MAXWELL'S EQUATIONS:

$$\left.\begin{aligned} \mathrm{div}\,\vec{D} &= \rho \\ \mathrm{div}\,\vec{B} &= 0 \\ \nabla\times\vec{E} &= -\partial\vec{B}/\partial t \\ \nabla\times\vec{H} &= \vec{J} + \partial\vec{D}/\partial t \end{aligned}\right\} \quad (1)$$

ρ = CHARGE DENSITY

$\vec{J}$ = CURRENT DENSITY

MAXWELL EQUATIONS IN A SOURCE-FREE REGION:

IN FREE SPACE OR DIELECTRIC, WE CAN SET $\rho = 0$ & $\vec{J} = 0$, IF THERE ARE NO FREE CHARGES IN THE NEIGHBORHOOD.

NOTE: EQS. (1) ARE MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM OR POINT FORM, i.e. THEY ARE VALID AT A GIVEN SPACE-TIME POINT. IF THE SOURCES (ρ & $\vec{J}$) ARE ZERO IN A REGION (FOR EXAMPLE, IN VACUUM FAR AWAY FROM ANTENNA), EQS. (1) BECOME:

$$\left.\begin{aligned} \mathrm{DIV}\,\vec{D} &= 0 \\ \mathrm{DIV}\,\vec{B} &= 0 \\ \nabla\times\vec{E} &= -\partial\vec{B}/\partial t \\ \nabla\times\vec{H} &= \partial\vec{D}/\partial t \end{aligned}\right\} \quad (2)$$

ELECTROMAGNETIC WAVE:

LET $\vec{E} = E_x\vec{x}$ & $\vec{H} = H_y\vec{y}$, $\vec{B} = \mu\vec{H} = \mu H_y\vec{y}$

$$\nabla\times\vec{E} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{bmatrix} = \frac{\partial E_x}{\partial z}\vec{y} - \frac{\partial E_x}{\partial y}\vec{z} \rightarrow (3)$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu\frac{\partial H_y}{\partial t}\vec{y} \rightarrow (4)$$

RHS OF (4) HAS ONLY y-COMPONENT WHILE CURL OF $\vec{E}$

HAS BOTH y- & z- COMPONENTS (EQ.(2)). COMPARING Y-COMPONENTS

$$\frac{\partial E_x}{\partial z}\,\vec{y} = -\mu \frac{\partial H_y}{\partial t}\,\vec{y} \longrightarrow (5)$$

NOTE : x-COMPONENT OF ELECTRIC FIELD CHANGING AS A FUNCTION OF Z LEADS TO CURL IN y-DIRECTION (RECALL WATER FLOW EXAMPLE)

COMPARING Z-COMPONENTS :

$$-\frac{\partial E_x}{\partial y} = 0$$

THIS MEANS THAT IN THIS SIMPLE CASE, ELECTRIC FIELD DOES NOT CHANGE AS A FUNCTION OF y.

NEXT, CONSIDER

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}, \quad \vec{D} = \epsilon \vec{E}, \quad \vec{E} = E_x \vec{x}$$

$$\vec{D} = \epsilon E_x \vec{x} \Rightarrow \frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial E_x}{\partial t}\vec{x}$$

$$\nabla \times \vec{H} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H_y & 0 \end{bmatrix} = -\frac{\partial H_y}{\partial z}\vec{x} + \frac{\partial H_y}{\partial x}\vec{z} = \epsilon \frac{\partial E_x}{\partial t}\vec{x}$$

COMPARING x- COMPONENTS,

$$-\frac{\partial H_y}{\partial z} = \epsilon \frac{\partial E_x}{\partial t} \longrightarrow (6)$$

YOU CAN ALSO COMPARE Z-COMPONENTS, BUT WE DO NOT NEED IT FOR TODAY'S DISCUSSION.

NOTE ON THE CURL CALCULATION:

OPTION #1 : CALCULATE THE CURL USING THE DETERMINANT APPROACH (AS ABOVE)

OPTION #2: $\nabla \times \vec{H} = \vec{F} = F_x \vec{x} + F_y \vec{y} + F_z \vec{z}$

OPTION #2: $\nabla \times \vec{H} = \vec{F} = F_x \vec{x} + F_y \vec{y} + F_z \vec{z}$

$$= -\frac{\partial H_y}{\partial z}\vec{x} + 0\,\vec{y} + \frac{\partial H_y}{\partial x}\vec{z}$$

$$F_x = -\frac{\partial H_y}{\partial z}, \quad F_z = \frac{\partial H_y}{\partial x} \quad \rightarrow (7)$$

FROM THE WATER FLOW EXAMPLE, RECALL THAT Y-COMPONENT CHANGING AS A FUNCTION OF Z, GIVES TO X COMPONENT OF CURL. HOWEVER, SIGN OF THE CURL CANNOT BE DETERMINED FROM THE WATER-FLOW EXAMPLE. SO, USE THIS APPROACH:

$$\begin{array}{ccccccccc} & - & & - & & - & & - & \\ x & & y & & z & & x & & y \\ & + & & + & & + & & + & \end{array}$$

IF THE X-COMPONENT CHANGES AS A FUNCTION OF Y, SIGN OF CURL IS -VE AND IT IS IN Z-DIRECTION. EXAMPLE 1:

$$\text{Z-COMPONENT OF } \nabla \times \vec{E} = -\frac{\partial E_x}{\partial y} \qquad x \;\; y$$

(SEE EQ.(3))

EXAMPLE 2: IF THE Y-COMPONENT OF $\vec{H}$ CHANGES AS A FUNCTION OF Z, SIGN OF CURL IS -VE AND IT IS IN X-DIRECTION

$$F_x = -\frac{\partial H_y}{\partial z} \qquad y \;\; z$$

(SEE EQ.(7))

EXAMPLE 3: IF THE Y-COMPONENT OF $\vec{H}$ CHANGES AS A FUNCTION OF X, THE SIGN OF CURL IS +VE & IT IS IN Z-DIRECTION.

$$F = +\frac{\partial H_y}{\partial x} \qquad x \;\; y$$

$$F_z = +\frac{\partial H_y}{\partial x}$$

x y
+

WE HAVE COUPLED EQUATIONS (EQS. (5) & (6))

$$-\frac{\partial H_y}{\partial z} = \epsilon \frac{\partial E_x}{\partial t} \rightarrow (6)$$

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \rightarrow (5)$$

WE WOULD LIKE TO ELIMINATE ONE OF THEM (SAY H_y).

DIFFERENTIATE (6) w.r.t. t

$$-\frac{\partial^2 H_y}{\partial t \partial z} = \epsilon \frac{\partial^2 E_x}{\partial t^2} \rightarrow (8)$$

DIFFERENTIATE (5) w.r.t. z

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu \frac{\partial}{\partial z}\left(\frac{\partial H_y}{\partial t}\right) = -\mu \frac{\partial^2 H_y}{\partial t \partial z} \rightarrow (9) \quad \left(\because \frac{\partial^2 H_y}{\partial t \partial z} = \frac{\partial^2 H_y}{\partial z \partial t}\right)$$

MULTIPLY (8) BY μ & ELIMINATE $\partial^2 H_y / \partial t \partial z$, AND USE (9):

FROM (9)

$$-\mu \frac{\partial^2 H_y}{\partial t \partial z} = \epsilon\mu \frac{\partial^2 E_x}{\partial t^2} = \frac{\partial^2 E_x}{\partial z^2}$$

or

WAVE EQUATION

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}} \rightarrow (10)$$

WAVE EQUATION FORMS THE BASIS FOR THE STUDY OF LIGHT WAVE PROPAGATION.

HOMEWORK: ELIMINATE E_x IN (5) & (6) TO OBTAIN

$$\frac{\partial^2 H_y}{\partial z^2} = \mu\epsilon \frac{\partial^2 H_y}{\partial t^2}$$

FREE-SPACE PROPAGATION:

IN THE VACUUM, $\epsilon = \epsilon_0 = 8.854 \times 10^{-12}\ C^2/Nm^2$

$\mu = \mu_0 = 4\pi \times 10^{-7}\ N/A^2$

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} \cong 3 \times 10^8\ m/s$$

$$= c = \text{SPEED OF LIGHT!}$$

$$\therefore \boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} \longrightarrow (11)$$

BEFORE MAXWELL'S TIME, ELECTROSTATICS, MAGNETOSTATICS & OPTICS WERE DIFFERENT BRANCHES OF PHYSICS WITH NO CONNECTIONS. MAXWELL COMBINED THESE FIELDS INTO A SINGLE FIELD, NOW KNOWN AS ELECTROMAGNETICS. THE EM WAVE PROPAGATES AT THE SPEED OF LIGHT & LIGHTWAVE IS AN EXAMPLE OF EM WAVE.

PROPAGATION IN A DIELECTRIC MEDIUM:

SPEED OF LIGHT IN THE VACUUM:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

IN A DIELECTRIC MEDIUM, LET

$$v = \frac{1}{\sqrt{\mu \epsilon}}, \longrightarrow (12)$$

LATER IT WILL BE SHOWN THAT v IS THE SPEED OF LIGHT IN THE MEDIUM.

$\mu = \mu_0 \mu_r$, μ_r = RELATIVE PERMEABILITY

$\epsilon = \epsilon_0 \epsilon_r$, ϵ_r = " PERMITTIVITY

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \underbrace{\frac{1}{\sqrt{\mu_0 \epsilon_0}}}_{c} \frac{1}{\sqrt{\mu_r \epsilon_r}}$$

$$= \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

DIELECTRIC MEDIUM IS A NON-MAGNETIC MEDIUM, i.e. $\mu_r = 1$

$$v = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{n} \longrightarrow (13)$$

WHERE $\boxed{n = \sqrt{\epsilon_r}}$ IS THE REFRACTIVE INDEX OF THE MEDIUM.

FOR ANY MEDIUM OTHER THAN THE VACUUM, $n > 1$.

$$\therefore \quad v < c$$

1-D WAVE EQUATION:

$$\frac{\partial^2 E_x}{\partial z^2} = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2}, \qquad v^2 = \frac{1}{\mu \epsilon}$$

$$\therefore \quad \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2} \longrightarrow (14)$$

TRY A TRIAL SOLUTION:

$$E_x = f(t + \alpha z)$$

$f(\,) \rightarrow$ ARBITRARY FUNCTION TO BE DETERMINED

α = CONST. TO BE DETERMINED

LET

$$u = t + \alpha z$$

$$\frac{\partial u}{\partial z} = \alpha, \quad \frac{\partial u}{\partial t} = 1$$

$$\frac{\partial E_x}{\partial z} = \frac{\partial E_x}{\partial u}\cdot\frac{\partial u}{\partial z} = \alpha\frac{\partial E_x}{\partial u}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial z}\right) = \alpha\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial u}\right) = \alpha\frac{\partial}{\partial u}\left(\frac{\partial E_x}{\partial u}\right)\cdot\frac{\partial u}{\partial z} = \alpha^2\frac{\partial^2 E_x}{\partial u^2}$$

$$\frac{\partial E_x}{\partial t} = \frac{\partial E_x}{\partial u}\cdot\frac{\partial u}{\partial t} = 1\cdot\frac{\partial E_x}{\partial u}$$

$$\frac{\partial^2 E_x}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial E_x}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial E_x}{\partial u}\right) = \frac{\partial}{\partial u}\left(\frac{\partial E_x}{\partial u}\right)\frac{\partial u}{\partial t} = \frac{\partial^2 E_x}{\partial u^2}$$

$$\left.\begin{aligned} \text{LHS OF (14)} &= \frac{\partial^2 E_x}{\partial z^2} = \alpha^2\frac{\partial^2 E_x}{\partial u^2} \\ \text{RHS OF (14)} &= \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial u^2} \end{aligned}\right\} \rightarrow \alpha^2 = \frac{1}{v^2}$$

$$\alpha = \pm 1/v$$

WITH + SIGN:

$$E_x = f(t + \alpha z) = f(t + z/v)$$

BACKWARD-PROPAGATING WAVE

WITH -VE SIGN

$$E_x = f(t - z/v)$$

FORWARD-PROPAGATING WAVE

OUR GOAL WAS TO DETERMINE BOTH α & $f(\,)$. BUT WE MANAGED TO FIND ONLY α. THIS MEANS THAT $f(\,)$ IS AN ARBITRARY FUNCTION DETERMINED BY THE BOUNDARY CONDITIONS.