MAGNETIC FIELDS

ELECTRIC FIELD

COULOMB'S LAW: $F \propto \frac{q_1 q_2}{r^2}$

LET $q_1 = q$, $q_2 = +1$

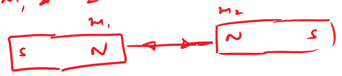
ELECTRIC FIELD INTENSITY $E \propto \frac{q \cdot (+1)}{r^2}$

FORCE ON A UNIT +VE ELECTRIC CHARGE = ELECTRIC FIELD INTENSITY

MAGNETIC FIELD

$F \propto \frac{m_1 m_2}{r^2}$

m_1 & m_2 ARE MAGNETIC CHARGES



LET $m_1 = m$, $m_2 = +1$

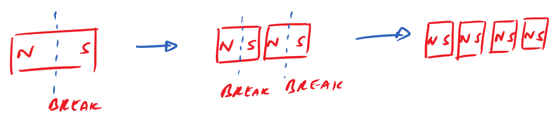
$H \propto \frac{m(+1)}{r^2}$

FORCE ON A UNIT +VE MAGNETIC CHARGE = MAG. FIELD INTENSITY, H

HOWEVER, ISOLATED MAGNETIC CHARGE DOES NOT EXIST. (NO MAGNETIC MONOPOLE)

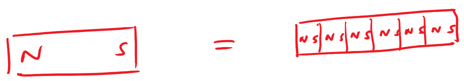
SO, THE ABOVE DEFINITION OF H IS NOT OF PRACTICAL INTEREST. NEVERTHELESS, IT SHOWS A STRONG ANALOGY BETWEEN E & H.

CONSIDER A MAGNET & BREAK IT INTO TWO PARTS



NO MATTER HOW MANY TIMES YOU BREAK A MAGNET. YOU WILL NEVER BE ABLE TO FIND AN ISOLATED NORTH POLE OR SOUTH POLE. MAGNETIC CHARGES ALWAYS OCCUR IN PAIRS.

A BAR MAGNET MAY BE IMAGINED AS A COLLECTION OF A LARGE NUMBER OF TINY MAGNETIC CHARGE PAIRS:



MAGNETIC FLUX DENSITY, B :

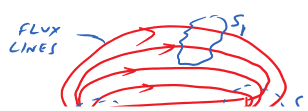
$$B = \mu H \quad \rightarrow (1)$$

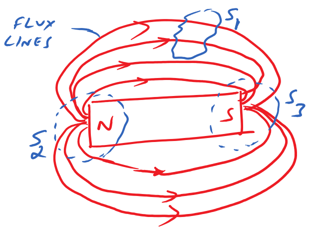
μ = PERMEABILITY OF THE MEDIUM

THIS IS THE ANALOG OF ELECTRIC FIELD EQUATION

$$D = \epsilon E$$

ELECTRIC FIELD	MAG. FIELD
E →	H
ϵ →	μ
$D = \epsilon E$ →	$B = \mu H$





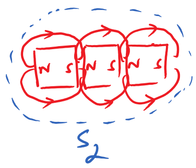
SURFACE S_1

NET FLUX CROSSING THE SURFACE S_1 = 0

$$\text{DIV } \vec{B} = 0$$

SURFACE S_2

NET FLUX CROSSING SURFACE S_2 APPEARS TO BE NON-ZERO. BUT N SHOWN IN THE FIGURE IS NOT AN ISOLATED CHARGE. NEAR N, IT WOULD LOOK LIKE THIS



$\therefore$ NET FLUX CROSSING THE SURFACE S_2 = 0

$\text{DIV } \vec{B} = 0$ ON S_2

SIMILARLY, YOU CAN SHOW THAT INSIDE SURFACE S_3,

$$\text{DIV } \vec{B} = 0$$

IN OTHERWORDS, $\boxed{\text{DIV } \vec{B} = 0}$ EVERYWHERE. $\rightarrow$ (2)

ELECTRIC FIELD	MAG. FIELD
$\text{DIV } \vec{D} = \rho$	$\text{DIV } \vec{B} = 0$

EQUIVALENT STATEMENTS:

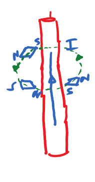
$\text{DIV } \vec{B} = 0$

NO MAGNETIC MONOPOLES

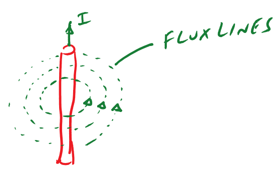
MAGNETIC FLUXLINES HAVE NO SOURCE OR SINK

MAGNETIC FLUXLINES FORM A CLOSED LOOP

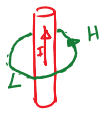
AMPERE'S LAW:



CONSIDER A CONDUCTOR CARRYING A DIRECT CURRENT I. IF YOU BRING A MAGNETIC COMPASS NEAR THE CONDUCTOR, IT WILL ORIENT IN THE DIRECTION SHOWN IN THE FIGURE. THIS MEAN THAT THE CURRENT-CARRYING CONDUCTOR PRODUCES MAGNETIC FIELD AROUND IT & COMPASS EXPERIENCES THIS FIELD. THE TRAJECTORY OF THE NORTH POLE OF THE COMPASS GIVES US THE MAG. FLUXLINES. NOTE THAT MAGNETIC FLUXLINES FORM A CLOSED LOOP.



AMPERE'S LAW:



THE LINE INTEGRAL OF $\vec{H}$ ABOUT ANY CLOSED PATH IS EQUAL TO THE CURRENT ENCLOSED BY THAT PATH.

$$\oint_L \vec{H}\cdot d\vec{l} = I \longrightarrow (3)$$

LET L BE A CIRCLE OF RADIUS r. DUE TO CYLINDRICAL SYMMETRY OF THE PROBLEM, YOU EXPECT THAT H IS CONSTANT AT A GIVEN RADIAL DISTANCE r FROM THE CONDUCTOR & DIRECTION OF H IS ALONG THE CIRCUMFERENCE.

$$\vec{H}\cdot d\vec{l} = H\,dl \quad (\because \vec{H} \text{ \& } d\vec{l} \text{ ARE IN THE SAME DIRECTION})$$

$$\oint_L \vec{H}\cdot d\vec{l} = \oint_L H\,dl = H\oint_L dl = H \times \text{CIRCUMFERENCE}$$

$$H \times 2\pi r = I$$

$$\boxed{H = \frac{I}{2\pi r}} \longrightarrow (4)$$

UNIT OF $I \rightarrow A$
UNIT OF $r \rightarrow m$
" " $H \rightarrow A/m$

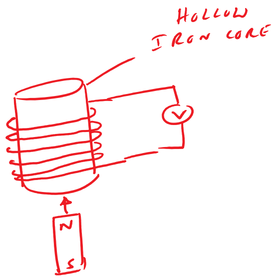
$H \propto$ CURRENT

$H \propto \dfrac{1}{\text{DISTANCE FROM THE CONDUCTOR}}$

	CAUSE	EFFECT
AMPERE'S LAW	I or V	H or B
	H or B	I or V ?

ACCORDING TO AMPERE'S LAW, CURRENT (CAUSE) PRODUCES MAGNETIC FIELD (EFFECT). CAN CAUSE & EFFECT BE REVERSED, i.e. CAN MAGNETIC FIELD PRODUCE VOLTAGE OR CURRENT? IF IT IS TRUE, WHAT KIND OF EXPERIMENT WILL YOU PERFORM TO PROVE OR DISPROVE?

FARADAY'S LAW:



CONSIDER AN IRON CORE WITH COPPER WINDING CONNECTED TO A VOLTMETER. IF YOU PUSH THE BAR MAGNET THROUGH THE HOLLOW IRON CORE, YOU WILL SEE A DEFLECTION IN THE VOLTMETER, INDICATING THAT AN EMF IS PRODUCED.

IF YOU STOP MOVING THE MAGNET → NO DEFLECTION
⇒ MAGNET IS STATIONARY W.R.T. IRON CORE → NO EMF

IF YOU PULL THE MAGNET OUT OF THE IRON CORE → DEFLECTION OF VOLTMETER IN THE OPPOSITE DIRECTION

BASED ON THESE THREE MEASUREMENTS, YOU CAN CONCLUDE THAT

RATE OF CHANGE OF MAGNETIC FLUX LINKED TO A COIL IS PROPORTIONAL TO THE EMF PRODUCED.

$$\text{EMF} \propto \frac{d\varphi}{dt} \rightarrow (5), \quad \varphi = \text{MAGNETIC FLUX LINKED TO COILS.}$$

$$\varphi = \iint \vec{B} \cdot ds$$

USING STOKES'S THEOREM, EQ. (5) MAY BE REWRITTEN AS

$$\text{curl } E = \nabla \times \vec{E} = -\frac{\partial B}{\partial t} \rightarrow (6)$$

EQ. (5) → FARADAY'S LAW IN INTEGRAL FORM
EQ. (6) → " " IN DIFFERENTIAL FORM

MEANING OF CURL:

THE CURL OF A VECTOR $\vec{A}$ IS DEFINED AS

$$\text{curl } \vec{A} = \nabla \times \vec{A} = F_x \vec{x} + F_y \vec{y} + F_z \vec{z}$$

$$\nabla \times \vec{A} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{bmatrix}$$

SIGN MATRIX

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

TO GET THE x-COMPONENT OF THE CURL, F_x:

SUPPRESS THE ROW & COLUMN CONTAINING $\vec{x}$, i.e. FIRST ROW & FIRST COLUMN

$$\begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{bmatrix} \rightarrow \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ A_y & A_z \end{vmatrix} = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$

$$F_x = \text{SIGN} \times \text{DETERMINANT}$$
$$= +\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)$$

TO GET F_y:

SUPPRESS THE ROW & COLUMN CONTAINING $\vec{y}$, i.e. FIRST ROW & SECOND COLUMN

$$\begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{bmatrix} \rightarrow \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ A_x & A_z \end{vmatrix} = \frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}$$

$$\begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{bmatrix} \rightarrow \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ A_x & A_z \end{vmatrix} = \frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

F_y = SIGN × DETERMINANT

$$= -\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)$$

TO GET F_z:

SUPPRESS THE FIRST ROW & THIRD COLUMN

$$\begin{bmatrix} x & y & z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{bmatrix} \longrightarrow \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ A_x & A_y \end{vmatrix} = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

$$F_z = +\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

INTERPRETATION OF CURL:

CONSIDER A VECTOR $\vec{A}$ THAT HAS ONLY x-COMPONENT

$$\vec{A} = A_x \vec{x} + 0\,\vec{y} + 0\,\vec{z}$$

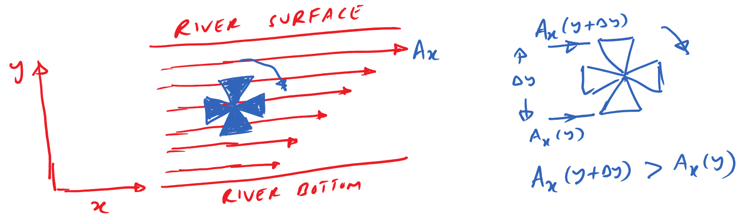
$$\vec{F} = \nabla \times \vec{A} = F_x \vec{x} + F_y \vec{y} + F_z \vec{z}$$

SUPPOSE THAT A_x IS A FUNCTION OF y ONLY.

LET US FOCUS ON THE z-COMPONENT OF THE CURL, F_z.

$$F_z = -\frac{\partial A_x}{\partial y} \quad (\because A_y = 0); \quad \vec{F} = F_z \vec{z}$$

CONSIDER THE WATER FLOW IN A RIVER. SUPPOSE THE SPEED OF WATER, A_x INCREASES AS WE GO FROM THE BOTTOM OF THE RIVER TO THE SURFACE, i.e. A_x IS CHANGING



AS A FUNCTION OF y. IF WE PLACE A PADDLE WHEEL WITH ITS AXIS PERPENDICULAR TO XY PLANE (z-AXIS), IT WILL TURN CLOCKWISE.

WE SAY THAT CURL EXISTS ALONG THE AXIS OF THE PADDLE (z-AXIS).

A LARGER SPEED OF THE PADDLE → LARGER VALUE OF CURL.

NOTE: CURL OF $\vec{A}$ IS A VECTOR $\vec{F}$. IT HAS BOTH MAGNITUDE & DIRECTION.

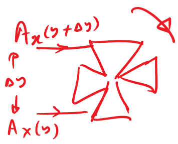
DIRECTION OF CURL → AXIS OF PADDLE WHEEL, i.e. z-AXIS

MAGNITUDE OF CURL $\longrightarrow$ SPEED OF PADDLE WHEEL

WHY?

$$\vec{F} = F_z \vec{z}$$

$$|\vec{F}| = |F_z| = \left|\frac{\partial A_x}{\partial y}\right|$$



FORCE ON THE UPPER PADDLE $\propto A_x(y+\Delta y)$

" " " LOWER " $\propto A_x(y)$

DIFFERENTIAL FORCE, $\Delta F \propto A_x(y+\Delta y) - A_x(y)$

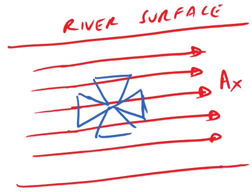
$$= \left(\frac{A_x(y+\Delta y) - A_x(y)}{\Delta y}\right)\Delta y$$

$$\simeq \frac{\partial A_x}{\partial y}\Delta y$$

ABS. SPEED OF PADDLE $\propto \left|\frac{\partial A_x}{\partial y}\right|$

SUPPOSE THE SPEED OF WATER IS THE SAME EVERYWHERE.

A_x IS CONSTANT.



IN THIS CASE, PADDLE WHEEL WILL NOT ROTATE SINCE THE FORCE ON THE UPPER PADDLE & LOWER PADDLE ARE EQUAL.

CURL OF $\vec{A} = \vec{F} = 0$