WAVE EQUATION:

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2}$$

v = SPEED OF LIGHT IN THE MEDIUM

IN VACUUM, $v = c = 3\times10^8$ m/s

$$E_x = f(t - z/v) \quad \text{or} \quad f(t + z/v)$$

f IS AN ARBITRARY FUNCTION TO BE DETERMINED BY THE BOUNDARY CONDITIONS.

FORWARD PROPAGATING WAVE

BACKWARD PROP. WAVE.

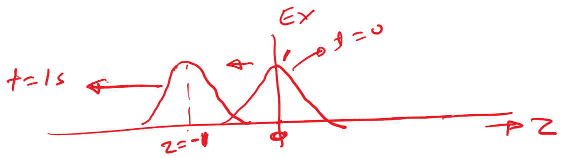
EXAMPLE:

LET $v = 1$ m/s

$$E_x = f(t+z)$$

LET $f(x) = e^{-x^2}$

AT $t=0$, $E_x = f(z) = e^{-z^2}$

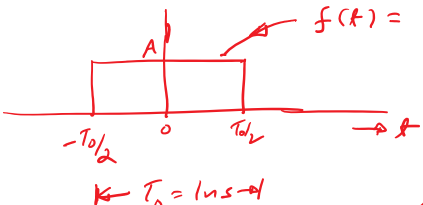


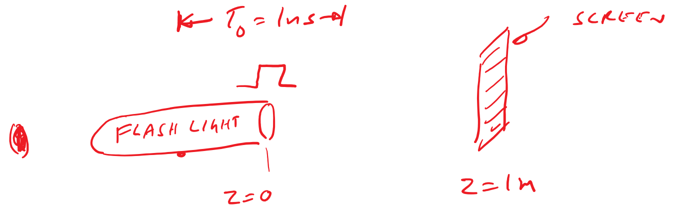
AT $t = 1$s, $E_x = f(1+z) = e^{-(1+z)^2}$

PULSE HAS MOVED TO THE LEFT ⇒ BACKWARD PROP. WAVE

EXAMPLE 1: A FLASH LIGHT IS TURNED ON FOR 1ns & THEN TURNED OFF, GENERATING A PULSE SHOWN BELOW

$f(t) = A\, \mathrm{rect}(t/T_0)$





$$E_x(t,0) = f(t) = A\,\mathrm{rect}(t/T_0)$$

$$\mathrm{rect}(x) = 1 \text{ if } |x| < 1/2$$
$$= 0 \quad \text{OTHERWISE}$$

NOTE: ACTUAL ELECTRIC FIELD HAS FAST OSCILLATIONS AT OPTICAL FREQUENCY WHICH IS IGNORED HERE.

SKETCH THE FIELD AT THE SCREEN.

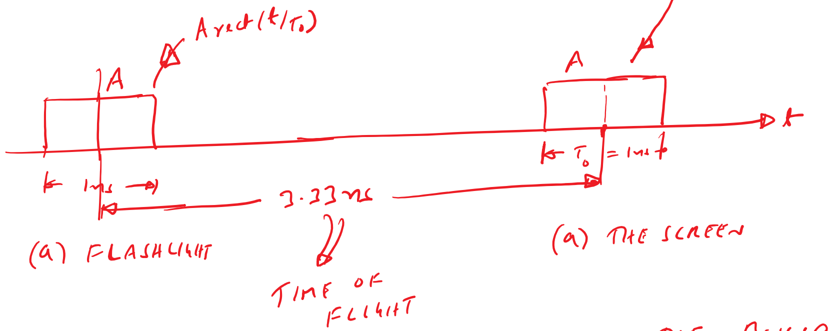
SOLUTION:

$$E_x(t,0) = f(t) = A\,\mathrm{rect}(t/T_0)$$
$$E_x(t,z) = f(t - z/v) = A\,\mathrm{rect}\left(\frac{t - z/v}{T_0}\right)$$

IN FREE SPACE, $v = c = 3\times10^8\,\mathrm{m/s}$

AT THE SCREEN, $z = 1\,\mathrm{m}$, $z/v = \dfrac{1\,\mathrm{m}}{3\times10^8\,\mathrm{m/s}} = 3.33\,\mathrm{ns}$ $\Rightarrow$ DECAY DUE TO PROPAGATION

$$E_x(t, z=1\mathrm{m}) = A\,\mathrm{rect}\left(\frac{t - 3.33\,\mathrm{ns}}{1\,\mathrm{ns}}\right)$$



OBSERVATIONS: (i) f IS DETERMINED BY THE BOUNDARY CONDITION

(ii) THE PULSE SHAPE DOES NOT CHANGE DUE TO PROPAGATION. $\Rightarrow$ IT IS SIMPLY DELAYED BY 3.33 ns

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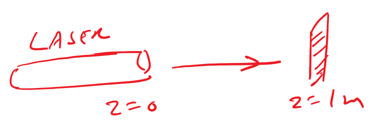
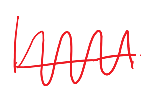
EX.2: REPEAT EX.1 BY REPLACING THE FLASHLIGHT BY A CW LASER OF FREQ. 190 THz

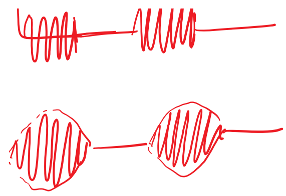


CW LASER $\Rightarrow$ CONTINUOUS WAVE LASER $\Rightarrow$



PULSED LASER $\Rightarrow$

PULSED LASER $\Rightarrow$



LASER OUTPUT IS

$$E_x(t,0) = A\cos(2\pi f_0 t) = f(t)$$

f_0 = LASER FREQ = 190 THz

$$E_x(t,z) = f(t - z/v)$$

$$= A\cos\left(2\pi f_0 (t - z/v)\right) \longrightarrow (*)$$

At $z = 1\,m$, $z/v = 3.33\,ns$

DELAY

$$E_x(t, z = 1m) = A\cos\left[2\pi f_0 (t - 3.33\,ns)\right]$$

$$= A\cos\left[2\pi f_0 t - 2\pi f_0 \times 3.33\,ns\right]$$

$$= A\cos\left[2\pi f_0 t - \theta\right]$$

PHASE-SHIFT DUE TO PROPAGATION

$$\theta = 2\pi f_0 z/v$$

1-D PLANE WAVE.

$$E_x(t,z) = A\cos\left[2\pi f (t - z/v)\right]$$

f = FREQ. OF THE WAVE

v = SPEED OF THE WAVE

$$f\lambda = v \Rightarrow f/v = 1/\lambda$$

WAVELENGTH

$$E_x(t,z) = A\cos\left[2\pi f t - 2\pi f/v \cdot z\right]$$

$$= A\cos\left[2\pi f t - \frac{2\pi}{\lambda} \cdot z\right] \longrightarrow (2)$$

$$\frac{2\pi}{\lambda} = k = \text{WAVENUMBER}$$

$$2\pi f = \text{ANGULAR FREQ.}$$

$$E_x(t,z) = A\cos[\omega t - kz] \longrightarrow (3)$$

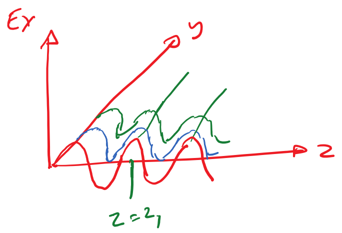
WHY IS IT CALLED A PLANE WAVE?

E_x DOES NOT DEPEND ON x & y

AT $t=t_1$, $z=z_1$, $x=0$ & $y=0$ $\Rightarrow$ $E_x = A\cos(\omega t_1 - kz_1)$

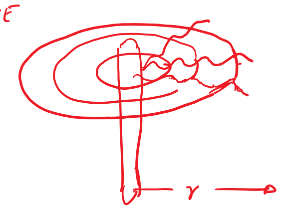
AT $t=t_1$ $z=z_1$, $x=5$ & $y=2$ $\Rightarrow$ $E_x = A\cos(\omega t_1 - kz_1)$

AT $t=t_1$:

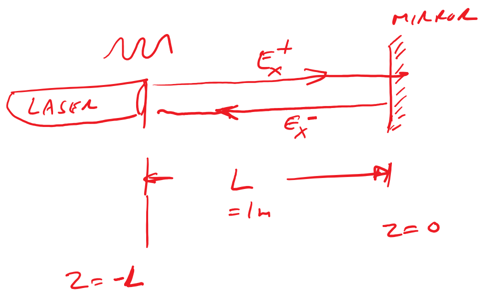


THE FIELD IS THE SAME ON A XY PLANE AT A GIVEN z_1 & t_1.

CYLINDRICAL WAVE



EX. 3:



FORWARD PROP. WAVE, $E_x^+ = A^+\cos[2\pi f_0(t - z/v)] \longrightarrow (4)$

BACKWARD " ", $E_x^- = A^-\cos[2\pi f_0(t + z/v)] \longrightarrow (5)$

TOTAL FIELD, $E_x^{TOT} = E_x^+ + E_x^-$

~~PET~~ MIRROR IS A CONDUCTOR.

IN A PERFECT CONDUCTOR, WHAT IS THE ELECTRIC FIELD? $= 0$

$$E_x^{TOT} = 0 \quad \text{AT } z = 0$$

$$E_x^+ + E_x^- = 0 \quad \text{AT } z = 0$$

$$A^+ \cos(2\pi f_0 t) + A^- \cos(2\pi f_0 t) = 0$$

$$\Rightarrow \quad A^+ + A^- = 0$$

$$A^- = -A^+$$

$$E_x = E_x^{tot} = A^+ \cos\left[2\pi f_0\left(t - z/v\right)\right] - A^+ \cos\left[2\pi f_0 (t + z/v)\right]$$

$$= A^+ \cos\left[\underset{A}{2\pi f_0 t} - \underset{B}{2\pi f_0 z/v}\right] - A^+ \cos\left[\underset{A}{2\pi f_0 t} + \underset{B}{2\pi f_0 z/v}\right]$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A + B) = \text{''} \quad \text{''} \quad - \quad \text{''} \quad \text{''}$$

$$E_x = A^+\left[\cancel{\cos(2\pi f_0 t)\cdot\cos(2\pi f_0 z/v)} + \sin(2\pi f_0 t)\sin(2\pi f_0 z/v)\right]$$

$$-A^+\left[\cancel{\cos(2\pi f_0 t)\cos(2\pi f_0 z/v)} - \sin(2\pi f_0 t)\cdot\sin(2\pi f_0 z/v)\right]$$

$$E_x = 2A^+ \sin(2\pi f_0 t)\sin(2\pi f_0 z/v)$$

$$f_0 \lambda = v$$

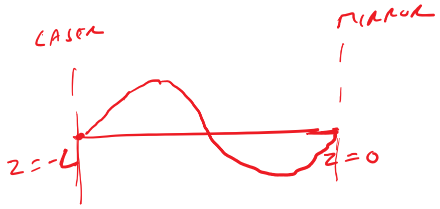
$$f/v = 1/\lambda$$

$$E_x = \underbrace{2A^+ \sin(2\pi f_0 t)}_{\phi} \sin(2\pi z/\lambda)$$

$$= B(t)\cdot\sin(2\pi z/\lambda)$$

$$B(t) = 2A^{+} \sin(2\pi f_0 t) \Rightarrow$$ TIME DEPENDENT AMPLITUDE

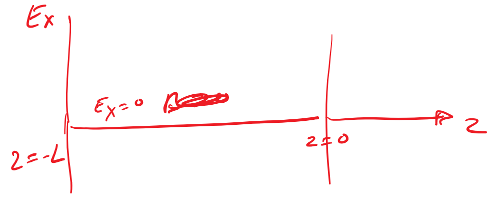
CASE (i): $L = \lambda$



At $z=0$,
$\sin(2\pi z/\lambda) = 0$

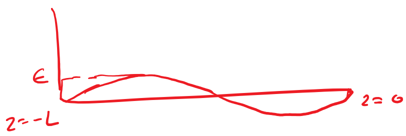
At $z = -L$
$\sin\left(\frac{2\pi(-L)}{L}\right) = 0$

At $t=0$, $B(t) = 2A^{+}\sin(0) = 0$, $B = 0$



At $t = \Delta t$ $B(t) = 2A^{+} \sin(2\pi f_0 \Delta t) = \epsilon$ ← NOTHING TO DO WITH PERMITTIVITY.

$$E_x = \epsilon \cdot \sin(2\pi z/\lambda)$$



At $2\pi f_0 t = \pi/2$ $t = \frac{\pi}{4\pi f_0}$

$$B(t) = 2A^{+} \sin(\pi/2) = 2A^{+}$$

$$E_x = 2A^{+} \cdot \sin(2\pi z/\lambda)$$

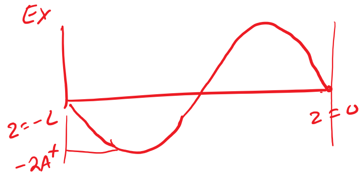
At $2\pi f_0 t = \pi$, $B(t) = 2A^{+} \sin(\pi) = 0$

$$E_x = 0$$

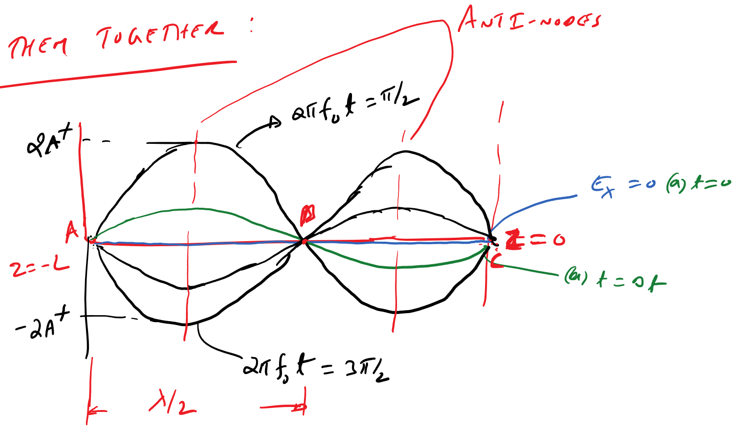
At $2\pi f_0 t = 3\pi/2$

$B(t) = 2A^+ \cdot \sin(3\pi/2) = -2A^+$

$E_x = -2A^+ \sin(2\pi z/\lambda)$



PUTTING THEM TOGETHER :



STANDING WAVES

WAVE 1 $\longrightarrow$ MOVING AT SPEED v

WAVE 2 $\longrightarrow$ " " " $-v$

NET SPEED $= 0$

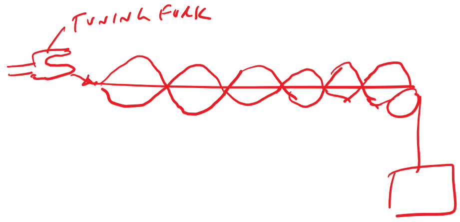
(i) AT SOME POINTS, THE FIELD IS ALWAYS ZERO.

THEY ARE KNOWN AS NODES

DISTANCE BETWEEN NODES $= \lambda/2$

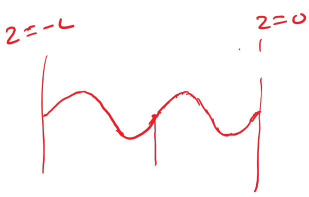
(ii) AT CERTAIN POINTS, FIELD VARIES FROM $-2A^+$ to $2A^+$.

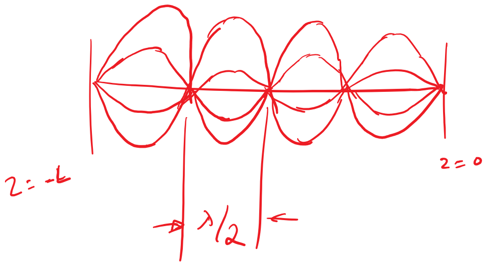
THESE ARE KNOWN AS ANTI-NODES.



CASE (ii) $\lambda = L/2$

$\sin\left(\frac{2\pi z}{L/2}\right) \longrightarrow$

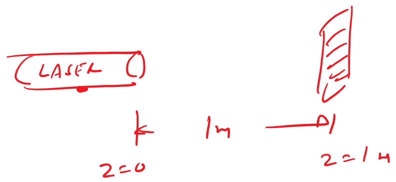


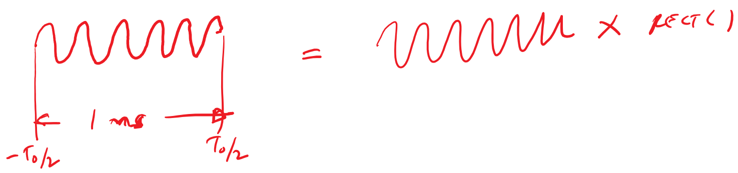


HOMEWORK:

A LASER IS TURNED ON FOR 1ns & THEN TURNED OFF

SKETCH THE ELECTRIC FIELD AT THE SCREEN.





$E_x(t,0) = \cos(2\pi f_0 t)\ \text{rect}(t/T_0)$