FARADAY'S LAW:

$$\text{CURL } \vec{E} = \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

LET $\vec{E} = E_x \vec{x} + 0\vec{y} + 0\vec{z}$

$$\vec{F} = \text{CURL } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{F} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

F_x : $\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ 0 & 0 \end{vmatrix}$ $\qquad F_x = 0$

F_y : $\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ E_x & 0 \end{vmatrix}$ $\qquad F_y = -(0 - \partial E_x/\partial z)$

$$= \frac{\partial E_x}{\partial z}$$

F_z : $\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ E_x & 0 \end{vmatrix}$ $\qquad F_z = +\left(-\frac{\partial E_x}{\partial y}\right)$

$$\vec{F} = -\frac{\partial \vec{B}}{\partial t}$$

LET $\vec{B} = B_x \vec{x} + B_y \vec{y} + B_z \vec{z}$

$$\frac{\partial \vec{B}}{\partial t} = \frac{\partial B_x}{\partial t}\vec{x} + \frac{\partial B_y}{\partial t}\vec{y} + \frac{\partial B_z}{\partial t}\vec{z}$$

$$\vec{F} = F_x \vec{x} + F_y \vec{y} + F_z \vec{z}$$

x-COMPONENT $\qquad F_x = 0 = -\frac{\partial B_x}{\partial t}$

y-COMPONENT $\qquad F_y = \frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$

z-COMPONENT $\qquad F_z = -\frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t}$

FOCUS ON THE z-COMPONENT

$$\boxed{F_z = \frac{\partial E_x}{\partial y} = \frac{\partial B_z}{\partial t}}$$

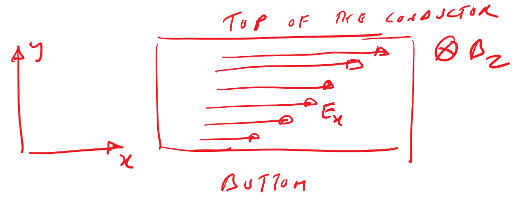
TIME VARYING MAGNETIC FIELD PRODUCES y-VARYING ELECTRIC FIELD.

IF B_z IS NOT TIME-VARYING, SAY $B_z = K$

$\frac{\partial B_z}{\partial t} = 0 \Rightarrow$ ELECTRIC FIELD DOES NOT CHANGE AS A FUNCTION OF y

$\frac{\partial E_x}{\partial y} = 0$ $\quad E_x = A\cos(\alpha x + \beta z)$

SUPPOSE THAT YOU ARE DOING AN EXPERIMENT IN THE LAB & OBSERVE THAT THE ELECTRIC FIELD IN A CONDUCTOR IS GRADUALLY INCREASING FROM THE BOTTOM OF THE CONDUCTOR TO THE TOP



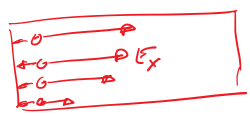
WHAT WOULD YOU INFER?

RECALL THE WATER FLOW EXAMPLE:

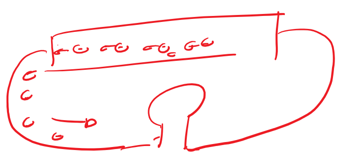
$\frac{\partial A_x}{\partial y} \neq 0 \Rightarrow$ CURL IN z-DIRECTION

$\frac{\partial E_x}{\partial y} \neq 0 \Rightarrow$ CURL IN z-DIRECTION

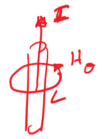
$$\frac{\partial E_x}{\partial y} = \frac{\partial B_z}{\partial t}$$



$F_x = q E_x$



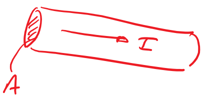
AMPERE'S LAW:



$$\oint_L \vec{H} \cdot d\vec{l} = I$$

$$I = \iint_S \vec{J} \cdot d\vec{S}$$

$\vec{J}$ = CURRENT DENSITY,



$$\text{CURRENT DENSITY} = \frac{\text{CURRENT}}{\text{AREA} \perp^r \text{ TO THE CURRENT FLOW}} = \frac{I}{A}$$

USING STOKES'S THEOREM,

CURL OF $\vec{H} = \nabla \times \vec{H} = \vec{J}$: AMPERE'S LAW DIFFERENTIAL FORM

$\text{DIV } D = \rho$ GAUSS'S LAW: ELECTRIC CHARGE

$\text{DIV } B = 0$ " " : MAGNETIC CHARGE

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ FARADAY'S LAW

$\nabla \times \vec{H} = \vec{J}$ AMPERE'S LAW

THESE LAWS WERE KNOWN PRIOR TO MAXWELL'S TIME.

MAXWELL'S CONTRIBUTIONS:

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$D = \epsilon E$ = ELECT. FLUX DENSITY.

$\frac{\partial \vec{D}}{\partial t} \Rightarrow$ DISPLACEMENT CURRENT DENSITY

$J \Rightarrow$ CONDUCTION " "

IF $J = 0$; $\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$

$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ → FARADAY'S LAW

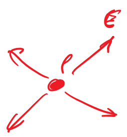
$\text{DIV } D = \rho$ → (1)

$$\text{DIV } B = 0 \rightarrow (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (3)$$

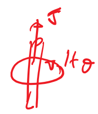
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow (4)$$

INTERPRETATION: Eq. (1): ρ PRODUCES ELECTRIC FIELD.



Eq. (4): TWO SOURCE TERM

(i) $\vec{J}$ & (ii) $\partial \vec{D}/\partial t$



PRACTICAL EXAMPLE:

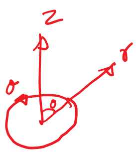
TX ANTENNA

$I = I_0 \cos(2\pi f_0 t)$

$$J = \frac{I}{A}$$

$$\left(\frac{I}{A}\right) = \left(\frac{I_0}{A}\right)\cos(2\pi f_0 t)$$

$$J = J_0 \cos(2\pi f_0 t)$$



(r, θ, z)

$$H_\theta = \frac{I}{2\pi r} = \frac{I_0 \cos(2\pi f_0 t)}{2\pi r}$$

NOTE: H_θ IS TIME-VARYING & IT IS ALSO SPACE-VARYING

$$H_\theta \propto \frac{1}{r}$$

AMPERE'S LAW: $\nabla \times \vec{H} = \vec{J}$

MAXWELL'S EQ: $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

IN A CONDUCTOR, $\frac{\partial \vec{D}}{\partial t} << J$

$$\nabla \times \vec{H} \cong \vec{J}$$

FARADAY'S LAW: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$

TIME-VARYING MAG. FIELD $\Rightarrow$ SPACE-VARYING ELECT. FIELD

$$\vec{H} = H_\theta \hat{\theta}$$

WHAT IS THE DIRECTION OF $\vec{E}$?

SUPPOSE $\vec{E} = E_z \vec{z}$

IF E_z IS CHANGING AS A FUNCTION OF r

$\frac{\partial E_z}{\partial r} \neq 0 \Rightarrow$ CURL OF $\vec{E}$ IS IN THE DIRECTION OF θ

$(\nabla \times E)_\theta$ = θ-COMPONENT OF $\nabla \times E$

THE RHS $= \mu \frac{\partial H_\theta}{\partial t} \cdot \vec{\theta}$ ✓

SUPPOSE $\vec{E} = E_r \vec{z}$ (HOMEWORK: SHOW THAT $\vec{H}$ HAS A COMPONENT IN θ-DIRECTION)

SUPPOSE $\vec{E} = E_\theta \vec{\theta}$,

CAN $\vec{H} = H_\theta \vec{\theta}$?

IF E_θ IS CHANGING AS A FUNCTION OF r

$\Rightarrow \frac{\partial E_\theta}{\partial r} \neq 0 \Rightarrow$ CURL IS IN z-DIRECTION

$\Rightarrow$ MAGNETIC FIELD H SHOULD IN z-DIRECTION

BUT WE KNOW THAT H IS IN θ-DIRECTION

IF E_θ IS CHANGING AS A FUNCTION OF z

$\frac{\partial E_\theta}{\partial z} \neq 0 \Rightarrow$ CURL IS IN THE DIRECTION OF r

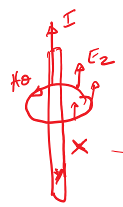
$\Rightarrow$ H IS IN r-DIRECTION ✗

$\vec{E}$ HAS NO COMPONENT IN θ-DIRECTION.

$\vec{E} = E_r \vec{r} + E_z \vec{z} + 0\,\vec{\theta}$

TODAY, I WILL FOCUS ONLY ON z-COMPONENT OF $\vec{E}$

$\vec{E} = E_z \vec{z}$



TIME-VARYING MAGNETIC FIELD H_θ PRODUCES SPACE-VARYING E_z

...SIDE THE TX. ANTENNA

CONSIDER A POINT X JUST OUTSIDE THE TX. ANTENNA

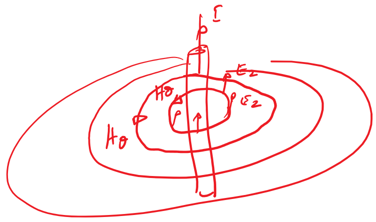
WHAT IS J AT X? $J = 0$ AT X

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

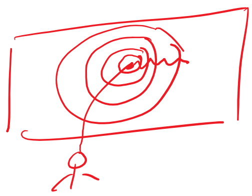
AT X: $J = 0 \Rightarrow \underline{\nabla \times H} = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t}$

AT Y: $J \neq 0$, $\nabla \times H \cong J$

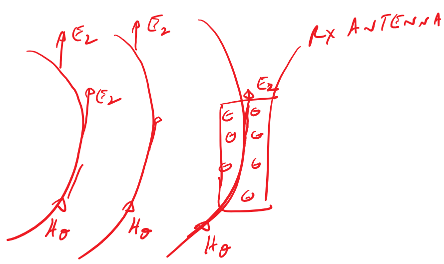
TIME CHANGING ELECTRIC FIELD PRODUCES SPACE-VARYING MAGNETIC FIELD.



EM WAVE IS SIMILAR TO THE WATER WAVE



SUPPOSE THE EM WAVE GENERATED AT TX PROPAGATES HUNDREDS OF KMS & ARRIVES AT THE RX ANTENNA



$E_z \propto \sin(2\pi f_0 t)$

$\underline{F_z = q E_z}$ $\Rightarrow$ ELECTRONS MOVE UP & DOWN

$\Rightarrow$ AC CURRENT

IF THE TX ANTENNA CURRENT $\rightarrow$ KA

RX " " $\rightarrow$ MA

Rx

QUESTIONS : (I) IF THE TX-ANTENNA CURRENT IS DC, CAN WE GET EM WAVE? NO

(II) IF MAXWELL HAD NOT INTRODUCED DISPLACEMENT CURRENT ~~DENS~~ DENSITY, WOULD WE HAVE EM WAVE?