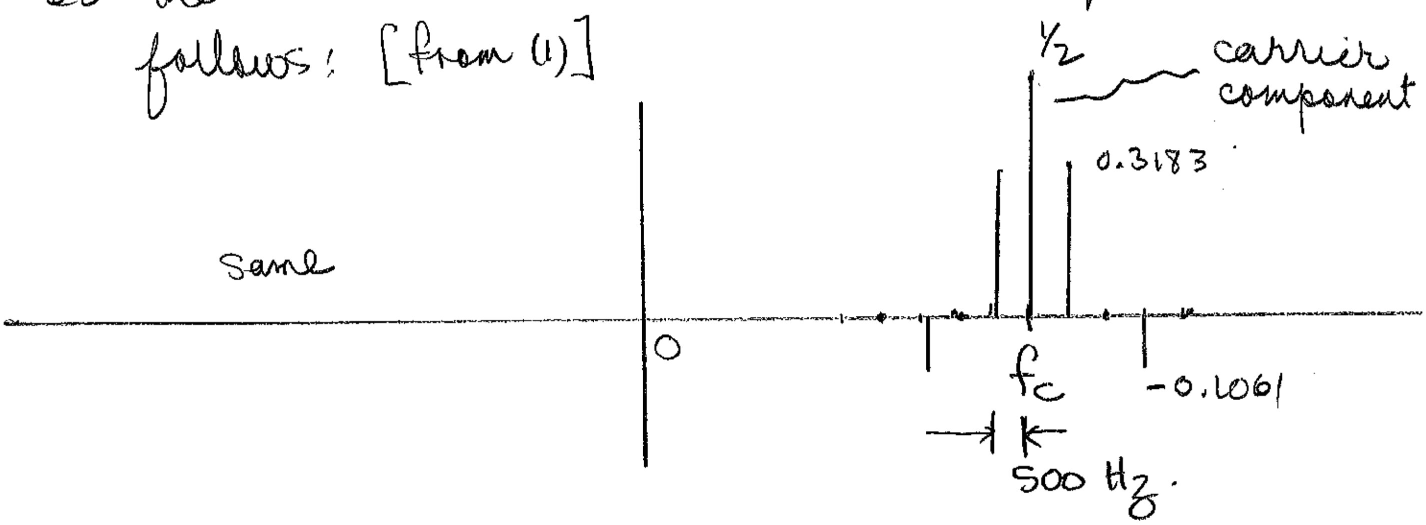
etc.

DC signal = 0.

DC component = 0.

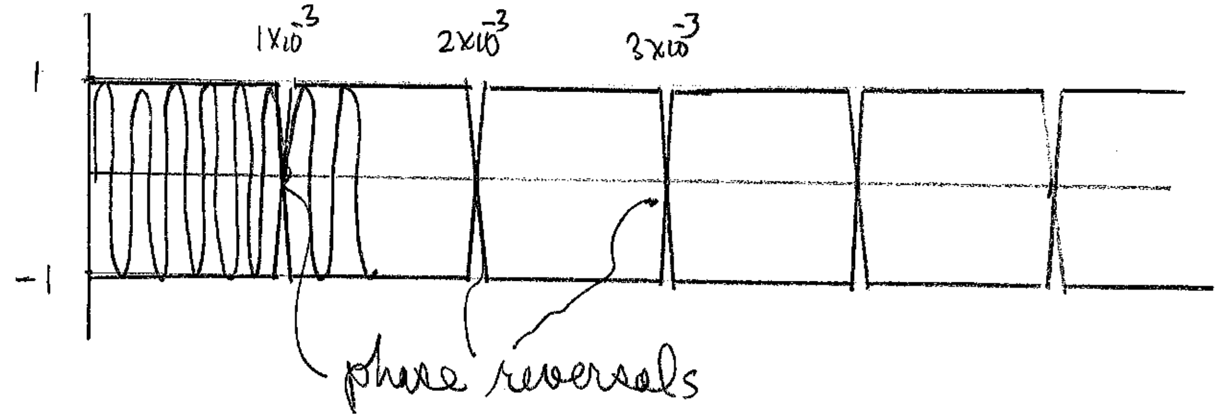


∴ the modulated AM wave has a spectrum as follows: [from (1)]



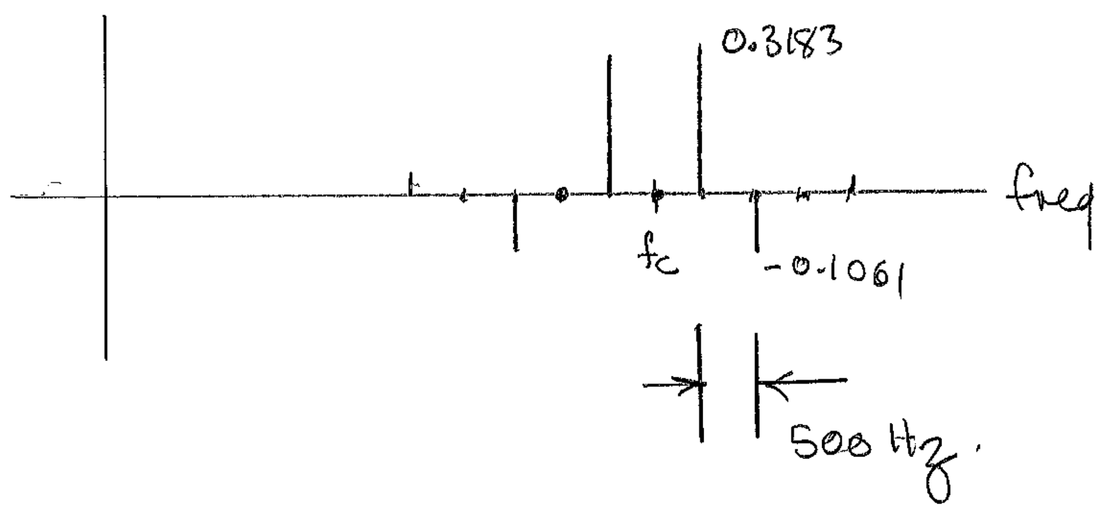
c. In this case, the DSB/SC waveform is described as

$$s(t) = m(t) \cos 2\pi f_c t$$



The spectrum is the same as in part b, except no carrier is present.

same.

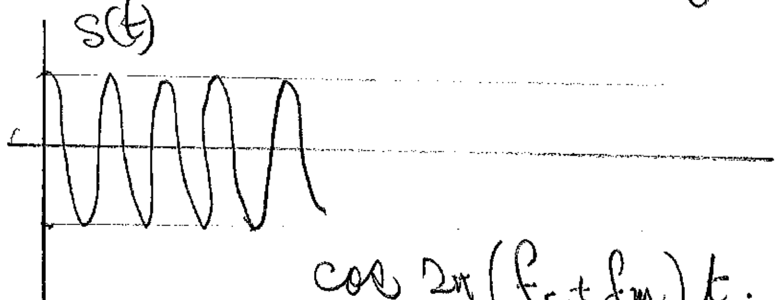


Question 2

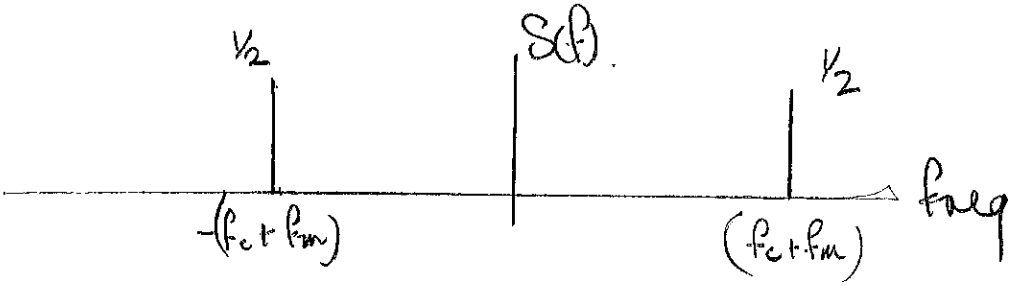
a. The Hilbert transform is a wideband 90° phase shifter. $\therefore$ the Hilbert transform of $A\cos 2\pi f_0 t = A \sin 2\pi f_0 t$.

- Since the two inputs $m_1(t)$ & $m_2(t)$ are a Hilbert transform pair, the configuration shown implements a single sideband modulation system.
- The SSB modulated signal for a sinusoidal message is a cosine - $\cos 2\pi(f_c \pm f_m)t$. In this case, the upper sideband is selected, due to the $-$ sign at the output adder.

b.



$\cos 2\pi(f_c + f_m)t$.



Checks:

1. The output waveform $s(t)$ according to the diagram is

$$s(t) = \cos(2\pi f_m t)\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t)$$

From the trig tables, this is equal to

$$s(t) = \cos 2\pi(f_m + f_c)t.$$

2. For the quadrature modulation system shown, the complex envelope $\tilde{m}(t) = m_1(t) + j\,m_2(t)$.

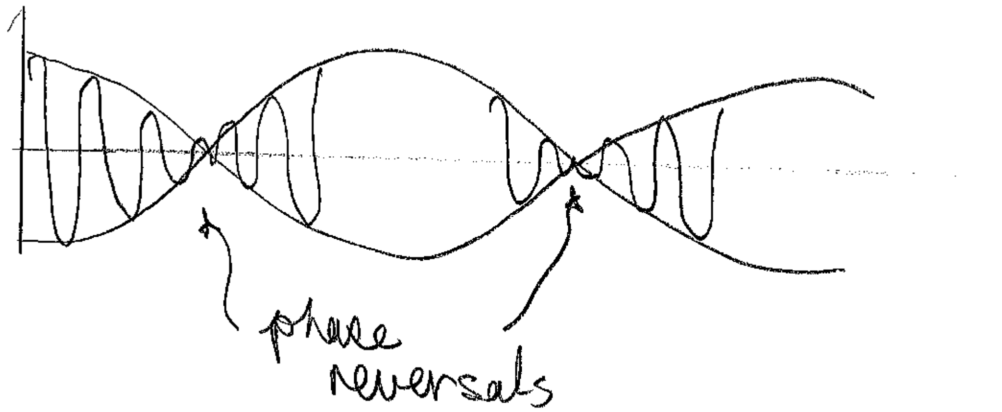
$$\therefore\ \tilde{m}(t) = \cos 2\pi f_m t + j \sin 2\pi f_m t = e^{j2\pi f_m t}.$$

The SSB signal is obtained by translating to f_c Hz and taking the real part

$$s(t) = \mathrm{Re}\left\{ e^{j2\pi f_m t} \cdot e^{j2\pi f_c t} \right\}$$

$$= \cos 2\pi(f_c + f_m)t.$$

2c. This signal is $\cos 2\pi f_m t \cdot \cos 2\pi f_c t$



Question 3

a) comparing this pdf with the Gaussian pdf expression given in the paper,

mean $= 0$, variance $\sigma^2 = 4$.

b). Expectation operations use ensemble averages – ie across samples of the r.p. at a specific time. (vertical averaging)

If the process is stationary, these expectations do not change with time. If the process is ergodic, then ensemble averages can be replaced with time averages.

We are given 100 samples of one realization of a process. For the $\overset{\mu}{\text{mean}}$, $f(x) = x$.

$$\text{ie} \quad \mu = E(x) = \underbrace{\int x\, p(x)\, dx}_{\text{ensemble avg}} \equiv \underbrace{\lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{\infty} x(n)}_{\text{time avg}}$$

Because we only have 100 samples available we take a finite sum to get an approximation:

$$\mu \simeq \frac{1}{100} \sum_{n=1}^{100} x(n)$$

For variance, $f(x) = (x-\mu)^2$

$$\therefore \sigma^2 \simeq \frac{1}{100} \sum_{n=1}^{100} (x-\mu)^2$$