Electrical Engineering EE3TR4

Midterm test: 1.5 Hours

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This examination paper includes 4 pages and 3 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) **If you want your paper to be considered for re-marking, then answer in pen and do not use white-out.**

(b) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted**.

(c) There are 3 questions. Attempt all three.

(d) You must show your work for full marks.

(e) **Make sure you read the entire paper over in its entirety before you start!**

(f) The tables of Fourier transforms and trigonometric identities at the back of this exam may be useful.

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1. The message signal shown in Figure 1 is modulated onto a carrier at frequency f_c Hz.

a. draw the corresponding amplitude modulated (AM) waveform $s_{\text{AM}}(t)$ for 100% modulation. Assume the carrier amplitude $A_c = 1$. (2 marks)

b. draw the magnitude spectrum $S_{\text{AM}}(f)$ of the modulated waveform above. Show all relevant values. (3 marks)

c. repeat parts **a.** and **b.** for DSB/SC modulation. (5 marks)

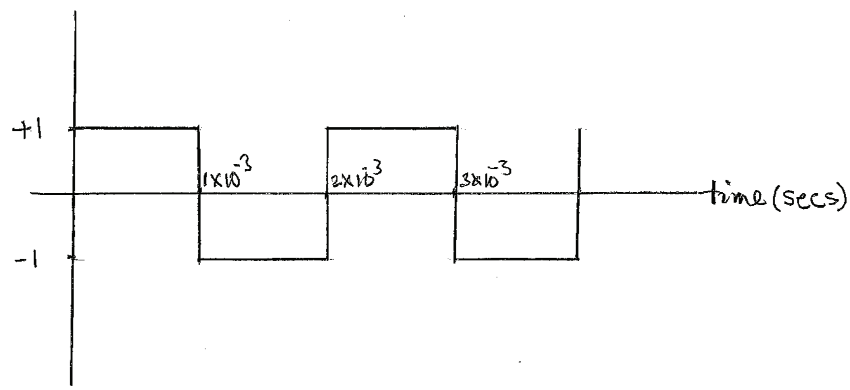

Figure 1: Message signal for Question 1.

2. Consider the configuration shown in Figure 2 below.

a. What is the Hilbert transform of $A \cos 2\pi f_o t$ (2 marks) ?

b. Draw the waveform $s(t)$ and the corresponding spectrum $S(f)$ of the output signal (5 marks). *Hint:* Check the trigonometric formulas at the end!

c. Sketch the time domain waveform at the output of the top multiplier in the figure (3 marks).

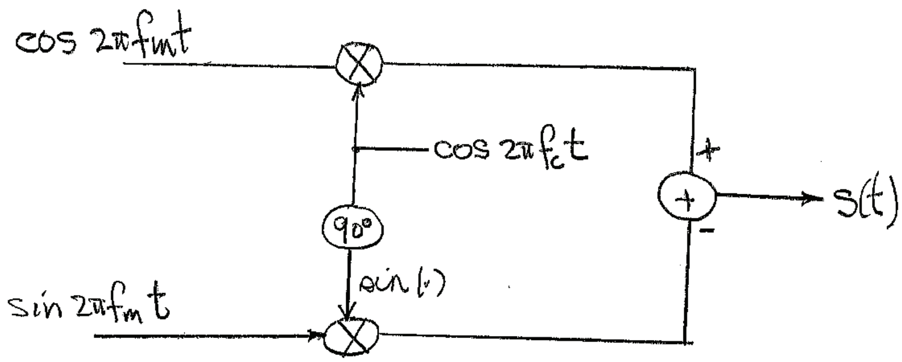

Figure 2: Modulation configuration for Question 2.

3. a. We are given a random process whose underlying pdf is a Gaussian distribution, given by

$$p(x) = \frac{1}{\sqrt{2\pi \cdot 4}} \exp\left(-\frac{1}{2} \cdot \frac{x^2}{4}\right). \tag{1}$$

What is the DC value and the power of this process? (Use the resistor value $R = 1$.) (4 marks)

b. We wish to take the expectation of some function $f(x)$ of a random process $x(t)$. Explain how the expectation is evaluated in the most general case. What happens to the expectation when the underlying process is non-stationary?

c. Let's say we are given a segment from a single realization (sample) of a random process; *i.e.* we are given $x(n), n = 1, \ldots, 100$. The mean and variance are unknown. Assume the process is stationary and ergodic. Carefully explain how we can estimate the mean and variance from this sample. (6 marks).

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Fourier Transform Pairs

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$\text{Tsinc}\big(\text{fT}\big)$
$\text{sinc}\big(2Wt\big)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]$

Trigonometric Identities

$$\cos(\theta) = \tfrac{1}{2}\big[\exp(j\theta) + \exp(-j\theta)\big]$$
$$\sin(\theta) = \tfrac{1}{2j}\big[\exp(j\theta) - \exp(-j\theta)\big]$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$
$$\cos^2(\theta) = \tfrac{1}{2}\big[1 + \cos(2\theta)\big]$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
$$\sin(\alpha)\sin(\beta) = \tfrac{1}{2}\big[\cos(\alpha - \beta) - \cos(\alpha + \beta)$$
$$\cos(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\cos(\alpha - \beta) + \cos(\alpha + \beta)$$
$$\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\sin(\alpha - \beta) + \sin(\alpha + \beta)$$

Note specifically that

$$\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) = \cos(\alpha + \beta)$$

Gaussian Distribution

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)$$

where μ, σ^2 are the mean and variance, respectively.

The End.