Solutions to EE3TR4 Midterm Feb 2010

J Reilly.

1. a. As you know from lab 1, the spectrum of the 1000 Hz square wave signal is discrete, with spectral components every $f_0 = 1000$ Hz. The amplitude of these components is well known to correspond to a sinc function; i.e. the amplitude c_n of the component at nf_0 Hz is given by:

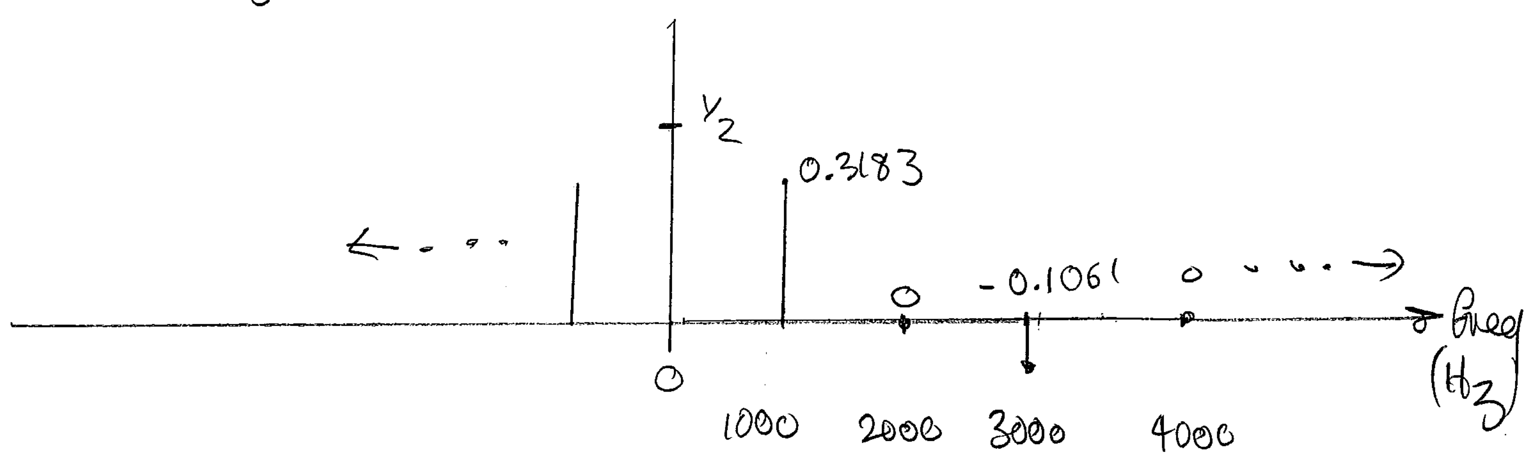
$$c_n = \frac{A}{2}\,\mathrm{sinc}(nf_0T)$$

where A is the amplitude and f_0T is the duty cycle ($\frac{1}{2}$ in this case)

The first 5 values are (for the amplitude $A = 1$)

$c_0 = \frac{1}{2}$, $c_1 = c_{-1} = 0.3183$ $c_2 = c_{-2} = 0$

$c_3 = c_{-3} = -0.1061$ $c_4 = c_{-4} = 0$, etc.



1b. Let the input to the filter be $x(t)$ and the output $y(t)$. Then

$$Y(f) = H(f)\,X(f)$$

We wish to evaluate the magnitude and phase of the spectral component of the output at $f = 1000$ Hz.

$$H(f)\Big|_{f=1000\text{ Hz}} = \frac{2}{\left(\frac{j2\pi 1000}{2\pi 1000}\right)^2 + \frac{j4\pi 1000}{2\pi 1000} - 1}$$

$$= \frac{2}{-1 + 2j - 1} = \frac{1}{-1+j}$$

$$= \frac{1}{\sqrt{2}} e^{-j135^\circ}$$

We have seen that the spectral component of $X(f)$ at 1000 Hz $= 0.3183 e^{j0}$.

$$\therefore\ Y(f)\Big|_{f=1000} = 0.3183 e^{j0} \cdot \frac{1}{\sqrt{2}} e^{-j135^\circ} = 0.2251 e^{-j135^\circ}$$

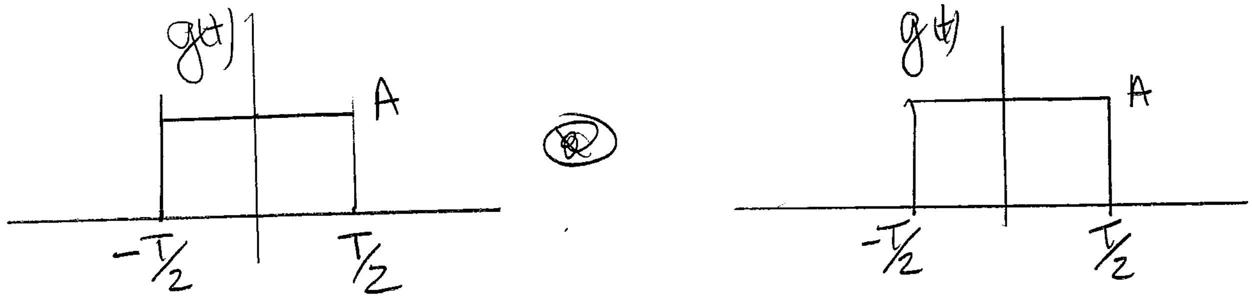
$\therefore$ the output component at 1000 Hz can be written as

$$0.2251 \cos\left(2\pi \times 1000 t - 135^{\circ}\right).$$

C. There is no frequency component in the input signal at 1500 Hz.

$\therefore$ the spectral component at 1500 Hz at the output is zero.

2. (a) The triangular pulse shown is the convolution of 2 rectangular pulses; i.e

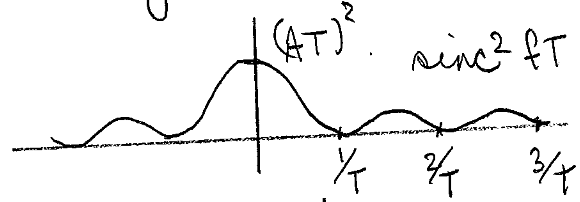


The rectangular pulses have a spectrum

$$g(t) \rightleftharpoons G(f) = AT \operatorname{sinc} fT.$$

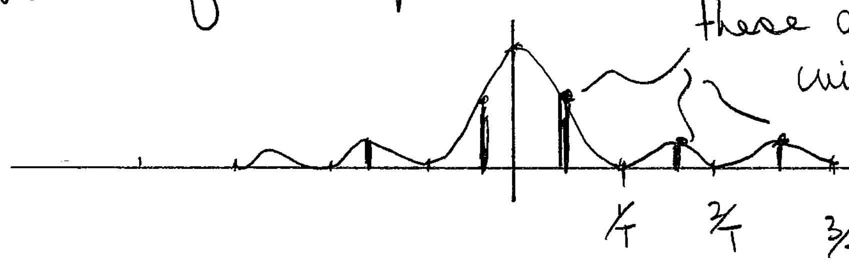
Since convolution in time $\rightleftharpoons$ mult. in freq, the spectrum of the triangular pulse is

$(AT)^2 \operatorname{sinc}^2(fT)$. $\rightarrow$



b). The spectrum in part b is a periodic version of that in part a, with period 2T.

$\therefore$ the spectrum becomes discrete with components every $\frac{1}{2T}$ Hz, with amplitudes given by the value of the spectrum in part a at $f = n/2T$ Hz



these are the spectral components, with amplitudes given by the function $(AT)^2 \operatorname{sinc}^2(fT)$.

3. The modulated AM wave is given as:

(a) $$s(t) = A_c\left[1 + k_a m(t)\right] \cos 2\pi f_c t$$

Here, $m(t) = A_m \cos 2\pi f_m t$. When the % modulation is 125%, then $k_a A_m = 1.25$.

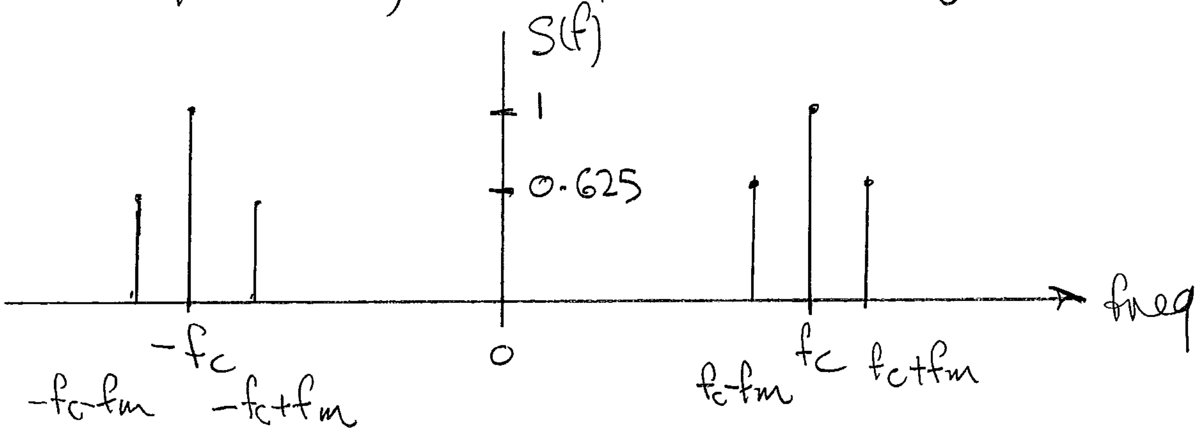
For $A_c = 2$, we have

$$s(t) = 2\left[1 + 1.25 \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

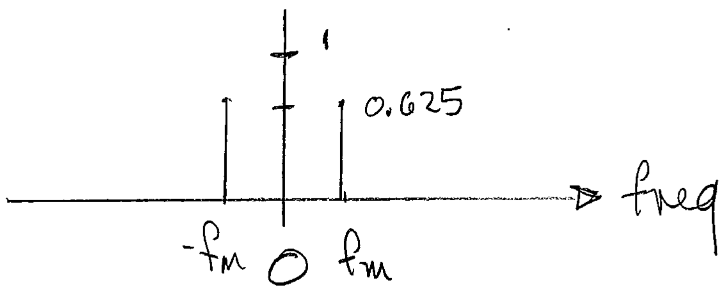
$$= 2 \cos 2\pi f_c t + 2.5 \cos 2\pi f_m t \cos 2\pi f_c t$$

$$= 2 \cos 2\pi f_c t + 1.25 \cos 2\pi (f_c + f_m) t + 1.25 \cos 2\pi (f_c - f_m) t$$

Because the spectrum of a sinusoid of amplitude A at frequency f_o consists of 2 spikes of amplitude A/2 at freqs. $\pm f_o$, the spectrum $S(f)$ of $s(t)$ is:



36. Note that if $s(t)$ were passed thru a product multiplier and then low-pass filtered, the resulting spectrum would be:



This is equivalent to the message + a DC component. The DC component can be removed by a blocking capacitor to recover the msg.