Solutions to Midterm EE3TR4 2009

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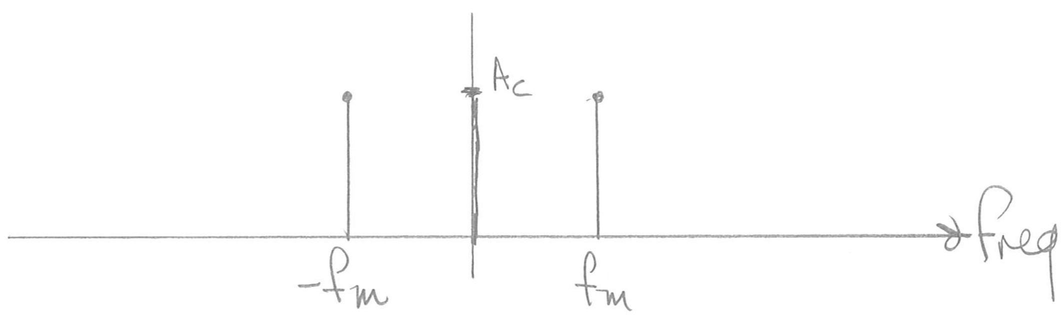
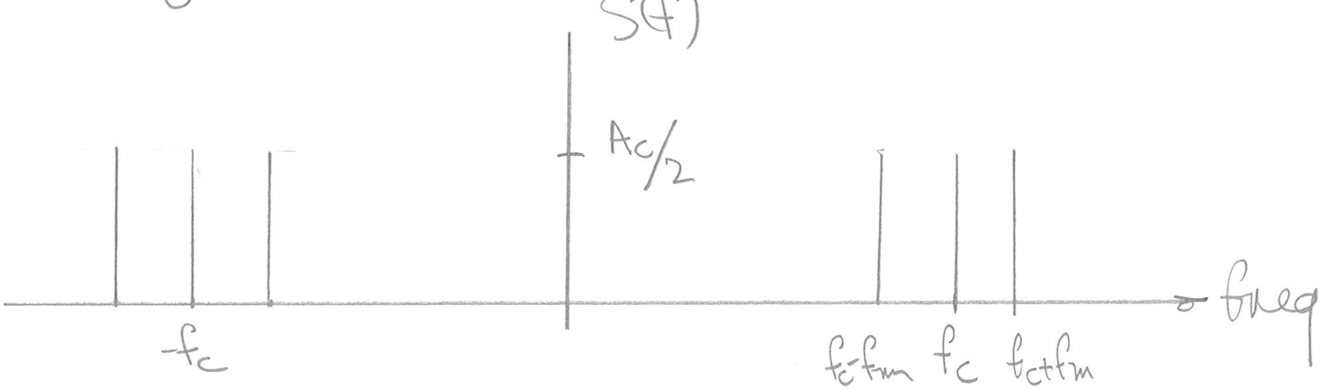
1 i) $$s(t) = \underbrace{A_c\left[1 + 2\cos 2\pi f_m t\right]}_{\text{envelope}} \cos 2\pi f_c t$$

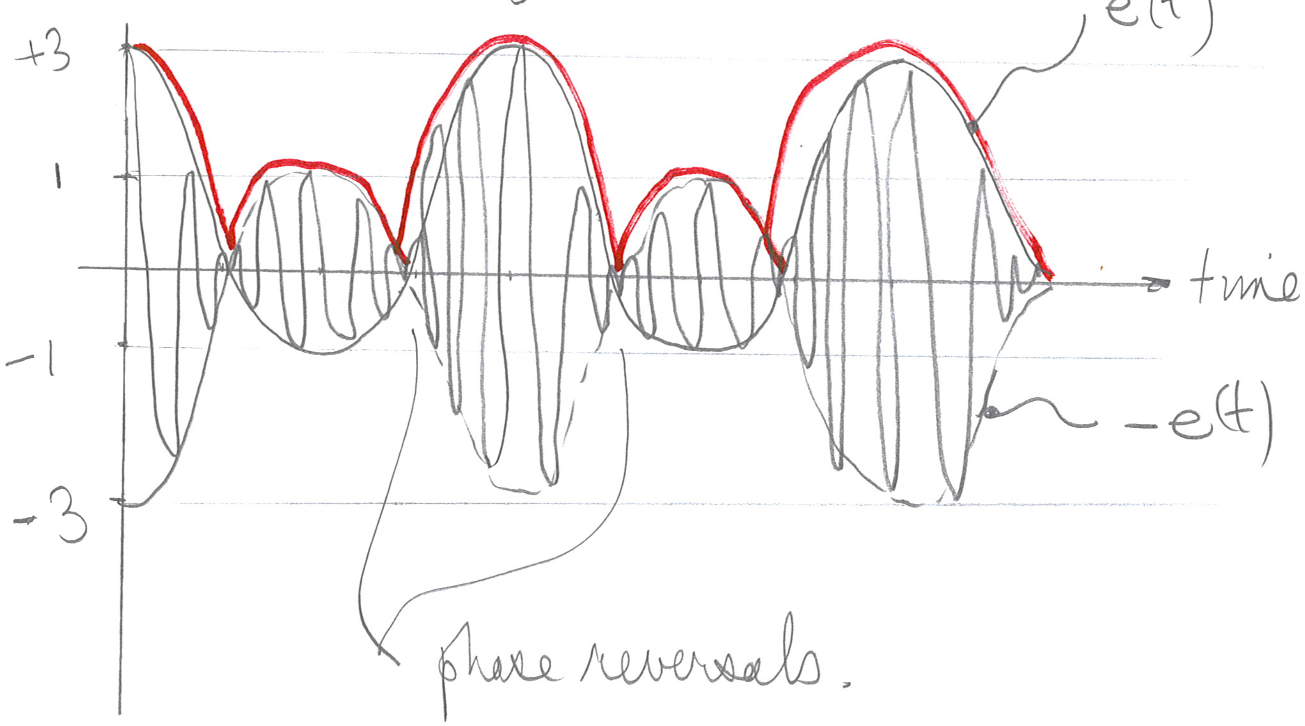
The spectrum of the envelope is:



After mult. in time by $\cos 2\pi f_c t$, the spectrum $S(f)$ of $s(t)$ becomes:

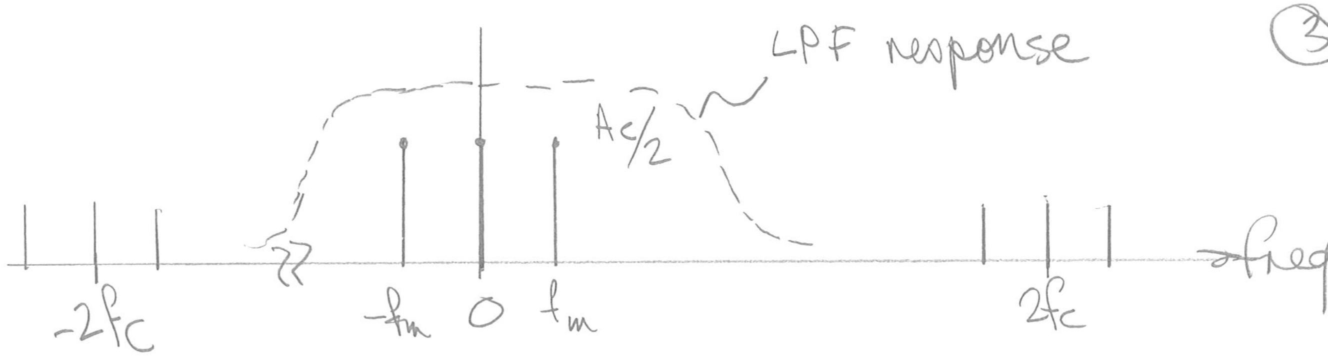


ii) The waveform of $s(t)$ is



iii) The output of the ideal envelope detector is shown above in red. It is a distorted version of the message which is a cosine. This is a result of the fact $s(t)$ has over 100% modulation.

iv) Here, we multiply $s(t)$ by $\cos 2\pi f_c t$. This has the effect of shifting the spectrum $S(f)$ of $s(t)$ up & down in frequency:



After passing through a low-pass filter, the output contains only the components around 0 Hz.

∴ the output of the demodulation process is

$$\frac{A_c}{2} + A_c \cos 2\pi f_m t .$$

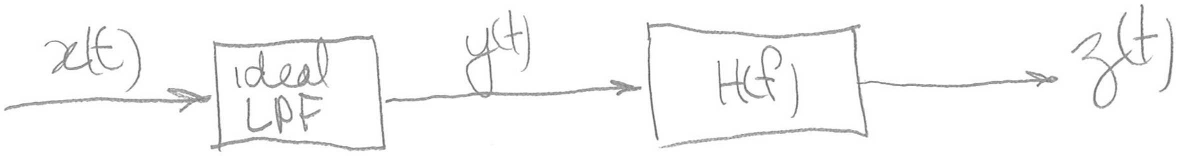
After removing the DC component (by a blocking capacitor), we are left with a cosine at f_m Hz, which is an undistorted version of the message.

2. The block diagram of the overall configuration is:

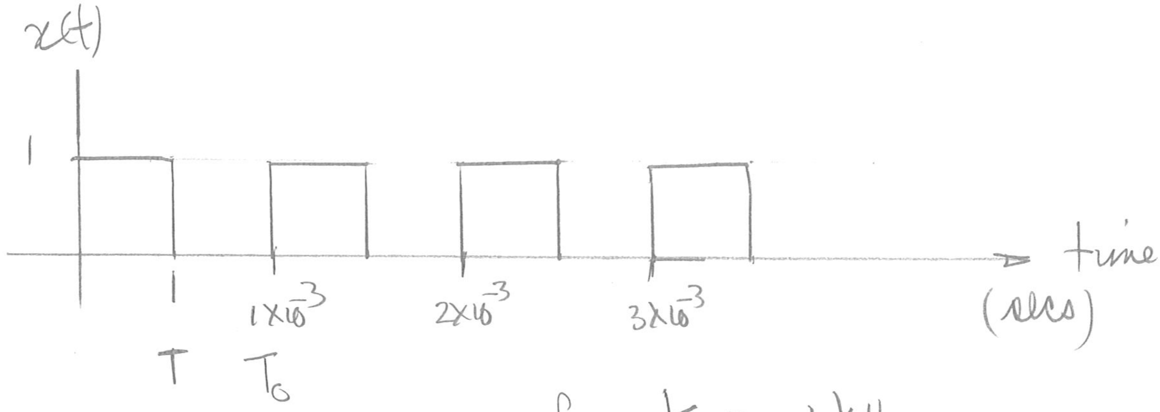
$x(t)$ = square wave
- assume duty cycle = $\frac{1}{2}$

cutoff
= 1000 Hz

If the blocks are linear, time-invariant (LTI) then they can be interchanged without affecting the output. Therefore, the above is equivalent to:



$x(t)$ is a square wave with freq. 1KHz and amplitude 1 V. Therefore the spectrum is discrete at 1KHz intervals. The weights of the spectral harmonics c_n are given as:



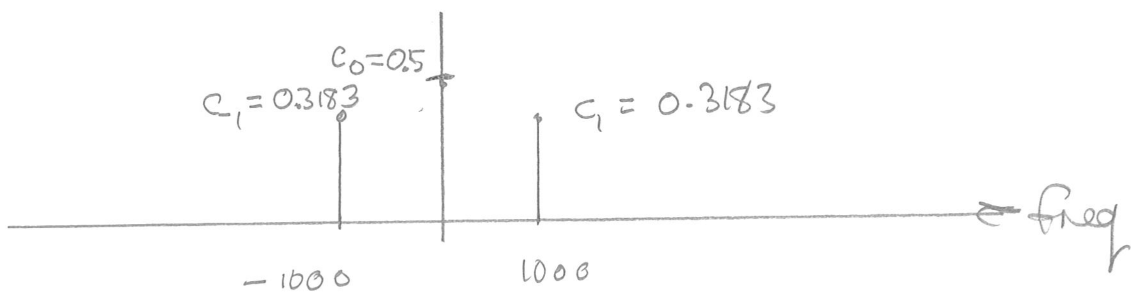
$$f_0 = \frac{1}{T_0} = 1\,kHz$$

$$\rightarrow \quad c_n = \frac{AT}{T_0}\,\text{sinc}\, nT/T_0$$

$$= \tfrac{1}{2}\,\text{sinc}\, n/2$$

$$\therefore \quad c_0 = \tfrac{1}{2} \qquad c_{\pm 1} = \tfrac{1}{2}\,\text{sinc}\, \tfrac{1}{2} = 0.3183$$

The spectrum $Y(f)$ of $y(t)$ is therefore



We must now evaluate the output $z(t)$ of the filter $H(f)$.

$$H(f) = \frac{1}{1 + j2\pi f/a}$$

$a = 2\pi \times 1000$

$$H(f) = \frac{1}{1 + j\,f/1000}$$

At $f = 0$, $H(0) = 1$

$f = 1000\ Hz$ $\quad H(1000) = \frac{1}{1+j} = \frac{1}{\sqrt{2}} e^{-j45^\circ}$

$f = -1000\ Hz$ $\quad H(-1000) = \frac{1}{1-j} = \frac{1}{\sqrt{2}} e^{+j45^\circ}$

The spectrum $Z(f) = Y(f) \cdot H(f)$

$\therefore$ $Z(f)$ consists only of 3 components: one at $0\ Hz$ and two at $\pm 1000\ Hz$.

$$Z(f) = 0.5\,\delta(f) \cdot H(0) + 0.3183\,\delta(f-1000) \cdot H(1000)$$
$$+ 0.3183\,\delta(f+1000) \cdot H(-1000)$$

$$= 0.5\,\delta(f) + \frac{0.3183}{\sqrt{2}}\,\delta(f-1000)\,e^{-j45^0}$$

$$+ \frac{0.3183}{\sqrt{2}}\,\delta(f+1000)\,e^{+j45^0}$$

$$\rightleftharpoons 0.5 + \frac{2 \times 0.3183}{\sqrt{2}} \cos\left(2\pi\, 1000 t - 45^0\right)$$

3. i) Each component at $\pm f_m$ Hz of $\cos 2\pi f_m t$ gets shifted by $\mp 90°$ when it is put thru a Hilbert transform.

$$\therefore \text{ H.T. of } \cos 2\pi f_m t = \cos(2\pi f_m t - 90°)$$

$$= \sin 2\pi f_m t.$$

ii) This question implies that $A_m = 1$ and then $\mu = k_a A_m = 1$.

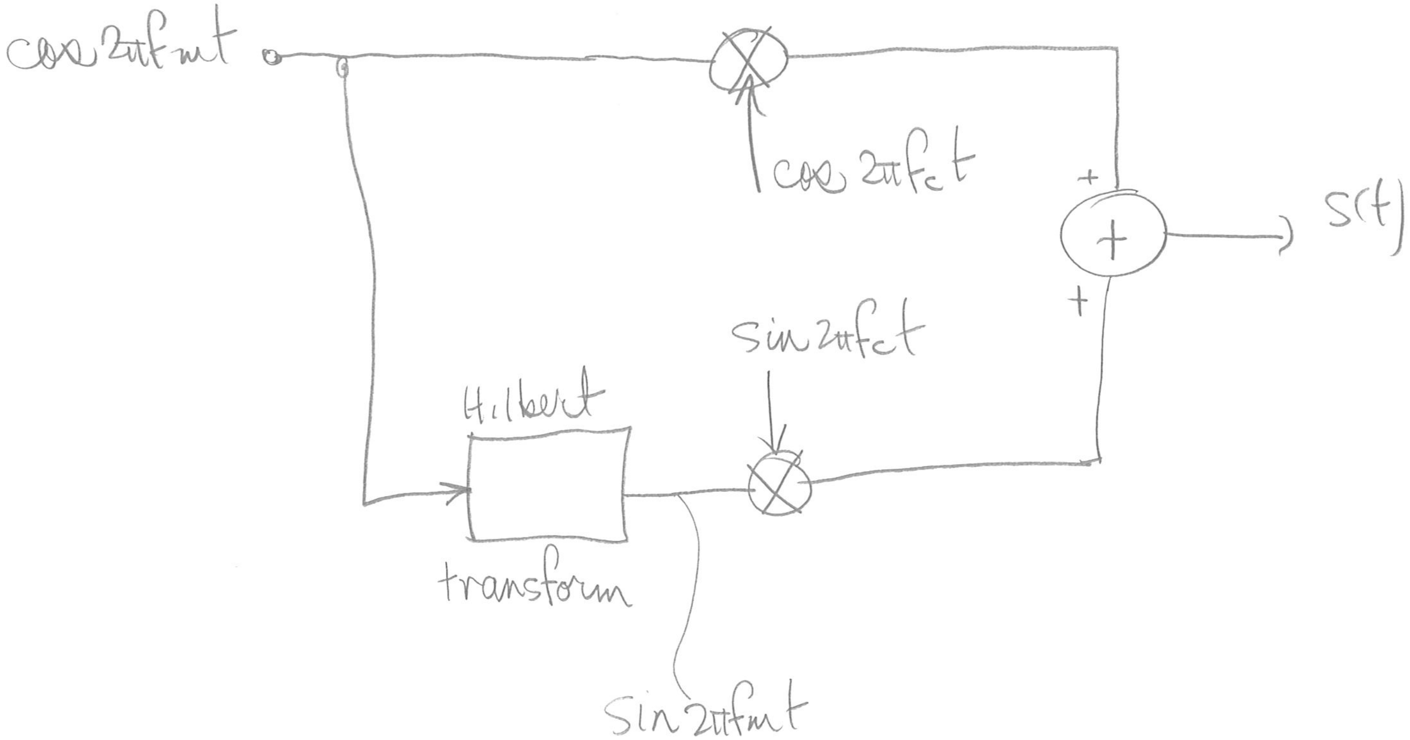
Then the % modulation is

$$\max \left| k_a A_m \cos 2\pi f_m t \right| \times 100\%$$

$$= 100\%.$$

In this case, the minimum value of the envelope $= 0$ and the envelope is not distorted, although this is the maximum value of μ for which undistorted demod. is possible.

iii) $m(t) = \cos 2\pi f_m t$



$$s(t) = \cos 2\pi f_m t \cos 2\pi f_c t + \sin 2\pi f_m t \sin 2\pi f_c t$$

"+" sign gives lower sideband

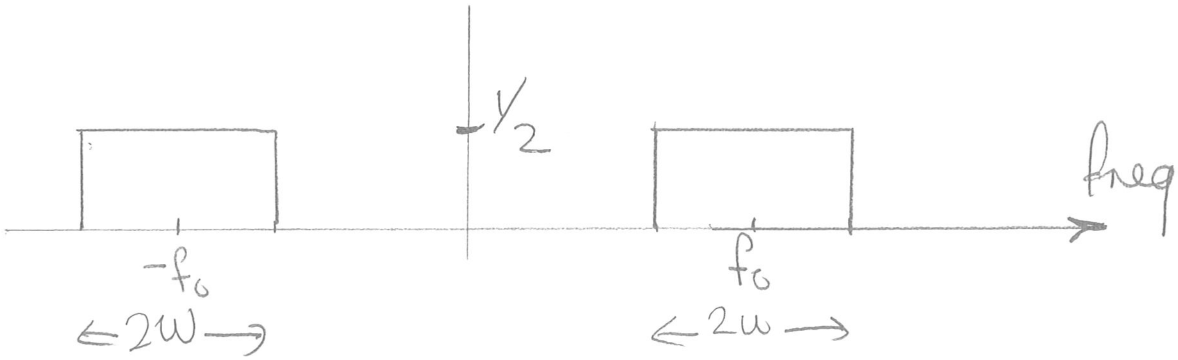
using $\cos A \cos B \mp \sin A \sin B = \cos(A \pm B)$

$$s(t) = \underline{\cos\left(2\pi(f_m + f_c)\,t\right)}$$

iv) $x(t) = 2W \,\mathrm{sinc}\, 2Wt$

$X(f) \rightleftharpoons \mathrm{rect}\, f/2W$ (from table)

when $x(t)$ is multiplied by $\cos 2\pi f_0 t$, the spectrum is shifted up and down by f_0 Hz:

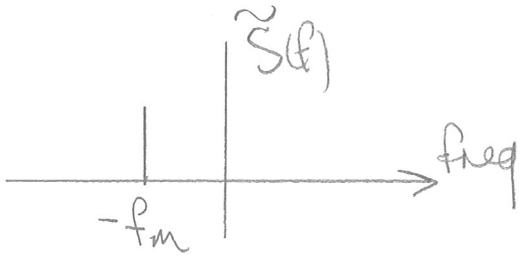


v) The complex envelope for part iii) is

$$\tilde{s}(t) = \cos 2\pi f_m t - j \sin 2\pi f_m t$$

"–" corresponds to lower sideband

$$= e^{-j2\pi f_m t}$$



This is the "wierd" spectrum which gives S(f) when translated to f_c Hz, after taking the real part.